## Revision questions

26. Find the equation of the parabola with vertex $(5, -4)$ that passes through the point $(1, 4)$.

27. Find the equation of the locus of a point that moves so that the ratio of PA to PB is 3:2, where $A = (2, 7)$ and $B = (-3, -4)$.

28. Find the equation of the parabola with focus $(0, -8)$ and directrix $y = 8$.

29. Find the equation of the tangent to the curve
$$x^2 = -4y$$
at the point where $x = 2$. Find the coordinates of the point where this tangent cuts the directrix.

30. Find the centre and radius of the circle whose equation is
$$x^2 - 10x + y^2 + 12y - 3 = 0$$

31. Describe the locus of a point moving so that it is always 2 units from the $x$-axis.

32. For the parabola $y^2 = -20x$ find:
(a) the coordinates of the focus
(b) the equation of the directrix
(c) the equation of the axis.

33. Find the equation of the locus of a point moving so that it is equidistant from the $x$-axis and the point $(-2, -3)$.

34. For the parabola $x^2 + 4x + 8y = 0$ find:
(a) the coordinates of the vertex
(b) the coordinates of the focus
(c) the equation of the directrix.

35. Find the locus of the point moving such that it is always 9 units from the point $(-6, 5)$.

## Challenge questions

36. Show that the line $x + y + 1 = 0$ is a tangent to the parabola
$$x^2 = 4y$$
Find its point of contact P with the parabola and find the equation of the line PF where F is the focus.

37. The points $A(2, -2)$ and $B(-4, -8)$ lie on the parabola
$$x^2 = -2y$$
The normals at A and B meet at point C. Find the equations of the normals and the coordinates of C.

38. Sketch the region
$$x^2 + y^2 - 4y - 5 \le 0$$

39. Find the equations of the tangents to the parabola
$$x^2 = 8y$$
at points $A(-4, 2)$ and $B(8, 8)$. Find Q, the point of intersection of these tangents. Find the midpoint M of AB and show that the line MQ is parallel to the axis of the parabola.

40. Find the equation of the locus of a point $P(x, y)$ given that PA is perpendicular to PB where $A = (3, 2)$ and $B = (-5, 1)$.

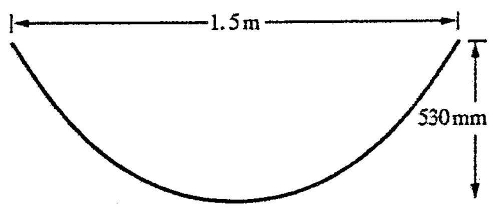
41. Find the coordinates of the focus and the equation of the directrix for the parabola
$$y = x^2 - 4x + 5.$$

42. Find the equation of the locus of a point that is equidistant from the lines L, $3x - 4y + 1 = 0$, and M, $3x - 4y - 5 = 0$.

43. Find the equation of the tangent to the curve $(x - 3)^2 = 16y$ at the point where $x = 5$.

44. (a) Find the equation of the circle with diameter AB where $A = (0, 6)$ and $B = (4, -2)$.
(b) Show that this circle is the equation of the locus of point $P(x, y)$ moving so that PA is perpendicular to PB.

45. A satellite dish is 1.5 m wide and 530 mm deep. Find the position of the focus from the vertex, to the nearest mm.



26. The general parabola has equation:
$$(x-h)^2 = 4a(y-k)$$
where vertex $= (h, k)$ and focal length $= a$.
$$(x-5)^2 = 4a(y+4)$$
Substitute (1, 4) into the equation:
$$(1-5)^2 = 4a(4+4)$$
$$16 = 32a$$
$$\tfrac{1}{2} = a$$
So $(x-5)^2 = 4(\tfrac{1}{2})(y+4)$
$$x^2 - 10x + 25 = 2(y+4)$$
$$= 2y + 8$$
$$x^2 - 10x - 2y + 17 = 0$$

27. *Hint:* $4PA^2 = 9PB^2$
$5x^2 + 70x + 5y^2 + 128y + 13 = 0$

28. Focus $= (0, -a) = (0, -8)$
Directrix: $y = a = 8$
Equation is of the form $x^2 = -4ay$ where $a = 8$.
So $x^2 = -4(8)y$
$x^2 = -32y$

29. $x + y - 1 = 0$, $(0, 1)$

30. $x^2 - 10x + y^2 + 12y = 3$
Completing the square:
$$x^2 - 10x + 25 + y^2 + 12y + 36 = 3 + 25 + 36$$
$$(x-5)^2 + (y+6)^2 = 64$$
Circle, centre $(5, -6)$ and radius 8.

31. Lines $y = \pm 2$

32. Equation is of the form $y^2 = -4ax$ where $a = 5$
(a) Focus $= (-a, 0) = (-5, 0)$
(b) Directrix: $x = a$
$\therefore x = 5$
(c) Axis: $y = 0$

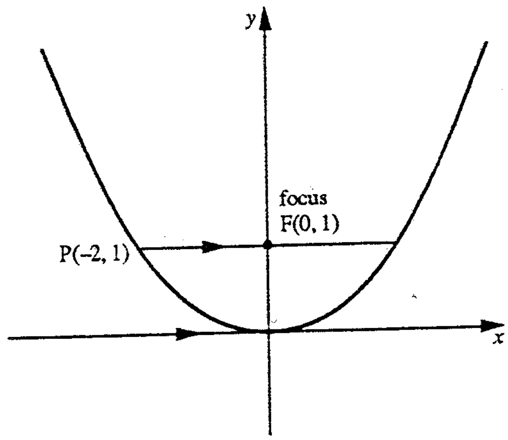
33. $x^2 + 4x + 6y + 13 = 0$

34. $x^2 + 4x = -8y$
$x^2 + 4x + 4 = -8y + 4$
$(x+2)^2 = -8(y - \tfrac{1}{2})$
(a) Vertex $= (-2, \tfrac{1}{2})$
(b) $4a = 8$
$a = 2$
Count down 2 units to the focus:
Focus $= (-2, -1\tfrac{1}{2})$
(c) Count up 2 units:
Directrix: $y = 2\tfrac{1}{2}$

35. $(x+6)^2 + (y-5)^2 = 81$
or $x^2 + 12x + y^2 - 10y - 20 = 0$

## Challenge questions

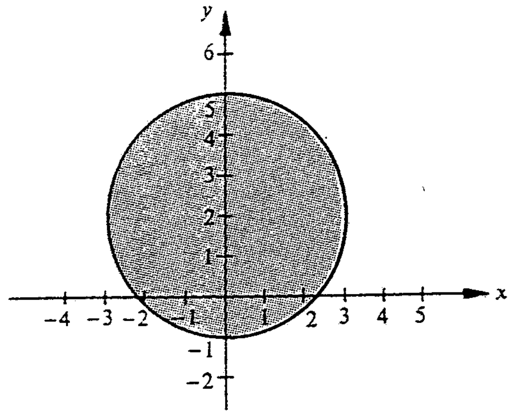
36. Solve simultaneous equations:
$$x^2 = 4y \qquad (1)$$
$$x + y + 1 = 0 \qquad (2)$$
(2): $y = -x - 1 \qquad (3)$
Substitute (3) into (1):
$$x^2 = 4(-x-1)$$
$$= -4x - 4$$
$$x^2 + 4x + 4 = 0$$
$$(x+2)^2 = 0$$
$$\therefore x = -2$$
There is only one point of intersection, when $x = -2$.
$\therefore$ Line is a tangent.
Substitute $x = -2$ in (3):
$$y = -(-2) - 1$$
$$= 1$$
So $P = (-2, 1)$
Now $4a = 4$
$\therefore a = 1$
$F = (0, a) = (0, 1)$




From the graph, line PF passes through the focus and is parallel to the $x$-axis.
$\therefore$ It has equation $y = 1$.

37. $x - 2y - 6 = 0$, $x + 4y + 36 = 0$
$C = (-8, -7)$

38. $x^2 + y^2 - 4y = 5$
$x^2 + y^2 - 4y + 4 = 5 + 4$
$x^2 + (y-2)^2 = 9$
Circle with centre (0, 2) and radius 3.
For $x^2 + y^2 - 4y - 5 \le 0$
choose any point inside, say (0, 2):
$0^2 + 2^2 - 4(2) - 5 \le 0$
$-9 \le 0$ (true)
$\therefore$ Region is inside the circle.




39. At A: $x + y + 2 = 0$
At B: $2x - y - 8 = 0$
$Q = (2, -4)$, $M = (2, 5)$
MQ has equation $x = 2$, which is a line parallel to $x = 0$ (axis of the parabola).

40. PA has gradient $m_1 = \frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{y - 2}{x - 3}$

PB has gradient $m_2 = \frac{y - 1}{x + 5}$

For ⊥ lines, $m_1 m_2 = -1$

that is, $\frac{y - 2}{x - 3} \times \frac{y - 1}{x + 5} = -1$

$$\frac{y^2 - y - 2y + 2}{x^2 + 5x - 3x - 15} = -1$$
$$\frac{y^2 - 3y + 2}{x^2 + 2x - 15} = -1$$
$$y^2 - 3y + 2 = -(x^2 + 2x - 15)$$
$$= -x^2 - 2x + 15$$
$$x^2 + 2x + y^2 - 3y - 13 = 0$$

41. Focus $= (2, 1\frac{1}{4})$
Directrix: $y = \frac{3}{4}$

42. Let $P = (x, y)$ be the moving point.
$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$
Distance P to L:
$$d = \frac{|3x - 4y + 1|}{\sqrt{3^2 + (-4)^2}}$$
$$= \frac{|3x - 4y + 1|}{5}$$
Distance P to M:
$$d = \frac{|3x - 4y - 5|}{\sqrt{3^2 + (-4)^2}}$$
$$= \frac{|3x - 4y - 5|}{5}$$

For P to be equidistant from L and M:
$$\frac{|3x - 4y - 1|}{5} = \frac{|3x - 4y - 5|}{5}$$
that is, $|3x - 4y - 1| = |3x - 4y - 5|$
(i) $3x - 4y - 1 = 3x - 4y - 5$
$-1 = -5$
∴ no solution.
(ii) $3x - 4y - 1 = -(3x - 4y - 5)$
$= -3x + 4y + 5$
$6x - 8y - 6 = 0$
$3x - 4y - 3 = 0.$

43. $x - 4y - 4 = 0$

44. (a) Centre: midpoint of AB
$$P = \left(\frac{0 + 4}{2}, \frac{6 + (-2)}{2}\right)$$
$= (2, 2)$
Radius $= \frac{1}{2}$ diameter AB.
$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(4 - 0)^2 + (-2 - 6)^2}$$
$$= \sqrt{16 + 64}$$
$$= \sqrt{80}$$
∴ radius $= \frac{1}{2}\sqrt{80}$
$= \frac{1}{2} \times \sqrt{16} \times \sqrt{5}$
$= \frac{1}{2} \times 4\sqrt{5}$
$= 2\sqrt{5}$

Equation:
$$(x - a)^2 + (y - b)^2 = r^2$$
$$(x - 2)^2 + (y - 2)^2 = (2\sqrt{5})^2$$
$$x^2 - 4x + 4 + y^2 - 4y + 4 = 4 \times 5$$
$$x^2 - 4x + y^2 - 4y + 8 - 20 = 0$$
$$x^2 - 4x + y^2 - 4y - 12 = 0$$

(b) PA has $m_1 = \frac{y - 6}{x - 0}$

PB has $m_2 = \frac{y + 2}{x - 4}$

For ⊥ lines, $m_1 m_2 = -1$
$$\therefore \frac{y - 6}{x - 0} \times \frac{y + 2}{x - 4} = -1$$
$$\frac{y^2 + 2y - 6y - 12}{x^2 - 4x} = -1$$
$$y^2 - 4y - 12 = -(x^2 - 4x)$$
$$= -x^2 + 4x$$
$$x^2 - 4x + y^2 - 4y - 12 = 0$$

45. 265 mm