TIME PAYMENT

One application of geometric progressions is the calculation of time payments.

Example:

Shafia borrowed \$250 000 from the bank to buy a house. She agreed to pay back the loan over 25 years at an interest rate of 6% per annum, reducible monthly. Calculate Shafia's monthly repayments.

Let the monthly repayments be R and the amount owing after n months be A_n

The rate of interest r is 6% p.a. = 0.5% per month.

At the end of the first month, after paying her first installment, Shafia owes

$A_1 = 250\,000 \times 1.005 - R$

At the end of the second month she owes $A_2 = A_1 \times 1.005 - R$

$A_2 = (250\,000 \times 1.005 - R) \times 1.005 - R$

$= 250\,000 \times 1.005^2 - 1.005R - R$

At the end of the third month she owes $A_3 = A_2 \times 1.005 - R$

$= (250\,000 \times 1.005^2 - 1.005R - R) \times 1.005 - R$

$= 250\,000 \times 1.005^3 - 1.005^2R - 1.005R - R$

By continuing the pattern for n months

$A_n = 250\,000 \times 1.005^n - 1.005^{(n-1)}R - 1.005^{(n-2)}R \ldots\ldots\ldots\ldots - R$

$A_n = 250\,000 \times 1.005^n - R(1 + 1.005 + 1.005^2 + \ldots\ldots\ldots\ldots + 1.005^{(n-1)})$

When the loan has been fully repaid n = 25 x 12 = 300 and $A_{300} = 0$.

$0 = 250\,000 \times 1.005^{300} - R(1 + 1.005 + 1.005^2 + 1.005^3 + \ldots\ldots\ldots + 1.005^{299})$

The part inside the brackets is a geometric series with a = 1 and r = 1.005

$$0 = 1116242.453 - R\,\frac{1(1.005^{300} - 1)}{0.005}$$

$0 = 1116242.453 - R \times 692.9939624$

$$R = \frac{1116242.453}{692.9939624} = 1610.75$$

Repayments = \$1610.75 per month.

Exercise.

Q.1. James borrowed \$10 000 to buy a car. The interest rate was 12% per annum, reducible monthly and the loan was to be repaid over 3 years. What were the monthly repayments?

Q.2. Stephanie borrowed \$50 000 to start a business. She didn't make any repayments for the first year and then repaid the loan in monthly installments over the next 5 years. If the interest rate was 9%, compounded monthly, what were Stephanie's monthly repayments?

Q.3. Mary and Norm borrowed \$100 000 over 15 years to buy a house. The interest rate was 8% per annum, compounded annually. What were their monthly repayments?

Q.4. Mac and Sue wanted to borrow \$200 000 to buy a house and pay it back over 20 years. One lender offered them a loan at 8½ % per annum, compounded annually and another lender offered them a loan at 9% per annum, compounded monthly.
Calculate the monthly repayments on each loan.

Answers: Q.1. \$332.14 Q.2. \$1135.28 Q.3. \$973.58
Q.4. 8 ½ % p.a. = \$1761.18 p.m. 9% p.a. = \$1799.45 p.m.

(ii) How much of the principal has Nathan paid back after 10 years? (2 mks)

$A_{10} = 200\,000 \times 1.005^{120} - R(1 + 1.005 + 1.005^2 + \dots\dots\dots + 1.005^{119})$

$A_{10} = 363879.3468 - 1433 \dfrac{1(1.005^{120} - 1)}{0.005}$

$A_{10} = 363879.3468 - 234839.104 = 129040.24$

Principal paid back = 200 000 – 129 040.24
= \$70960 (nearest \$1 based on repayments of \$1433.00

(iii) If Nathan increases his repayments to \$1500 per month, how long will it take him to pay off the loan? (2 mks)

220 months, = 18 years & 4 months

$0 = 200\,000 \times 1.005^n - 1500(1 + 1.005 + 1.005^2 + \dots\dots\dots + 1.005^{(n-1)})$

Dividing by 1500

$0 = 133.333333 \times 1.005^n - \dfrac{1(1.005^n - 1)}{.005}$

Multiplying by 0.005

$0.6666666667 \times 1.005^n - 1.005^n + 1 = 0$

$0.3333333333 \times 1.005^n = 1$

Multiplying by 3

$1.005^n = 3$

Taking logs of both sides

$n \log 1.005 = \log 3$

$2.166 \times 10^{-3}\, n = 0.47712$

$n = \dfrac{0.47712}{2.166 x 10^{-3}} = 220.28$

= 220 months (to nearest month) = 18 years 4 months.