Sydney Girls High School

2011

TRIAL HIGHER SCHOOL

CERTIFICATE EXAMINATION

Mathematics Extension One

General Instructions

- Reading time: 5 minutes
- Working time: 2 hours
- Attempt all questions
- All questions are of equal value.
- All necessary working should be shown in every question.
- Marks may be deducted for careless or badly arranged work.
- Standard integrals are supplied
- Board-approved calculators may be used.
- Diagrams are not to scale
- Each question attempted should be started on a new sheet. Write on one side of the paper only.

This is a trial paper ONLY. It does not necessarily reflect the format or the contents of the 2011 HSC Examination Paper in this subject.

Name.____________________

Teacher.____________________

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

Question One (12 marks) **Marks**

a) Find $\int \frac{dx}{9+x^2}$ [1]

b) Given $f(x)=\frac{1}{2}\sin^{-1}2x$:

i) State the range of $f(x)$ [1]

ii) State the domain of $f(x)$ [1]

iii) Sketch $f(x)$ [1]

c) Solve $\frac{5}{x-4}<1$ [2]

d) Evaluate $\int_0^1 x\sqrt{1-x^2}\,dx$ using the substitution $u=1-x^2$ [3]

e) i) Show that there is a solution to $x^3=x+1$ between $x=1$ and $x=2$ [1]

ii) Use one application of Newton's method and $x=1.5$ to find a further approximation correct to one decimal place. [2]

Question Two (12 marks) **Marks**

a) The point $P(2,3)$ divides the interval AB internally in the ratio $2:3$. If A has coordinates $(-1,6)$ find the coordinates of B. [2]

b) A function is defined $f(x)=\frac{3}{x}-4$.

i) Find f^{-1} [1]

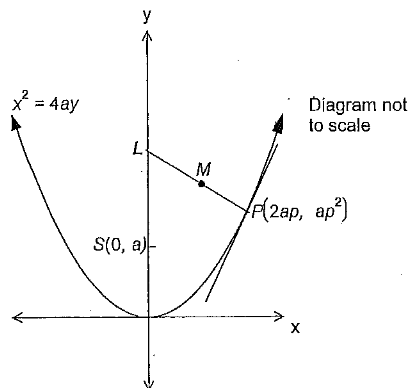
ii) Evaluate $f^{-1}(4)$ [1]

c) Given $P(x)=2x^3-17x^2+7x+8$:

i) Show that $(x-1)$ is a factor of $P(x)$ [1]

ii) Hence fully factorise $P(x)$ [2]

d) The diagram shows the parabola $x^2=4ay$. The point $P(2ap,ap^2)$ where $p\neq 0$ lies on the parabola. The normal at P cuts the y-axis at L. M is the midpoint of LP.



i) Show that the equation of the normal to the parabola at P is $x+py=ap^3+2ap$. [2]

ii) Find the coordinates of L, the point where the normal cuts the y-axis. [1]

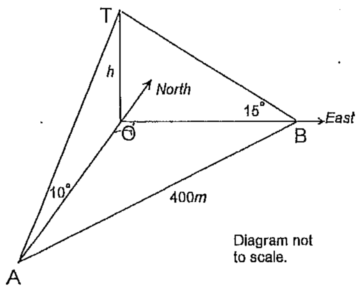
iii) Show that SM is parallel to the tangent at P. [2]

Question Three (12 marks) **Marks**

a) Find the size of the acute angle between the lines whose equations are [3]

$x-2y-1=0$ and $x+3y+2=0$

b) A tower TO is due north of an observer at A. The angle of elevation from A to the top of the top of the tower T is 10°. From a point B due east of the tower, the angle of elevation to the top of the tower is 15°. The distance from A to B is 400m



i) Find an expression for AO in terms of h. [1]
ii) Calculate the height h of the tower. [3]
iii) Find the bearing of A from B [2]

c) The polynomial $P(x)$ is defined as $P(x)=x^3+ax^2+2ax+b$ where a and b are constants. The zeros of $P(x)$ are $2,-3$ and γ. Find the values a,b and γ [3]

Question Four (12 marks) **Marks**

a) Find $\int 2\cos^2 4x\,dx$ [2]

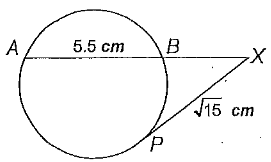
b) The velocity of a particle is given by $\dot{x}=2-3e^{-t}$ where x is the displacement in metres and t is the time in seconds. Initially the particle is at the origin.

i) Find an expression for the acceleration $\ddot{x}$ of the particle at any time t. [1]
ii) Find an expression for the displacement x of the particle at any time t. [2]
iii) Find the time when the particle is next at rest (give exact answer). [2]
iv) Explain what happens to the acceleration and hence the velocity as t becomes very large. [2]

c) Prove by mathematical induction that $2\times5^{n-1}+12^n$ is divisible by 7 for all integers $n\geq1$ [3]

Question Five (12 marks) **Marks**

a) In the diagram below the tangent at P meets AB at X.



If $AB = 5.5cm$ and $PX = \sqrt{15}cm$ find the length of BX. [2]

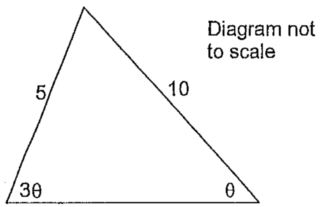
b) Evaluate $\lim_{x\to\infty} \dfrac{2x^2-3x+1}{5x^2+x-4}$ [1]

c) i) Write $\sqrt{12}\sin x + 2\cos x$ in the form $r\sin(x+\alpha)$ [2]

ii) Hence or otherwise solve $\sqrt{12}\sin x + 2\cos x = -2\sqrt{2}$ for $0 \le x \le 2\pi$ [3]

d) i) Prove $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$ [2]

ii)



Hence find the value of θ in the triangle [2]

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Question Six (12 marks) **Marks**

a) If the displacement of a particle is given by $x = 2\sin 2t + 3\cos 2t$, show that the motion of the particle is simple harmonic. [2]

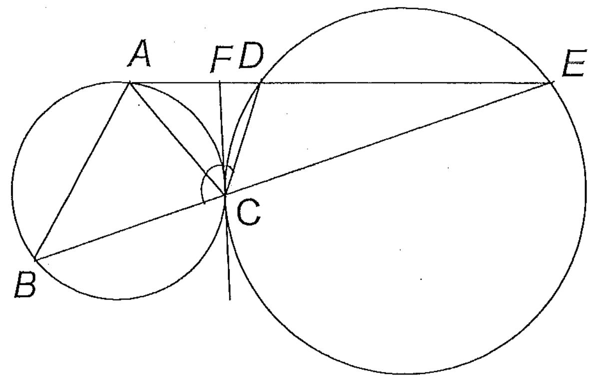
b) Jane is inflating balloons for the Year 12 Formal. Each empty balloon is being inflated so that its volume increases at the rate of $8cm^3/s$.

i) Show that the radius at any time t is $r = \sqrt[3]{\dfrac{6t}{\pi}}$ [2]

ii) Find the rate of increase of the surface area after 4 seconds [2]

iii) The balloon will burst when the surface area reaches $3000cm^2$. After how many seconds should Jane cease inflation? [3] 2.

c) Two circles touch each other externally at C. The tangent to the smaller circle at A meets the larger circle at D and E. EC meets the smaller circle at B. FC is the common tangent to both circles. Copy or trace the diagram.



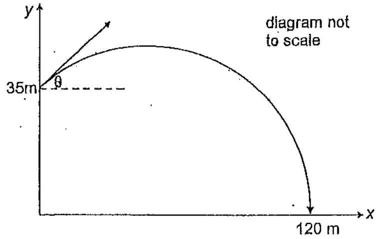
i) Prove $\angle FAC = \angle FCA$ [2]

ii) Prove $\angle ACD = \angle ACB$ [2]

Question Seven (12 Marks) **Marks**

a) Find the gradient of the tangent to $y=\sin^{-1}(\tan x)$ at $x=0$. [2]

b)

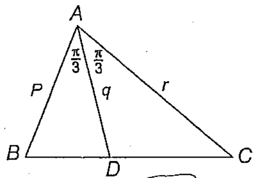


A particle is projected from the top of a tower with a velocity of $30ms^{-1}$ to hit an object that is 120 metres away in the horizontal direction and 35 metres below in the vertical direction (as shown above). The components of its displacement after t seconds are:

$$\left.\begin{aligned} x &= 30t\cos\theta \\ y &= 30t\sin\theta - 5t^2 + 35 \end{aligned}\right\} \text{Do not prove these.}$$

i) If the particle hits the object prove $80\sec^2\theta - 120\tan\theta - 35 = 0$ [3]
ii) Find the angle of projection to the nearest minute [3]
iii) Find the time taken for the particle to reach the object [2]

c) In triangle ABC below $AB = p$, $AD = q$, $AC = r$, $\angle BAD = \frac{\pi}{3} = \angle DAC$ [2]



Show that $\frac{1}{p} + \frac{1}{r} = \frac{1}{q}$

End of paper

Extension One Mathematics
TRIAL HSC 2011 SOLUTIONS

SGHS

Question One:

a) $\frac{1}{3}\tan^{-1}\frac{x}{3} + c$ ✓

b) Let $y = \frac{1}{2}\sin^{-1}2x$

$\therefore 2y = \sin^{-1}2x$.

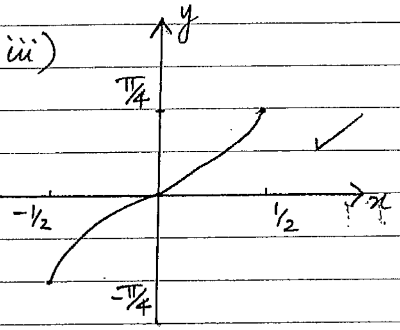
i) $-\frac{\pi}{2} \le 2y \le \frac{\pi}{2}$

$\therefore -\frac{\pi}{4} \le y \le \frac{\pi}{4}$ ✓

ii) $-1 \le 2x \le 1$

$-\frac{1}{2} \le x \le \frac{1}{2}$ ✓

iii)



✓

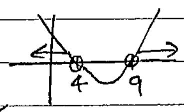
c) $\frac{5}{x-4} < 1$

$5(x-4) < (x-4)^2$

$5x - 20 < x^2 - 8x + 16$

$0 < x^2 - 13x + 36$

$0 < (x-4)(x-9)$



$x < 4$ or $x > 9$ ✓✓

d) $\int_0^1 x\sqrt{1-x^2}\,dx$

$u = 1 - x^2$

$\frac{du}{dx} = -2x$

$\frac{du}{-2x} = dx$.

when $x=0$, $u=1$

when $x=1$, $u=0$

$= \int_0^1 x.\sqrt{u}\,.\frac{du}{-2x}$ ✓

$= -\frac{1}{2}\int_1^0 u^{1/2}\,du$

$= -\frac{1}{2}\left[\frac{2u^{3/2}}{3}\right]_1^0$ ✓

$= -\frac{1}{2}\left[0 - \frac{2}{3}\right]$

$= \frac{1}{3}$ ✓

e) i) Let $y = x^3 - x - 1$

when $x = 1$, $y = -1 < 0$

when $x = 2$, $y = 5 > 0$

Since there is a change of sign then there is a solution between $x=1$ and $x=2$ ✓

ii) $x_1 = x - \frac{f(x)}{f'(x)}$

$= 1.5 - \frac{f(1.5)}{f'(1.5)}$ ✓

$= 1.5 - \frac{0.875}{3.5}$

$= 1.25$

$\therefore x_1 = 1.3$ (1 dec.pl.) ✓

Question Two (12 marks).

a) $P(2,3)$ $\quad m:n \quad$ $A(-1,6)$ $\quad B(x_2, y_2)$

$x\,y \qquad 2:3 \qquad x_1 y_1$

$x = \dfrac{nx_1 + mx_2}{m+n}$ $\qquad y = \dfrac{ny_1 + my_2}{m+n}$

$2 = \dfrac{3(-1) + 2(x_2)}{2+3}$ $\qquad 3 = \dfrac{3(6) + 2(y_2)}{2+3}$

$10 = -3 + 2x_2$ $\qquad 15 = 18 + 2y_2$

$13 = 2x_2$

$\therefore x_2 = 6\frac{1}{2}$ $\qquad -3 = 2y_2$

$y_2 = -\frac{3}{2}$

$\therefore B(6\frac{1}{2}, -3/2)$. 2

b) $f(x) = \frac{3}{x} - 4$

i) let $y = \frac{3}{x} - 4$

$\therefore f^{-1}: x = \frac{3}{y} - 4$

$x + 4 = \frac{3}{y}$

$y = \dfrac{3}{x+4}$ 1

ii) Evaluate

$f^{-1}(4) = \dfrac{3}{x+4}$

$= \dfrac{3}{4+4}$

$f^{-1}(4) = 3/8$ 1

c) i) $P(x) = 2x^3 - 17x^2 + 7x + 8$ $\quad (x-1)$ is a factor

$\therefore P(1) = 0$

$= 2(1)^3 - 17(1)^2 + 7(1) + 8$

$= 2 - 17 + 7 + 8$

$P(1) = 0$ $\quad \therefore (x-1)$ is a factor 1

c) ii)

$$
\begin{array}{r}
2x^2 - 15x - 8 \\
x-1 \,\overline{)\, 2x^3 - 17x^2 + 7x + 8} \\
\underline{2x^3 - 2x^2} \\
-15x^2 + 7x + 8 \\
\underline{-15x^2 + 15x} \\
-8x + 8 \\
\underline{-8x + 8} \\
0
\end{array}
$$

$\therefore P(x) = (x-1)(2x^2 - 15x - 8)$

$= (x-1)(2x+1)(x-8)$ 2

d) i) Gradient of tangent at $P = p$ $\qquad P(2ap, ap^2)$

Gradient of normal at $P = -\frac{1}{p}$

$\therefore$ Equation of normal at P: $\quad y - y_1 = m(x - x_1)$

$y - ap^2 = -\frac{1}{p}(x - 2ap)$

$py - ap^3 = -x + 2ap$

$\therefore x + py = ap^3 + 2ap$. 2

ii) Coordinates of L at $x = 0$

$x + py = ap^3 + 2ap$

$py = ap^3 + 2ap$ $\qquad \therefore L(0, ap^2 + 2a)$

$py = p(ap^2 + 2a)$ 1

$y = ap^2 + 2a$

iii) Midpoint $M\left(\dfrac{0 + 2ap}{2}, \dfrac{ap^2 + 2a + ap^2}{2}\right)$

$L(0, ap^2 + 2a)$

$2(2ap, ap^2)$ $\qquad M\left(ap, \dfrac{2ap^2 + 2a}{2}\right)$

$M(ap, ap^2 + a)$ $\qquad S(0, a)$

Gradient of $SM = \dfrac{y_2 - y_1}{x_2 - x_1}$ $\qquad = \dfrac{ap^2}{ap}$ 2

$= \dfrac{ap^2 + a - a}{ap - 0}$ $\qquad m = p$

$\therefore$ Gradient of $SM =$ Gradient of tangent at $P = p$

$\therefore SM \parallel$ to tangent at P.

Question 3

a) $x - 2y - 1 = 0 \Rightarrow y = \frac{1}{2}x - \frac{1}{2}$ $\quad m_1 = \frac{1}{2}$ ✓

$x + 3y + 2 = 0 \Rightarrow y = -\frac{1}{3}x - \frac{2}{3}$ $\quad m_2 = -\frac{1}{3}$

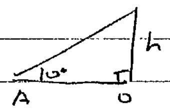
$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$ ✓

$= \left|\left(\frac{1}{2} - \frac{-1}{3}\right) \div \left(1 - \frac{1}{6}\right)\right|$

$= |1|$

$\theta = 45^\circ$ or $\frac{\pi}{4}$ ✓ (3)

b) i)



$AO = \frac{h}{\tan 10^\circ}$ or $AO = h\cot 10^\circ$ ✓ (1)

ii) $OB = \frac{h}{\tan 15^\circ}$ or $OB = h\cot 15^\circ$ ✓

Now $OA^2 + OB^2 = 400^2$

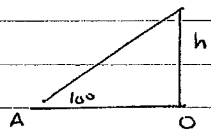
$\frac{h^2}{\tan^2 10^\circ} + \frac{h^2}{\tan^2 15^\circ} = 400^2$

$h^2\left(\frac{1}{\tan^2 10^\circ} + \frac{1}{\tan^2 15^\circ}\right) = 400^2$ ✓

$h = 400 \div \sqrt{\frac{1}{\tan^2 10^\circ} + \frac{1}{\tan^2 15^\circ}}$ (3)

$= 58.9\text{m}$ ✓

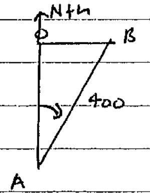
iii)



$\frac{h}{AO} = \tan 10^\circ$

$AO = \frac{h}{\tan 10^\circ}$

$= 334.03$



$\cos\angle OAB = \frac{OA}{400}$

$= \frac{334.03..}{400}$

$\angle OAB = 33^\circ$ or $\angle OOA =$ (2) ✓

$\therefore$ Bearing $= 180^\circ + 33^\circ$

$= 213^\circ$ ✓

c) $P(x) = x^3 + ax^2 + 2ax + b$

when $x = 2$, $8 + 4a + 4a + b = 0$

$8a + b = -8$ (1)

when $x = -3$, $-27 + 9a - 6a + b = 0$

$3a + b = 27$ (2)

(1) − (2) $\quad 5a = -35$

$a = -7$ ✓

$b = 48$ ✓ (the constant term)

Now $(x-2)(x+3)(x-\alpha) = 0$

$\therefore 6\alpha = 48$

$\alpha = 8$ ✓ (3)

2011 Ext 1 Trial – Solution to Question 4

(a)	$\cos 2x = 2\cos^2 x - 1 \quad \Rightarrow 2\cos^2 4x = \cos 8x + 1$ $\int 2\cos^2 4x\,dx = \int(\cos 8x + 1)\,dx$ $= \dfrac{\sin 8x}{8} + x + C$
(b)(i)	$\dot{x} = 2 - 3e^{-t} \quad \therefore \ddot{x} = 3e^{-t}$
(b)(ii)	$\dot{x} = 2 - 3e^{-t} \quad x = \int(2 - 3e^{-t})\,dt = 2t + 3e^{-t} + C$ $x = 0$ when $t = 0 \quad 0 = 2(0) + 3e^0 + C \Rightarrow C = -3$ $\therefore x = 2t + 3e^{-t} - 3$
(b)(iii)	$\dot{x} = 2 - 3e^{-t} \quad \dot{x} = 0 \quad \Rightarrow 0 = 2 - 3e^{-t}$ $3e^{-t} = 2 \quad e^{-t} = \dfrac{2}{3} \quad \Rightarrow -t = \ln\dfrac{2}{3}$ $\therefore t = -\ln\dfrac{2}{3} = \ln\dfrac{3}{2}$ s
(b)(iv)	as $t \to \infty$ $\ddot{x} \to 0^+$ and $\dot{x} \to 2^-$ i.e. acceleration approaches 0 and velocity approaches 2 m/s
(c)	Step 1 : Prove true for $n = 1$. $2 \times 5^{1-1} + 12^1 = 2 + 12 = 14 = 2 \times 7$ $\therefore$ Divisible by 7 for $n = 1$ Step 2 : Assume true for $n = k$. $2 \times 5^{k-1} + 12^k = 7A$ where A is some integer. Step 3 : Prove true for $n = k + 1$. i.e. prove $2 \times 5^{k+1-1} + 12^{k+1} = 7B$ $LHS = 2 \times 5^{k+1-1} + 12^{k+1}$ $= 5^1 \times 2 \times 5^{k-1} + 12 \times 12^k$ $= 5 \times (7A - 12^k) + 12 \times 12^k$ $= 35A - 5 \times 12^k + 12 \times 12^k$ $= 35A + 7 \times 12^k$ $= 7(5A + 12^k)$ $= 7B$ where $B = 5A + 12^k$ $LHS = RHS$ Step 4 : If true for $n = k$, then proven true for $n = k + 1$. Since proven true for $n = 1$, must be true for $n = 2$. Since true for $n = 2$, must be true for $n = 3$, etc. Hence, proven true by mathematical induction for all positive integers.

Extension 1 Solutions   Question 5

a) let $BX = x$

$x(x+5.5) = (\sqrt{15})^2$

$x^2 + 5.5x - 15 = 0$

$2x^2 + 11x - 30 = 0$

$(2x+15)(x-2) = 0$

$\therefore x = -7.5, 2$

but $x > 0$

$\therefore x = 2$.

b) $\lim_{x\to\infty} \dfrac{2x^2-3x+1}{5x^2+x-4} = \lim_{x\to\infty} \dfrac{2-\frac{3}{x}+\frac{1}{x^2}}{5+\frac{1}{x}-\frac{4}{x^2}}$

$= \frac{2}{5}$

c) i) $\frac{\sqrt{12}}{r}\sin x + \frac{2}{r}\cos x = \sin x\cos\alpha + \cos x\sin\alpha$

$\cos\alpha = \frac{\sqrt{12}}{r}$ $\sin\alpha = \frac{2}{r}$

$\cos^2\alpha = \frac{12}{r^2}$ $\sin^2\alpha = \frac{4}{r^2}$

$r^2 = 16$

$r = 4$

$\sin\alpha = \frac{2}{4}$

$\alpha = \frac{\pi}{6}$

$\therefore \sqrt{12}\sin x + 2\cos x = 4\sin\left(x+\frac{\pi}{6}\right)$

ii) $4\sin\left(x+\frac{\pi}{6}\right) = -2\sqrt{2}$

$\sin\left(x+\frac{\pi}{6}\right) = -\frac{\sqrt{2}}{2}$

$x + \frac{\pi}{6} = \frac{5\pi}{4}, \frac{7\pi}{4}$

$x = \frac{13\pi}{12}, \frac{19\pi}{12}$

d) i) LHS $= \sin 3\theta$

$= \sin(2\theta+\theta)$

$= \sin 2\theta\cos\theta + \sin\theta\cos 2\theta$

$= 2\sin\theta\cos^2\theta + \sin\theta(1-2\sin^2\theta)$

$= 2\sin\theta(1-\sin^2\theta) + \sin\theta(1-2\sin^2\theta)$

$= 3\sin\theta - 4\sin^3\theta$

$=$ RHS

ii) $\frac{\sin 3\theta}{10} = \frac{\sin\theta}{5}$

$\sin 3\theta = 2\sin\theta$

$3\sin\theta - 4\sin^3\theta = 2\sin\theta$

$\sin\theta - 4\sin^3\theta = 0$

$\sin\theta(1-4\sin^2\theta) = 0$

$\sin\theta = 0$ or $\sin^2\theta = \frac{1}{4}$

not a solution to this problem $\sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

however only $\theta = \frac{\pi}{6}$ satisfies the problem.

6 a) $\dot{x} = 4\cos 2t - 6\sin 2t$

$\ddot{x} = -8\sin 2t - 12\cos 2t$

$= -4x$ ✓✓

b) i) $V = \dfrac{4\pi r^3}{3}$

$\dfrac{dV}{dt} = \dfrac{dV}{dr} \times \dfrac{dr}{dt}$

$8 = 4\pi r^2 \times \dfrac{dr}{dt}$

$\dfrac{dt}{dr} = \dfrac{\pi r^2}{2}$

$t = \dfrac{\pi r^3}{6}$

$\dfrac{6t}{\pi} = r^3$

$r = \sqrt[3]{\dfrac{6t}{\pi}}$ ✓✓

ii) when $t = 4$

$r = \sqrt[3]{\dfrac{24}{\pi}}$

$S = 4\pi r^2$

$\dfrac{dS}{dt} = \dfrac{dS}{dr} \times \dfrac{dr}{dt}$

$= 8\pi r \times \dfrac{2}{\pi r^2}$

$= \dfrac{16}{r}$

$= \dfrac{16}{\sqrt[3]{\dfrac{24}{\pi}}}$ ✓✓

iii) $4\pi r^2 = 3000$

$r^2 = \dfrac{750}{\pi}$

$r = \sqrt{\dfrac{750}{\pi}}$

$t = \dfrac{\pi \times \left(\sqrt{\dfrac{750}{\pi}}\right)^3}{6}$ ✓✓

c) i) $F\hat{A}C = \hat{B}$ ($\angle$s in alternate segments)

$F\hat{C}A = \hat{B}$ (" " " ") ✓✓

ii) $A\hat{C}B = F\hat{A}C + \hat{E}$ (exterior $\angle$ of a Δ)

$F\hat{C}D = \hat{E}$ ($\angle$s in alternate segment)

$\therefore A\hat{C}B = F\hat{A}C + F\hat{C}D$

$= A\hat{C}D$ ✓✓

Question 7:

a) let $u = \tan x$ $\qquad y = \sin^{-1} u$

$\frac{du}{dx} = \sec^2 x \qquad \frac{dy}{du} = \frac{1}{\sqrt{1-u^2}}$

$$\frac{dy}{dx} = \sec^2 x \cdot \frac{1}{\sqrt{1-u^2}}$$

$$= \frac{\sec^2 x}{\sqrt{1-\tan^2 x}} \qquad 1$$

when $x = 0$ $\quad m_T = \frac{\sec^2 0}{\sqrt{1-0^2}} = 1 \qquad 1$

b) When particle hits object, $x = 120$:

$$120 = 30t\cos\theta$$

$$t = \frac{4}{\cos\theta} \quad 1 \qquad 1$$

When particle hits object, $y = 0$:

$$0 = 30t\sin\theta - 5t^2 + 35 \quad 2$$

Sub 1 into 2:

$$0 = 30 \times \frac{4}{\cos\theta}\sin\theta - 5\left(\frac{4}{\cos\theta}\right)^2 + 35 \qquad 1$$

$$= 120\tan\theta - 80\sec^2\theta + 35$$

$$80\sec^2\theta - 120\tan\theta - 35 = 0 \qquad 1$$

b) $80(1+\tan^2\theta) - 120\tan\theta - 35 = 0 \qquad 1$

$$80\tan^2\theta - 120\tan\theta + 45 = 0$$

$$16\tan^2\theta - 24\tan\theta + 9 = 0$$

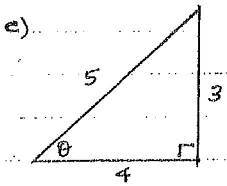
$$(4\tan\theta - 3)^2 = 0 \qquad 1$$

$$4\tan\theta = 3$$

$$\tan\theta = \frac{3}{4}$$

$\therefore$ $\theta = 36°52'$ (nearest minute) $\quad 1$

e)



$$x = 30t\cos\theta$$

$$120 = 30 \times t \times \frac{4}{5}$$

$$t = 5\text{s} \qquad 2$$

c) In ΔABD:

$$\text{Area } \Delta ABD = \frac{1}{2}pq\sin\frac{\pi}{3}$$

In ΔADC:

$$\text{Area } \Delta ADC = \frac{1}{2}qr\sin\frac{\pi}{3}$$

In ΔABC:

$$\text{Area } \Delta ABC = \frac{1}{2}pr\sin\frac{2\pi}{3}$$

1

$$\text{Area } \Delta ABC = \text{Area } \Delta ABD + \text{Area } \Delta ADC$$

$$\frac{1}{2}pr\sin\frac{2\pi}{3} = \frac{1}{2}pq\sin\frac{\pi}{3} + \frac{1}{2}qr\sin\frac{\pi}{3}$$

$$pr\sin\frac{\pi}{3} = pq\sin\frac{\pi}{3} + qr\sin\frac{\pi}{3}$$

$$pr = pq + qr$$

$$\therefore \frac{1}{q} = \frac{1}{r} + \frac{1}{p}$$

1