C.E.M. TUITION

Name : ______________________________

Review Paper No. 1

Relations & Functions

Year 11 - 2 Unit

1. Find the largest possible domain of each of the following functions.

   (i) $y=\sqrt{1+2x}$ (ii) $y=\dfrac{1}{\sqrt{2-x}}$

   (iii) $y=\sqrt{1+2x}+\dfrac{1}{\sqrt{2-x}}$

2. Use the graph $y=|x|$ to sketch the graphs (i) $y=|x-1|$

   (ii) $y=|x|-1$

3. Show that the function $f(x) = x^3 - x$ is odd. Sketch the graph of the function and find the values of x for which $f(x)$ is negative.

4. Sketch the graph of the function $y = \dfrac{x+1}{x+2}$ and state its domain and range.

5. Sketch the graph of the function

$$f(x) = \begin{cases} 4 & , \quad x \le 0 \\ (x-2)^2 & , \quad 0 < x \le 3 \\ 1 & , \quad x > 3 \end{cases}$$

Find the range of the function.

6. On the same axes sketch the graphs $y = x^2 - 1$ and $y = \frac{-1}{x}$. By inspection of the graph, state the number of solutions of $x^2 - 1 = \frac{-1}{x}$. (Do not attempt to find these solutions or the intersection points of the graphs.)

7. Sketch the graph of the circle $(x-3)^2+(y-4)^2=25$. Shade the region in the first quadrant contained on or inside the circle, and state the three inequalities satisfied simultaneously by the points in the shaded region.

8. On the same axes sketch the graphs $y=\sqrt{4-x^2}$ and $y=x-2$. Shade the region where $y \le \sqrt{4-x^2}$, $x \ge 0$ and $y \ge x-2$.

1. (i) $\sqrt{(\quad)}$ is only defined for $(\quad) \geq 0$. Hence the domain of the function $y = \sqrt{1+2x}$ is given by

$$1 + 2x \geq 0$$
$$2x \geq -1$$
$$x \geq -\tfrac{1}{2}$$

Hence domain is $\{x : x \geq -\tfrac{1}{2}\}$.

(ii) $\dfrac{1}{\sqrt{(\quad)}}$ is only defined for $(\quad) > 0$.
Hence the domain of the function $y = \dfrac{1}{\sqrt{2-x}}$ is given by

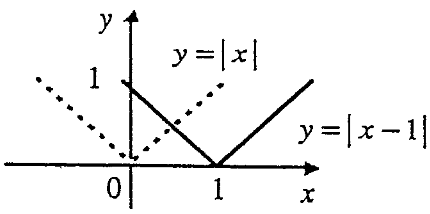
$$2 - x > 0$$
$$x < 2$$

Hence domain is $\{x : x < 2\}$.

(iii) Since both the restrictions $x \geq -\tfrac{1}{2}$ and $x < 2$ must be satisfied if both terms are to be defined, the domain of $y = \sqrt{1+2x} + \dfrac{1}{\sqrt{2-x}}$ is $\{x : -\tfrac{1}{2} \leq x < 2\}$.

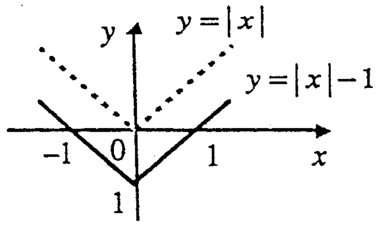
2. (i) The graph $y = |x-1|$ has the same shape as $y = |x|$, with its vertex at x such that $x - 1 = 0$ rather than $x = 0$. Hence the vertex has coordinates $(1, 0)$ and the graph of $y = |x|$ is translated 1 unit to the right.

Figure 14.17



(ii) The graph $y = |x| - 1$ has the same shape as $y = |x|$, but is translated downwards by 1 unit since every y value is decreased by 1.

Figure 14.18



3. f is an odd function if $f(-x) = -f(x)$.

$$f(x) = x^3 - x$$
$$f(-x) = (-x)^3 - (-x)$$
$$= -x^3 + x$$
$$= -(x^3 - x)$$
$$= -f(x)$$

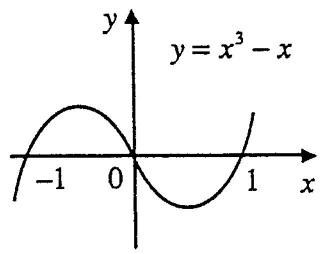
$\therefore$ $f(x)$ is an odd function.

y-intercept: $x = 0 \Rightarrow y = 0$

x-intercepts: $y = 0 \Rightarrow \begin{cases} x^3 - x = 0 \\ x(x^2 - 1) = 0 \\ x(x-1)(x+1) = 0 \\ x = 0, \quad x = \pm 1 \end{cases}$

Consider the function $f(x)$ for $x > 0$.
For $0 < x < 1$, $x^3 < x \Rightarrow f(x) < 0$.
For $x > 1$, $x^3 > x \Rightarrow f(x) > 0$.
Since $f(x)$ is an odd function, the graph $y = f(x)$ has point symmetry in the origin, and we can use the graph of $y = f(x)$ for $x > 0$ to sketch $y = f(x)$ for $x < 0$.

Figure 14.19



By inspection of the graph, $f(x) < 0$ for $x < -1$ or $0 < x < 1$.

4. Graphs of functions of the form $y = \frac{ax+b}{cx+d}$ are always hyperbolas. To graph such a function, we need to find the vertical and horizontal asymptotes.

Consider $y = \frac{x+1}{x+2}$.

vertical asymptote: $y \to \infty$ as $x \to -2$

$\therefore x = -2$ is a vertical asymptote.

horizontal asymptote:

As $x \to \infty$, $y = \frac{x+1}{x+2} = \frac{1+\frac{1}{x}}{1+\frac{2}{x}} \to \frac{1+0}{1+0}$.

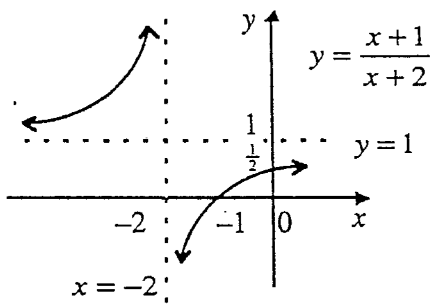
Hence $y \to 1$ as $x \to \infty$, and $y = 1$ is a horizontal asymptote.

To decide in which quadrants relative to these asymptotes the hyperbola lies, we can find the intercepts on the coordinate axes.

y-intercept: $x = 0 \Rightarrow y = \frac{1}{2}$

x-intercept: $y = 0 \Rightarrow x = -1$

Figure 14.20



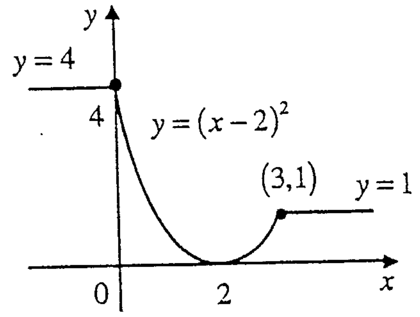
By inspection of the graph, $y = \frac{x+1}{x+2}$ has domain $\{x : x \neq -2\}$, range $\{y : y \neq 1\}$.

5. The graph of the function

$$f(x) = \begin{cases} 4 & , \quad x \le 0 \\ (x-2)^2 & , \quad 0 < x \le 3 \\ 1 & , \quad x > 3 \end{cases}$$

consists of a section of the straight line $y = 4$ for $x \le 0$, the section of the concave up parabola $y = (x-2)^2$ lying between the points $(0, 4)$ and $(3, 1)$ with vertex at $(2, 0)$, and the section of straight line $y = 1$ for $x > 3$.

Figure 14.21



By inspection of the graph, the range of the function *f* is $\{y : 0 \le y \le 4\}$.

6. $y = \frac{-1}{x}$ is a hyperbola with the coordinate axes as asymptotes.

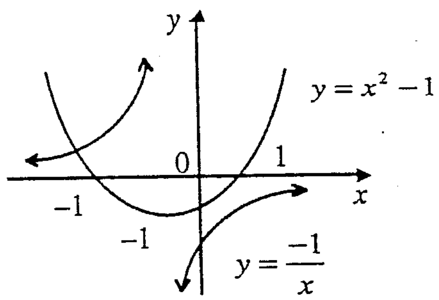
$xy = -1 \Rightarrow$ x and y have different signs, hence this hyperbola lies in the second and fourth quadrants.

$y = x^2 - 1$ is the same shape as the parabola $y = x^2$ translated downwards by 1 unit, with intercepts on the coordinate axes given by

y-intercept: $x = 0 \Rightarrow y = -1$

x-intercepts: $y = 0 \Rightarrow x = \pm 1$

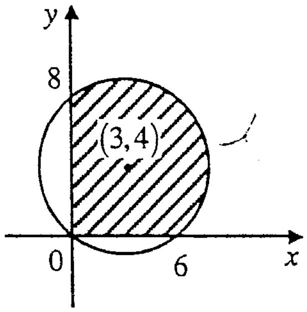
Figure 14.22



The solutions of the equation $x^2 - 1 = \frac{-1}{x}$ are the x-coordinates of the intersection points of the graphs $y = x^2 - 1$ and $y = \frac{-1}{x}$. Since these graphs intersect exactly once, the equation has exactly one solution.

7. The circle $(x-3)^2+(y-4)^2=25$ has centre $(3,4)$ and radius 5, and passes through $(0,0)$, $(0,8)$ and $(6,0)$.

Figure 14.23



Points in the shaded region satisfy the three inequalities $x \geq 0$, $y \geq 0$ and $(x-3)^2+(y-4)^2 \leq 25$.

8. $y=\sqrt{4-x^2}$ is the upper semicircle $x^2+y^2=4,\ y \geq 0$ with centre $(0,0)$ and radius 2.
$y=x-2$ is the straight line with y-intercept -2 and x-intercept 2.

Test $(0,0)$:

$$\left.\begin{array}{l} y=\sqrt{4-x^2} \\ 0<\sqrt{4-0} \end{array}\right\} \Rightarrow \begin{array}{c} (0,0) \text{ lies in region} \\ y \leq \sqrt{4-x^2} \end{array}$$

$$\left.\begin{array}{l} y=x-2 \\ 0>0-2 \end{array}\right\} \Rightarrow \begin{array}{c} (0,0) \text{ lies in region} \\ y \geq x-2 \end{array}$$

$y \leq \sqrt{4-x^2}$ has domain $-2 \leq x \leq 2$. Hence points in the required region satisfy $-2 \leq x \leq 2$, lie below the semicircle $y=\sqrt{4-x^2}$ and above the line $y=x-2$. All boundaries are included in the region and are shown as firm lines or curves.

Figure 14.24