BOSTES | BOARD OF STUDIES
TEACHING &
EDUCATIONAL
STANDARDS NSW

2014 HIGHER SCHOOL CERTIFICATE
EXAMINATION

Mathematics General 2

General Instructions
- Reading time – 5 minutes
- Working time – $2\frac{1}{2}$ hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- A formulae and data sheet is provided at the back of this paper
- In Questions 26–30, show relevant mathematical reasoning and/or calculations

Total marks – 100

(Section I) Pages 2–12

25 marks
- Attempt Questions 1–25
- Allow about 35 minutes for this section

(Section II) Pages 13–34

75 marks
- Attempt Questions 26–30
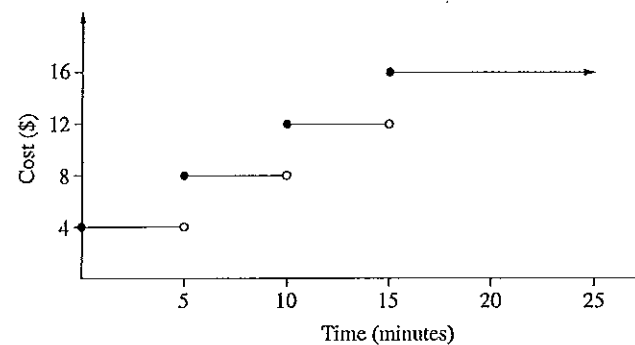- Allow about 1 hour and 55 minutes for this section

Section I

25 marks
Attempt Questions 1–25
Allow about 35 minutes for this section

Use the multiple-choice answer sheet for Questions 1–25.

1 The step graph shows the cost of telephone calls to China.

Time (minutes)

Bella made a telephone call at 9.50 pm to a friend in China and spoke to him until 10.07 pm.
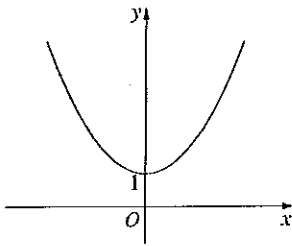
How much did this telephone call cost?

(A) $4

(B) $8

(C) $12

(D) $16

2 A measurement of 72 cm is increased by 20% and then the result is decreased by 20%.

What is the new measurement, correct to the nearest centimetre?

(A) 46 cm

(B) 69 cm

(C) 72 cm

(D) 104 cm

3   The diagram shows the graph of an equation.

Which of the following equations does the graph best represent?

(A) $y = \dfrac{3}{x} + 1$

(B) $y = 3^x + 1$

(C) $y = 3x^2 + 1$

(D) $y = 3x^3 + 1$

4   Young's formula below is used to calculate the required dosages of medicine for children aged 1–12 years.

$$\text{Dosage} = \frac{\text{age of child (in years)} \times \text{adult dosage}}{\text{age of child (in years)} + 12}$$

How much of the medicine should be given to an 18-month-old child in a 24-hour period if each adult dosage is 45 mL? The medicine is to be taken every 6 hours by both adults and children.

(A)   5 mL

(B)   20 mL

(C)   27 mL

(D)   30 mL

5   How many kilobytes are there in 2 gigabytes?

(A) 2^{20}
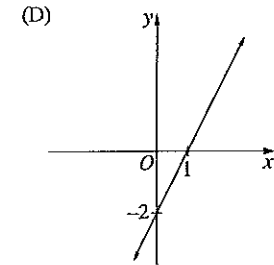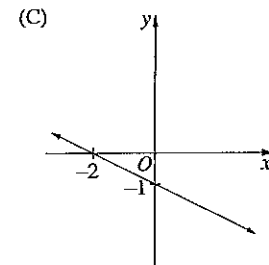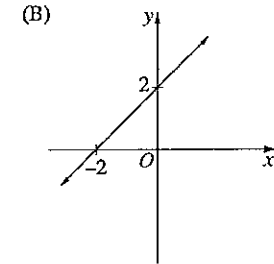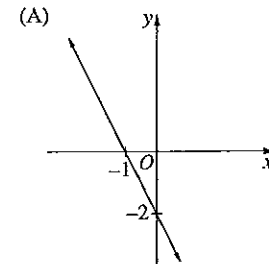
(B) 2^{21}

(C) 2^{30}

(D) 2^{31}

6   A cafe menu has 3 entrees, 5 main courses and 2 desserts. Ariana is choosing a three-course meal consisting of an entree, a main course and a dessert.

How many different three-course meals can Ariana choose?

(A)   3

(B)   10

(C)   15

(D)   30

7   Which of the following is the graph of $y = 2x - 2$?

(A)

(B)

(C)

(D)

8　A group of 150 people was surveyed and the results recorded.

Survey results

	Owns a mobile	Does not own a mobile	Total
Male	42	28	70
Female	63	17	80
	105	45	150

A person is selected at random from the surveyed group.

What is the probability that the person selected is a male who does not own a mobile?

(A)　$\dfrac{28}{150}$

(B)　$\dfrac{45}{150}$

(C)　$\dfrac{28}{70}$

(D)　$\dfrac{45}{70}$

9　A car is bought for $19 990. It will depreciate at 18% per annum.

Using the declining balance method, what will be the salvage value of the car after 3 years, to the nearest dollar?

(A)　$8968

(B)　$9195

(C)　$11 022

(D)　$16 392

10　The top of the Sydney Harbour Bridge is measured to be 138.4 m above sea level.

What is the percentage error in this measurement?

(A)　0.036%

(B)　0.050%

(C)　0.072%

(D)　0.289%

11　Simplify $6w^4 \times \dfrac{1}{3}w^2$.

(A)　$2w^6$

(B)　$2w^8$

(C)　$18w^6$

(D)　$18w^8$

12　A path 1.5 metres wide surrounds a circular lawn of radius 3 metres.

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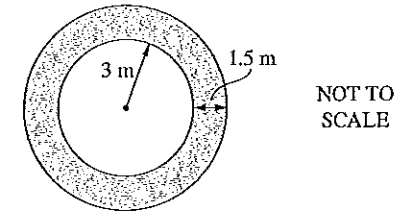
What is the approximate area of the path?

(A)　7.1 m^2

(B)　21.2 m^2

(C)　35.3 m^2

(D)　56.5 m^2

13　Jane sells jewellery. Her commission is based on a sliding scale of 6% on the first $2000 of her sales, 3.5% on the next $1000, and 2% thereafter.

What is Jane's commission when her total sales are $5670?

(A)　$188.40

(B)　$208.40

(C)　$321.85

(D)　$652.05

14   Twenty Year 12 students were surveyed. These students were asked how many hours of sport they play per week, to the nearest hour.

The results are shown in the frequency table.

Hours per week	Frequency
0–2	5
3–5	10
6–8	3
9–11	2

What is the mean number of hours of sport played by the students per week?

(A)   3.3

(B)   4.3

(C)   5.0

(D)   5.3

15   Which expression will give the shortest distance, in kilometres, between Mount Isa (20°S 140°E) and Tokyo (35°N 140°E)?

(A) $\dfrac{15}{360} \times 2 \times \pi \times 6400$

(B) $\dfrac{55}{360} \times 2 \times \pi \times 6400$

(C) $\dfrac{140}{360} \times 2 \times \pi \times 6400$

(D) $\dfrac{305}{360} \times 2 \times \pi \times 6400$

16   In Mathsville, there are on average eight rainy days in October.

Which expression could be used to find a value for the probability that it will rain on two consecutive days in October in Mathsville?

(A) $\dfrac{8}{31} \times \dfrac{7}{30}$

(B) $\dfrac{8}{31} \times \dfrac{7}{31}$

(C) $\dfrac{8}{31} \times \dfrac{8}{30}$

(D) $\dfrac{8}{31} \times \dfrac{8}{31}$

17   A child who weighs 14 kg needs to be given 15 mg of paracetamol for every 2 kg of body weight. Every 10 mL of a particular medicine contains 120 mg of paracetamol.

What is the correct dosage of this medicine for the child?

(A)   5.6 mL

(B)   8.75 mL

(C)   11.43 mL

(D)   17.5 mL

18   The average NSW annual water consumption from the residential sector is equal to 90 340 litres per person per year. The Building Sustainability Index (BASIX) uses this as the benchmark to set a target for reducing water consumption by up to 40%.

A new building, planned to house 50 people, has been designed to meet a 25% reduction on this water consumption benchmark.

How much water per year, to the nearest kilolitre, is this building designed to save when fully occupied?

(A)   1129

(B)   1807

(C)   2710

(D)   3388

19 Jaz has 2 bags of apples.

Bag *A* contains 4 red apples and 3 green apples.

Bag *B* contains 3 red apples and 1 green apple.

Jaz chooses an apple from one of the bags.

Which tree diagram could be used to determine the probability that Jaz chooses a red apple?

(A)

(B)

(C)

(D)
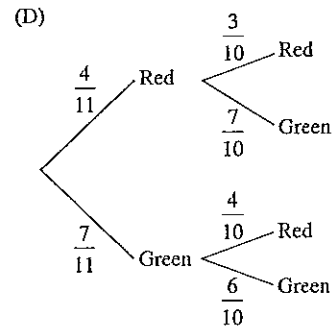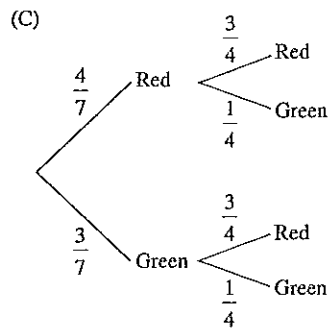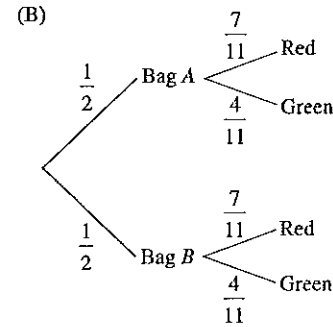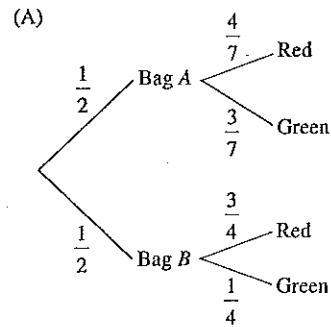
20 In a household of 4, each member uses an average of 13 minutes of hot water per day.

The household uses a 9 kW hot water unit.

Electricity is charged at 11.97c/kWh when the hot water unit is being used.

What is the electricity cost for the hot water used by this household in one week?

(A) $1.63

(B) $6.54

(C) $392.14

(D) $653.56

21 A table of future value interest factors is shown.

Table of future value interest factors

Period	Interest rate per period				
	1%	*2%*	*3%*	*4%*	*5%*
1	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0200	2.0300	2.0400	2.0500
3	3.0301	3.0604	3.0909	3.1216	3.1525
4	4.0604	4.1216	4.1836	4.2465	4.3101

A certain annuity involves making equal contributions of $25 000 into an account every 6 months for 2 years at an interest rate of 4% per annum.

Based on the information provided, what is the future value of this annuity?

(A) $50 500

(B) $51 000

(C) $103 040

(D) $106 162

22 Heather's car uses fuel at the rate of 6.6 L per 100 km for long-distance driving and 8.9 L per 100 km for short-distance driving.

She used the car to make a journey of 560 km, which included 65 km of short-distance driving.

Approximately how much fuel did Heather's car use on the journey?

(A) 37 L

(B) 38 L

(C) 48 L

(D) 50 L

23 The following information is given about the locations of three towns X, Y and Z:

- X is due east of Z
- X is on a bearing of 145° from Y
- Y is on a bearing of 060° from Z.

Which diagram best represents this information?

(A)

(B)

NOT TO SCALE

(C)

(D)
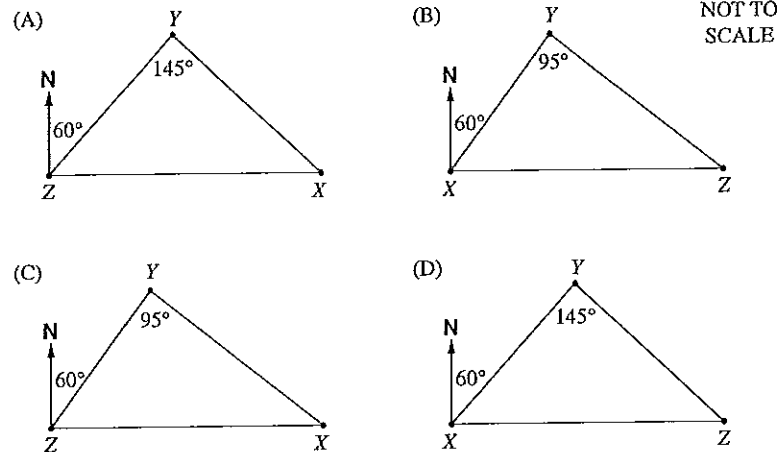
24 The weights of 10 000 newborn babies in NSW are normally distributed. These weights have a mean of 3.1 kg and a standard deviation of 0.35 kg.

How many of these newborn babies have a weight between 2.75 kg and 4.15 kg?

(A) 4985

(B) 6570

(C) 8370

(D) 8385

25 A grain silo is made up of a cylinder with a hemisphere (half a sphere) on top. The outside of the silo is to be painted.
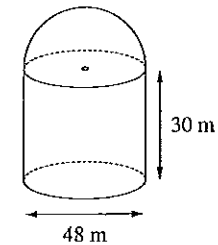
What is the area to be painted?

(A) 8143 m^2

(B) 11 762 m^2

(C) 12 667 m^2

(D) 23 524 m^2

Mathematics General 2

Section II

Centre Number

Student Number

75 marks
Attempt Questions 26–30
Allow about 1 hour and 55 minutes
for this section

Answer the questions in the spaces provided.

Your responses should include relevant mathematical reasoning and/or calculations.

Extra writing space is provided on pages 33 and 34. If you use this space, clearly indicate which question you are answering.

Write your Centre Number and Student Number at the top of this page.

Please turn over

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Question 26 (15 marks)

(a) Expand $4x\left(7x^4 - x^2\right)$.   **1**

(b) Calculate the value of h correct to two decimal places.   **2**

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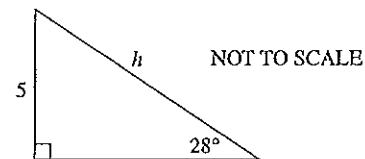
(c) Solve the equation $\dfrac{5x+1}{3} - 4 = 5 - 7x$.   **3**

Question 26 continues on page 15

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(d) Solve these simultaneous equations to find the values of x and y. **3**

$$y = 2x + 1$$
$$x - 2y - 4 = 0$$

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(e) The times taken for 160 music downloads were recorded, grouped into classes and then displayed using the cumulative frequency histogram shown. **2**

Time taken for download

Download time (seconds)
(Class centres)

On the diagram, draw the lines that are needed to find the median download time.

– 15 –

(f) The weight of an object on the moon varies directly with its weight on Earth. An astronaut who weighs 84 kg on Earth weighs only 14 kg on the moon. **2**

A lunar landing craft weighs 2449 kg when on the moon. Calculate the weight of this landing craft when on Earth.

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Question 26 (continued)

(g)   Singapore is located at 1°N 104°E and Sydney is located at 34°S 151°E.   2

What is the time difference between Singapore and Sydney? (Ignore daylight saving.)

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End of Question 26

Please turn over

Question 27 (15 marks)

(a)   Alex is buying a used car which has a sale price of $13 380. In addition to the sale price there are the following costs:

Transfer of registration $30
Stamp Duty   ....

(i)   Stamp Duty for this car is calculated at $3 for every $100, or part thereof, of the sale price.   1

Calculate the Stamp Duty payable.

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(ii) Alex borrows the total amount to be paid for the car including Stamp Duty and transfer of registration. Interest on the loan is charged at a flat rate of 7.5% per annum. The loan is to be repaid in equal monthly instalments over 3 years.   4

Calculate Alex's monthly repayments.

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(iii) Alex wishes to take out comprehensive insurance for the car for 12 months. The cost of comprehensive insurance is calculated using the following: **3**

Base rate	$845
Fire Service Levy (FSL)	1% of base rate
Stamp Duty	5.5% of the total of base rate and FSL
GST	10% of the total of base rate and FSL.

Find the total amount that Alex will need to pay for comprehensive insurance.

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(iv) Alex has decided he will take out the comprehensive car insurance rather than the less expensive non-compulsory third-party car insurance. **1**

What extra cover is provided by the comprehensive car insurance?

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(b) Xuso is comparing the costs of two different ways of travelling to university. **2**

Xuso's motorcycle uses one litre of fuel for every 17 km travelled. The cost of fuel is $1.67/L and the distance from her home to the university car park is 34 km. The cost of travelling by bus is $36.40 for 10 single trips.

Which way of travelling is cheaper and by how much? Support your answer with calculations.
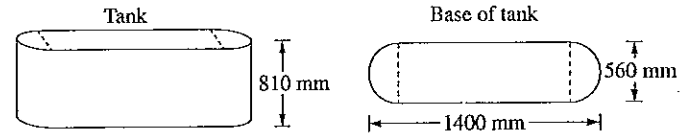
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(c) The base of a water tank is in the shape of a rectangle with a semicircle at each end, as shown. **4**

The tank is 1400 mm long, 560 mm wide, and has a height of 810 mm.

Tank — 810 mm

Base of tank — 560 mm — 1400 mm

NOT TO SCALE

What is the capacity of the tank, to the nearest litre?

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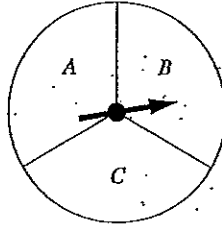
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Question 28 (15 marks)

(a) James plays a game involving a spinner with sectors of equal size labelled A, B and C, as shown.

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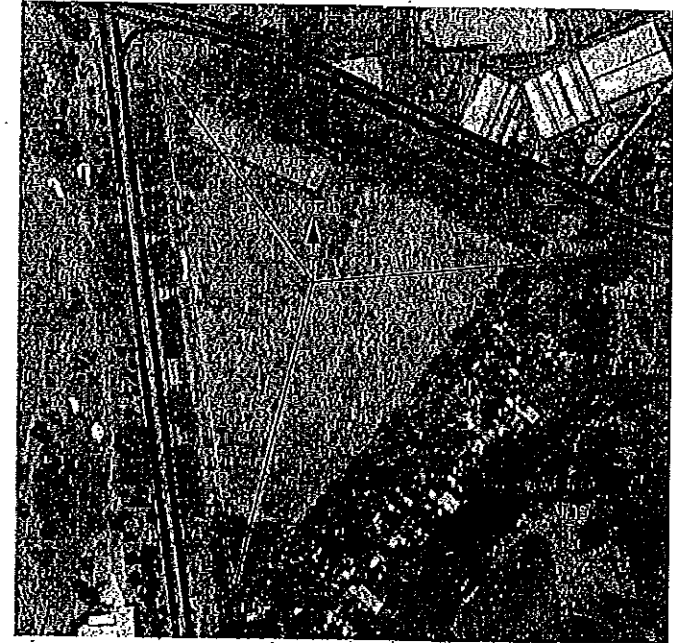He pays $2 to play the game. He wins $5 if the spinner stops in A and 50 cents if it stops in B or C.

Calculate James's financial expectation for the game.

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Question 28 (continued)

(b) A radial compass survey of a sports centre is shown in the diagram.

(i) Show that the size of angle AOB is 114°.

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(ii) Calculate the length of the boundary *AB*, to the nearest metre.　2

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(iii) Find the area of triangle *AOB* in hectares, correct to two significant figures.　3

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(c) A fair coin is tossed three times. Using a tree diagram, or otherwise, calculate the probability of obtaining two heads and a tail in any order.　2

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(d) An aerial diagram of a swimming pool is shown.
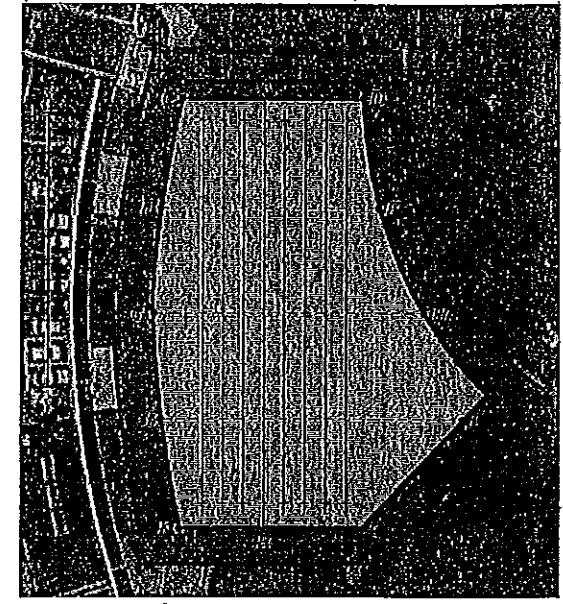
The swimming pool is a standard length of 50 metres but is not in the shape of a rectangle.

(i) By measuring the length *AB*, determine the scale of the diagram.　1

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$$1\,cm = \underline{\qquad}\ m$$

(ii) Using this scale, calculate the length *XY* of the car park, in metres.　1

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Question 28 (continued)

(iii) In the diagram of the swimming pool, the five widths are measured to be: **3**

$CD = 21.88$ m

$EF = 25.63$ m

$GH = 31.88$ m

$IJ = 36.25$ m

$KL = 21.88$ m

The average depth of the pool is 1.2 m.

Calculate the approximate volume of the swimming pool, in cubic metres. In your calculations, use TWO applications of Simpson's Rule.

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End of Question 28

Question 29 (15 marks)

(a) The cost of hiring an open space for a music festival is \$120 000. The cost will be shared equally by the people attending the festival, so that C (in dollars) is the cost per person when n people attend the festival.

(i) Complete the table below by filling in the THREE missing values.   **1**

Number of people (n)	500	1000	1500	2000	2500	3000
Cost per person (C)				60	48	40

(ii) Using the values from the table, draw the graph showing the relationship between n and C.   **2**
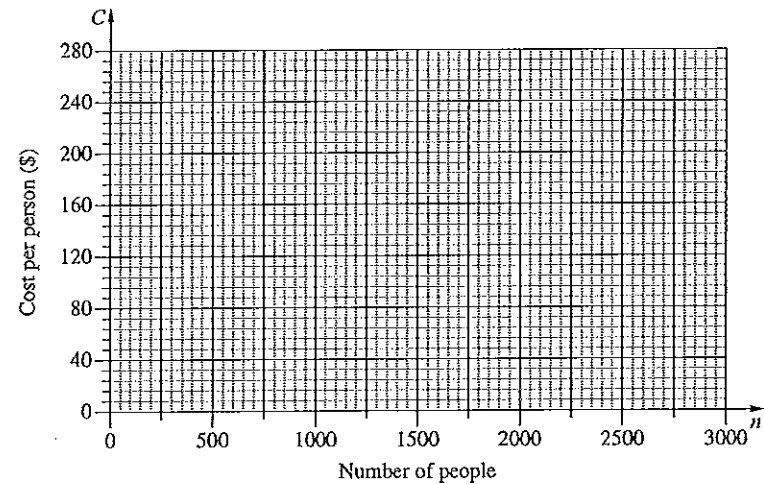
(iii) What equation represents the relationship between n and C?   **1**

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Question 29 (continued)

(iv) Give ONE limitation of this equation in relation to this context.　　　1

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(v) Is it possible for the cost per person to be $94? Support your answer with　　1
appropriate calculations.

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(b) What is the maximum number of standard drinks that a male weighing 84 kg　　3
can consume over 4 hours in order to maintain a blood alcohol content (BAC)
of less than 0.05?

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Question 29 (continued)

(c) Terry and Kim each sat twenty class tests. Terry's results on the tests are
displayed in the box-and-whisker plot shown in part (i).

(i) Kim's 5-number summary for the tests is 67, 69, 71, 73, 75.　　　1

Draw a box-and-whisker plot to display Kim's results below that of
Terry's results.
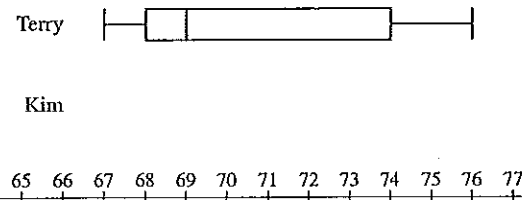
(ii) What percentage of Terry's results were below 69?　　　1

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(iii) Terry claims that his results were better than Kim's. Is he correct? Justify　　4
your answer by referring to the summary statistics and the skewness of
the distributions.

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End of Question 29

Question 30 (15 marks)

(a) Chandra and Sascha plan to have $20 000 in an investment account in 15 years   **3**
time for their grandchild's university fees.

The interest rate for the investment account will be fixed at 3% per annum
compounded monthly.

Calculate the amount that they will need to deposit into the account now in order
to achieve their plan.

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Question 30 (continued)

(b) The scatterplot shows the relationship between expenditure per primary school
student, as a percentage of a country's Gross Domestic Product (GDP), and the
life expectancy in years for 15 countries.

Expenditure per primary school student and life expectancy
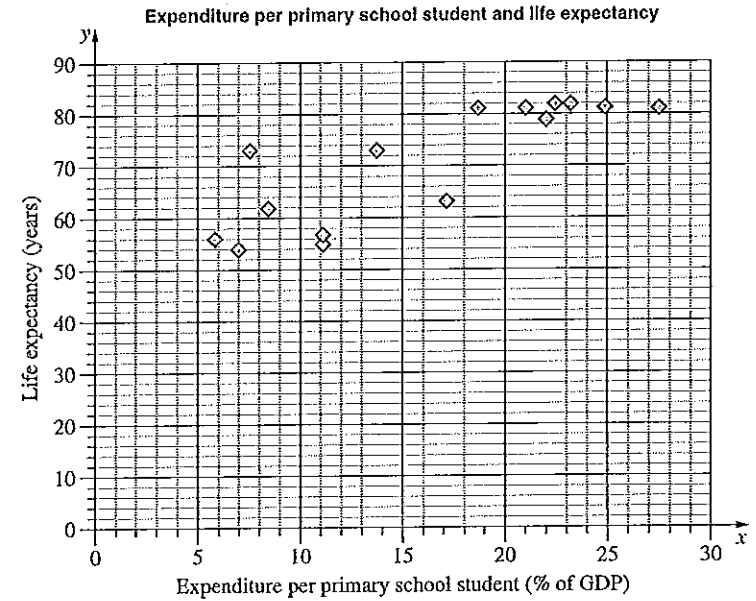
(i) For the given data, the correlation coefficient, r, is 0.83. What does this   **1**
indicate about the relationship between expenditure per primary school
student and life expectancy for the 15 countries?

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(ii) For the data representing expenditure per primary school student, Q_L is 8.4 and Q_U is 22.5.

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What is the interquartile range?

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(iii) Another country has an expenditure per primary school student of 47.6% of its GDP. Would this country be an outlier for this set of data? Justify your answer with calculations.

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(iv) The expenditures per primary school student for the 15 countries in the scatterplot are:

2

5.9, 7, 7.6, 8.4, 11.2, 11.2, 13.7, 17.1, 18.7, 21.1, 22, 22.5, 23.2, 24.9, 27.6

Complete the table below by calculating the mean, $\bar{x}$, and the standard deviation, σ_x, of these data. Calculate both values to two decimal places.

The table also shows the mean, $\bar{y}$, and the standard deviation, σ_y, of life expectancy for the same 15 countries.

	Mean	*Standard deviation*
Expenditure per primary school student	$\bar{x} =$	$\sigma_x =$
Life expectancy	$\bar{y} = 70.73$	$\sigma_y = 10.94$

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(v) Using the values from the table in part (iv), show that the equation of the least-squares line of best fit is

2

$$y = 1.\dot{2}9x + 49.9.$$

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(vi) On the scatterplot on page 30, draw the least-squares line of best fit, $y = 1.29x + 49.9$.

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(vii) Using this line, or otherwise, estimate the life expectancy in a country which has an expenditure per primary school student of 18% of its GDP.

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(viii) Why is this line NOT useful for predicting life expectancy in a country which has expenditure per primary school student of 60% of its GDP?

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Section II extra writing space

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Mathematics General 2

FORMULAE AND DATA SHEET

Financial Mathematics

Simple Interest

$$I = Prn$$

P Is initial amount

r Is interest rate per period, expressed as a decimal

n Is number of periods

Compound Interest

$$A = P(1+r)^n$$

A Is final amount

P Is initial amount

r Is interest rate per period, expressed as a decimal

n Is number of compounding periods

Present value and future value

$$PV = \frac{FV}{(1+r)^n}, \qquad FV = PV(1+r)^n$$

r Is interest rate per period, expressed as a decimal

n Is number of compounding periods

Straight-line method of depreciation

$$S = V_0 - Dn$$

S Is salvage value of asset after n periods

V_0 Is initial value of asset

D Is amount of depreciation per period

n Is number of periods

Declining-balance method of depreciation

$$S = V_0(1-r)^n$$

S Is salvage value of asset after n periods

V_0 Is initial value of asset

r Is depreciation rate per period, expressed as a decimal

n Is number of periods

Data Analysis

Mean of a sample

$$\bar{x} = \frac{\text{sum of scores}}{\text{number of scores}}$$

z-score

For any score x,

$$z = \frac{x - \bar{x}}{s}$$

$\bar{x}$ Is mean

s Is standard deviation

Outlier(s)

score(s) less than $Q_L - 1.5 \times IQR$

or

score(s) more than $Q_U + 1.5 \times IQR$

Q_L Is lower quartile

Q_U Is upper quartile

IQR Is interquartile range

Least-squares line of best fit

$$y = \text{gradient} \times x + y\text{-intercept}$$

$$\text{gradient} = r \times \frac{\text{standard deviation of } y \text{ scores}}{\text{standard deviation of } x \text{ scores}}$$

$$y\text{-intercept} = \bar{y} - (\text{gradient} \times \bar{x})$$

r Is correlation coefficient

$\bar{x}$ Is mean of x scores

$\bar{y}$ Is mean of y scores

Normal distribution

- approximately 68% of scores have z-scores between −1 and 1
- approximately 95% of scores have z-scores between −2 and 2
- approximately 99.7% of scores have z-scores between −3 and 3

Spherical Geometry

Circumference of a circle

$$C = 2\pi r \quad \text{or} \quad C = \pi D$$

r Is radius

D Is diameter

Arc length of a circle

$$l = \frac{\theta}{360} 2\pi r$$

r Is radius

θ Is number of degrees in central angle

Radius of Earth

(taken as) 6400 km

Time differences

For calculation of time differences using longitude:

$15° = 1$ hour time difference

Area

Circle

$$A = \pi r^2$$

r Is radius

Sector

$$A = \frac{\theta}{360} \pi r^2$$

r Is radius

θ Is number of degrees in central angle

Annulus

$$A = \pi(R^2 - r^2)$$

R Is radius of outer circle

r Is radius of inner circle

Trapezium

$$A = \frac{h}{2}(a+b)$$

h Is perpendicular height

a and b are the lengths of the parallel sides

Area of land and catchment areas

unit conversion: $1 \text{ ha} = 10\,000 \text{ m}^2$

Surface Area

Sphere

$$A = 4\pi r^2$$

r Is radius

Closed cylinder

$$A = 2\pi r^2 + 2\pi rh$$

r Is radius

h Is perpendicular height

Volume

Prism or cylinder

$$V = Ah$$

A Is area of base

h Is perpendicular height

Pyramid or cone

$$V = \frac{1}{3}Ah$$

A Is area of base

h Is perpendicular height

Volume and capacity

unit conversion: $1 \text{ m}^3 = 1000 \text{ L}$

Approximation Using Simpson's Rule

Area

$$A \approx \frac{h}{3}\left(d_f + 4d_m + d_l\right)$$

h Is distance between successive measurements

d_f Is first measurement

d_m Is middle measurement

d_l Is last measurement

Volume

$$V \approx \frac{h}{3}\left\{A_L + 4A_M + A_R\right\}$$

h Is distance between successive measurements

A_L Is area of left end

A_M Is area of middle

A_R Is area of right end

Trigonometric Ratios
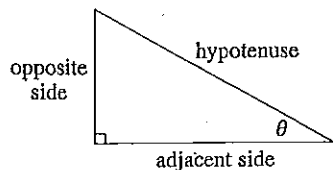
$$\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\tan \theta = \frac{\text{opposite side}}{\text{adjacent side}}$$

Sine rule

In $\triangle ABC$,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Area of a triangle

In $\triangle ABC$,

$$A = \frac{1}{2} ab \sin C$$

Cosine rule

In $\triangle ABC$,

$$c^2 = a^2 + b^2 - 2ab \cos C$$

or

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Units of Memory and File Size

1 byte = 8 bits

1 kilobyte = 2^{10} bytes = 1024 bytes

1 megabyte = 2^{20} bytes = 1024 kilobytes

1 gigabyte = 2^{30} bytes = 1024 megabytes

1 terabyte = 2^{40} bytes = 1024 gigabytes

Blood Alcohol Content Estimates

$$BAC_{male} = \frac{10N - 7.5H}{6.8M}$$

or

$$BAC_{female} = \frac{10N - 7.5H}{5.5M}$$

N is number of standard drinks consumed

H is number of hours of drinking

M is person's mass in kilograms

Distance, Speed and Time

$$D = ST, \quad S = \frac{D}{T}, \quad T = \frac{D}{S}$$

$$\text{average speed} = \frac{\text{total distance travelled}}{\text{total time taken}}$$

$$\text{stopping distance} = \left\{ \begin{array}{c} \text{reaction-time} \\ \text{distance} \end{array} \right\} + \left\{ \begin{array}{c} \text{braking} \\ \text{distance} \end{array} \right\}$$

Probability of an Event

The probability of an event where outcomes are equally likely is given by:

$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

Straight Lines

Gradient

$$m = \frac{\text{vertical change in position}}{\text{horizontal change in position}}$$

Gradient–intercept form

$$y = mx + b$$

m is gradient

b is y-intercept

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2014 Higher School Certificate Solutions
Mathematics General 2

SECTION I

Summary

1	D	6	D	11	A	16	D	21	C
2	B	7	D	12	C	17	B	22	B
3	C	8	A	13	B	18	A	23	C
4	B	9	C	14	B	19	A	24	D
5	B	10	A	15	B	20	B	25	A

SECTION I

1 (D) The call duration was 17 minutes. The graph indicates that this call would cost $16.

2 (B) $72 \times 1.2 \times 0.8 = 69\,\text{cm}$ (nearest cm).

3 (C) The graph looks like a parabola. The only equation on the list which is a parabola is $y = 3x^2 + 1$.

4 (B)
$$\text{Dosage for 6 hours} = \frac{1.5 \times 45}{1.5 + 12}$$
$$= 5\,\text{mL}$$
$$\therefore \text{Dosage for 24 hours} = 4 \times 5\,\text{mL}$$
$$= 20\,\text{mL}.$$

5 (B) From the Formulae Sheet,
$$1 \text{ gigabyte} = 2^{10} \text{ megabytes}$$
$$\therefore 2 \text{ gigabytes} = 2 \times 2^{10} \text{ megabytes}$$
$$= 2^{11} \text{ megabytes}$$
From the Formulae Sheet,
$$1 \text{ megabyte} = 2^{10} \text{ kilobytes}$$
$$\therefore 2^{11} \text{ megabytes} = 2^{11} \times 2^{10} \text{ kilobytes}$$
$$= 2^{21} \text{ kilobytes}.$$

6 (D) By multiplying the number of choices at each stage, the number of three-course meals is given by:
$3 \times 5 \times 2 = 30$.

7 (D) The line $y = 2x - 2$ has a positive gradient ($m = 2$), and a negative y-intercept ($b = -2$). Graph D is the only one with these properties.

8 (A) From the 150 people available for selection, there are 28 males who do not own a mobile.

9 (C) From the Formulae Sheet, declining balance is given by:
$$S = V_0(1 - r)^n$$
$$= 19990(1 - 0.18)^3$$
$$= \$11022 \text{ (nearest dollar)}.$$

10 (A) The measured height is $138.4\,\text{m} \pm 0.05\,\text{m}$ (to 1 decimal place)
$$\therefore \text{Percentage error} = \pm \frac{0.05}{138.4} \times 100\%$$
$$= \pm 0.036\%.$$

11 (A) $6w^4 \times \frac{1}{3}w^2 = \frac{6w^4 \times w^2}{3}$
$$= 2w^6.$$

12 (C) From the Formulae Sheet,
A = area of annulus
$$A = \pi(R^2 - r^2)$$
$$= \pi(4.5^2 - 3^2)$$
$$= 35.3 \text{ m}^2 (1 \text{ dp}).$$

13 (B)
$$6\% \text{ of } 2000 = 0.06 \times 2000$$
$$= 120$$
$$3.5\% \text{ of } 1000 = 0.035 \times 1000$$
$$= 35$$
$$5670 - 3000 = 2670$$
$$2\% \text{ of } 2670 = 0.02 \times 2670$$
$$= 53.40$$
$$\$120 + \$35 + \$53.40 = \$208.40.$$

14 (B)

Class centre	Frequency	
1	5	1 x 5 = 5
4	10	4 x 10 = 40
7	3	7 x 3 = 21
10	2	10 x 2 = 20
Total	20	86

From the Formulae Sheet,
$$\text{Mean} = \frac{\text{sum of scores}}{\text{number of scores}}$$
$$= \frac{86}{20}$$
$$= 4.3.$$

15 (B) From the Formulae Sheet,
Radius of Earth $= 6400\,\text{km}$
Distance is given by:
l = arc length of circle
$$l = \frac{\theta}{360} 2\pi r$$
$$= \frac{20 + 35}{360} \times 2 \times \pi \times 6400$$
$$= \frac{55}{360} \times 2 \times \pi \times 6400.$$

16 (D) Based on previous experience, the probability that it will rain on ANY day in Mathsville is given by:
$$P(R) = \frac{8}{31}$$
Therefore the probability of raining on any two consecutive days is:
$$P(RR) = \frac{8}{31} \times \frac{8}{31}.$$

17 (B)

Mass of child	Paracetamol	Medicine
2 kg	15 mg	
∴ 14 kg	105 mg	??? mL
	120 mg	10 mL
∴ 1 mg		$\frac{10}{120}$ mL
∴ 105 mg		8.75 mL

The dosage is 8.75mL.

18 (A)
$$1000 \text{ litres} = 1 \text{ kilolitre}$$
$$\therefore 90340 \text{ litres} = 90.34 \text{ kilolitres}$$
$$90.34 \text{ kilolitres per person per year}$$
$$90.34 \times 50 = 4517\,\text{kL per year}$$
$$25\% \text{ of } 4517 = 1129 \text{ kL per year}$$
(nearest kL).

19 (A) Firstly Jaż must choose a bag. The two bags are equally likely to be chosen, so:
$$P(A) = \frac{1}{2} \text{ and } P(B) = \frac{1}{2}$$
Bag A contains 7 apples, so
$$P(\text{Red}) = \frac{4}{7} \text{ and } P(\text{Green}) = \frac{3}{7}$$
Bag B contains 4 apples, so
$$P(\text{Red}) = \frac{3}{4} \text{ and } P(\text{Green}) = \frac{1}{4}$$
The only tree diagram with this information is A.

20 (B) 4 people for 13 minutes each for 7 days per week
$$4 \times 13 \times 7 = 364 \text{ minutes}$$
Number of hours:
$$364 \div 60 = 6.0666\ldots$$
$$\text{Cost} = 9 \times 6.066\ldots \times \$0.1197$$
$$= \$6.54 \text{ (nearest cent)}.$$

21 (C)
$$2 \text{ years} = 4 \text{ periods}$$
$$\text{Interest rate} = 4\% \text{ per annum}$$
$$= 2\% \text{ per half year}$$
$$\text{Future value} = 4.1216 \times \$25000$$
$$= \$103040.$$

22 (B) Of the 560 km driven, 65 km was short-distance and the remainder, 495 km, was long-distance.

Long distance:

$$\text{Consumption} = \frac{495}{100} \times 6.6$$

$$= 32.67$$

Short distance:

$$\text{Consumption} = \frac{65}{100} \times 8.9$$

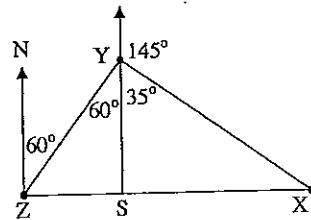$$= 5.785$$

$$\text{Total litres used} = 32.67 + 5.785$$

$$= 38.455$$

$$= 38 \text{ (nearest litre)}.$$

23 (C) X is due East of Z, so options B and D are incorrect.

In the diagram below:
- alternate angles in parallel lines have been used to show that angle ZYS is 60 degrees.
- YS is a straight line, so angle SYX is 35 degrees

Therefore angle ZYX = 95 degrees.

24 (D) Given that the mean is 3.1 kg and the standard deviation is 0.35 kg, the z-scores are:

-3	-2	-1	0	1	2	3
na	na	2.75	3.1	3.45	3.8	4.15

Therefore we require the number of babies with z-scores from -1 to 3.

Method 1:

From the Formulae Sheet,
Approximately 99.7% of scores have z-scores between -3 and 3.
By symmetry, the percentage of scores which have z-scores between 0 and 3 is given by:
$99.7\% \div 2 = 49.85\%$

Also, approximately 68% of scores have z-scores between -1 and 1.
By symmetry, the percentage of scores which have z-scores between -1 and 0 is given by:
$68\% \div 2 = 34\%$

Therefore the required percentage is $49.85\% + 34\% = 83.85\%$
So the number of newborns is:
83.85% of $10\,000 = 8\,385$.

Method 2:
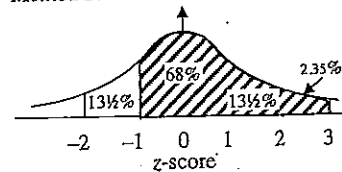
We require the shaded percentage
$68\% + 13.5\% + 2.35\% = 83.85\%$
So the number of newborns is:
83.85% of $10\,000 = 8\,385$.

25 (A) From the Formulae Sheet:

Surface area of sphere is $4\pi r^2$

$\therefore$ Surface area of hemisphere is $2\pi r^2$

Surface area of curved surface of closed cylinder is $2\pi rh$

$\therefore$ Total area to be painted is:

$$A = 2\pi r^2 + 2\pi rh$$

$$= 2 \times \pi \times 24^2 + 2 \times \pi \times 24 \times 30$$

$$= 2592\pi$$

$$= 8143 \, \text{m}^2 \text{ (nearest square metre)}.$$

SECTION II

Question 26

(a) $4x(7x^4 - x^2) = 28x^5 - 4x^3$.

(b)   Using the sine ratio,

$$\sin 28° = \frac{5}{h}$$

$$h = \frac{5}{\sin 28°}$$

$$= 10.6502......$$

$$= 10.65 \text{ (2 dp)}.$$

(c)

$$\frac{5x+1}{3} - 4 = 5 - 7x$$

$$\frac{5x+1}{3} = 9 - 7x$$

$$5x + 1 = 27 - 21x$$

$$26x = 26$$

$$x = 1.$$

(d) $y = 2x + 1$ ...(1)

$x - 2y - 4 = 0$ ...(2)

Substitute (1) into (2):

$x - 2 \times (2x+1) - 4 = 0$

$x - 4x - 2 - 4 = 0$

$-3x = 6$

$x = -2$

Substitute $x = -2$ into (1):

$y = 2x + 1$

$y = 2 \times (-2) + 1$

$= -3$

$\therefore x = -2, y = -3.$

(e)

Time taken for download

(f)   Let M be the weight on the moon.
Let E be the weight on Earth.
M varies directly with E.

$\therefore$ $M = k \times E$

$14 = k \times 84$

$84k = 14$

$k = \frac{14}{84}$

$= \frac{1}{6}$

$\therefore M = \frac{1}{6} \times E$

$2449 = \frac{1}{6} \times E$

$\frac{E}{6} = 2449$

$E = 2449 \times 6$

$= 14\,694 \, \text{kg}$

The weight of the landing craft when on Earth is 14 694 kg.

(g)   Singapore and Sydney are both East of the prime meridian.
Difference in longitude is given by:
$151° - 104° = 47°$

Method 1:
15 degrees of longitude makes a 1 hour time difference, so:
$47 \div 15 = 3.133333....\text{hours}$
$= 3 \text{ hours } 8 \text{ minutes}.$

Method 2:
1 degree of longitude makes a 4 minute time difference, so:
$47 \times 4 = 188 \text{ minutes}$
$= 3 \text{ hours } 8 \text{ minutes}.$

Question 27

(a) (i)   The phrase 'every hundred dollars or part thereof' implies that 13380 needs to be rounded up to 13400, which is 134 hundreds.
$\therefore$ Stamp Duty $= 134 \times \$3$
$= \$402.$

(ii) Total cost $= \$13380 + \$30 + \$402$

$= \$13812$

$\text{Interest} = \dfrac{7.5}{100} \times \$13812 \times 3$

$= \$3107.70$

Total amount to be repaid:

$\$13812 + \$3107.70 = \$16\,919.70$

There are 36 months in 3 years, so the monthly repayments will be:

$\$16\,919.70 \div 36 = \$469.99$ (nearest cent).

(iii) FSL:

1% of $\$845 = \dfrac{1}{100} \times \$845$

$= \$8.45$

Base rate + FSL:

$\$845 + \$8.45 = \$853.45$

Stamp Duty:

$\dfrac{5.5}{100} \times \$853.45 = \$46.94$ (nearest cent)

GST:

$\dfrac{10}{100} \times \$853.45 = \$85.35$ (nearest cent)

Total amount:

$\$853.45 + \$46.94 + \$85.35 = \$985.74$.

Note: In the real world, these amounts are rounded off to the nearest cent before they are added together, as has been done above. If the unrounded amounts were added, the total amount would be $985.73.

(iv) Comprehensive car insurance also covers:
- damage to the insured car
- theft of the insured car.

(b) **Motorcycle:**
The cost of fuel for 1 trip can be calculated as follows:
17 km requires 1 L which costs $1.67.

Therefore 34 km requires 2 L which costs $3.34.

Note: There may also be other costs such as parking, tolls and fines.

Bus:
$\$36.40 \div 10 = \$3.64$

Therefore, motorcycle is cheaper by $3.64 - 3.34 = \$0.30$

$= 30$ cents.

(c) The base of the tank is composed of a rectangle and two semi-circles (calculate these as a full circle).

Length of the rectangle:
$1400 - 560 = 840$

Length $= 840$ mm

$= 0.84$ m

Area of the rectangle:
Area $= 0.84 \times 0.56$

$= 0.4704 \text{ m}^2$

Radius of the circle:
Radius $= 560 \text{ mm} \div 2$

$= 280$ mm

$= 0.28$ m

Area of the circle:
$A = \pi \times r^2$

$= \pi \times 0.28^2$

Area (in square metres) of the base of the tank:
$0.4704 + \pi \times 0.28^2 = 0.7167......$

Volume (in cubic metres) of the tank:
area of base $\times$ height $= 0.7167.... \times 0.81$

$= 0.580527.....$

Every cubic metre holds 1000 L, so the capacity of the tank (in litres) is given by:
$0.580527..... \times 1000 = 580.527.....$

Therefore the capacity of the tank is 581 litres (to the nearest litre).

Question 28

(a)

Outcome	Bet	Collect	Profit	Probability
A	$2	$5	+$3	$\frac{1}{3}$
B or C	$2	$0.50	-$1.50	$\frac{2}{3}$

Method 1:
Financial Expectation:

$\$3 \times \dfrac{1}{3} + (-\$1.50) \times \dfrac{2}{3} = \$0$

∴ James expects to break even in the long run.

Method 2:
Expected return:

$\$5 \times \dfrac{1}{3} + \$0.50 \times \dfrac{2}{3} - \$2 = \$0$

∴ James expects to break even in the long run.

(b) (i) $\angle AOB = (360° - 320°) + 74°$

$= 40° + 74°$

$= 114°$.

(ii) From the Formulae Sheet, the cosine rule states that:

$c^2 = a^2 + b^2 - 2ab\cos C$

Therefore:

$AB^2 = 287^2 + 211^2 - 2 \times 287 \times 211 \times \cos 114°$

$= 176151.501.......$

∴ $AB = \sqrt{176151.501.......}$

$= 419.704...$

$= 420$ m (nearest metre).

(iii) From the Formulae Sheet, the area of a triangle is:

$A = \dfrac{1}{2} ab \sin C$

$A = \dfrac{1}{2} \times 287 \times 211 \times \sin 114°$

$= 27660.786...$ square metres

1 ha = 10 000 square metres, so the required area is given by:
$27660.786... \div 10000 = 2.7660786...$

Therefore the required area is 2.8 hectares, correct to two significant figures.

(c) Using the tree diagram or ordered list below, there are eight outcomes. They are equally likely. Three of the eight outcomes contain two heads and a tail.

Therefore the required probability is $\dfrac{3}{8}$.

Method 1: Tree diagram
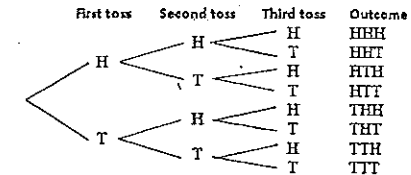
Method 2: Ordered list of outcomes

First toss	Second toss	Third toss	Outcome
H	H	H	HHH
H	H	T	HHT
H	T	H	HTH
H	T	T	HTT
T	H	H	THH
T	H	T	THT
T	T	H	TTH
T	T	T	TTT

(d) (i) By measurement, AB is approximately 8 cm long on the diagram, so 8 cm represents 50 m.
∴ 1 cm represents 6.25 metres.

(ii) By measurement, XY is approximately 5.1 cm long on the diagram. Using the scale factor gives
$5.1 \times 6.25 = 31.875$
So XY is approximately 31.875 m.

(iii) From the Formulae Sheet, Simpson's Rule states that:

$A \approx \dfrac{h}{3}(d_F + 4d_M + d_L)$

From the information provided,
$h = 50 \div 4 = 6.25$

Area of $CDFHGE$ is given by:

$$A \approx \frac{12.5}{3}(21.88 + 4 \times 25.63 + 31.88)$$

Area of $GHJLKI$ is given by:

$$A \approx \frac{12.5}{3}(31.88 + 4 \times 36.25 + 21.88)$$

∴ Total area (in sq. metres) is given by:

$$651.166... + 828.166..... = 1479.333....$$

The average depth is 1.2 metres, so the volume can be approximated using the formula

$V \approx$ approx. area of surface × ave. depth
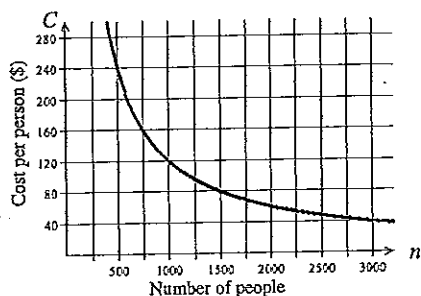
$$\approx 1479.33.... \times 1.2$$

$$= 1775.2$$

So the volume is approximately 1775.2 cubic metres.

Question 29

(a) (i)

n	500	1000	1500	2000	2500	3000
C	240	120	80	60	48	40

(ii)

(iii) The cost per person is equal to $120000 divided by the number of people, so:

$$C = \frac{120000}{n}$$

OR

$$n \times C = 120000$$

OR

$$nC = 120000.$$

(iv) There might be a restriction on the total number of people that can fit in the open space.

(v) Let $C = 94$ in $C = \dfrac{120000}{n}$

$$94 = \frac{120000}{n}$$

$$n = \frac{120000}{94}$$

$$= 1276.5957......$$

Since n represents the number of people it must be a whole number, but 1276.5957... is not a whole number so the cost per person can't be $94.

(b)

$$BAC_{male} = \frac{10N - 7.5H}{6.8M}$$

$$0.05 = \frac{10N - 7.5 \times 4}{6.8 \times 84}$$

$$\frac{10N - 7.5 \times 4}{571.2} = 0.05$$

$$10N - 30 = 28.56$$

$$10N = 58.56$$

$$N = 5.856$$

∴ The maximum whole number of standard drinks that can be consumed is 5. The 6th drink, if consumed completely, will put his BAC over 0.05.

(c) (i)
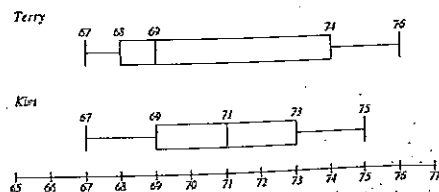
(ii) The graph indicates that Terry's median score is 69. If he did not score 69 in any tests, exactly 50% of his scores were below 69.

Note: It is possible that Terry may

have scored 69 in one or more tests, so it is impossible to give a definite answer to this question.
For example, Terry's scores may have been as follows:

67	67	67	67	68
68	69	69	69	69
69	69	69	69	73
75	75	75	75	76

In this example only 5 of the 20 scores are below 69. This is 25% of the scores.

Note: The following solution assumes that all of the test results are whole numbers.

(iii) Terry and Kim both scored 67 as their lowest result. Terry's highest result was 76 and Kim's was 75. Terry's five highest results (on the right whisker) range from 74 to 76. They appear to be slightly better than Kim's five highest results, which range from 73 to 75, but it is impossible to be sure unless we see the raw data.

It is possible that Terry's five best results were: 74, 74, 74, 74, 76 and Kim's were: 75, 75, 75, 75, 75.

It is also possible that Terry's five best results were 76, 76, 76, 76, 76 and Kim's were 73, 73, 73, 73, 75

Kim's median mark (71) was greater than Terry's (69). Terry's marks have a positive skew, whereas Kim's appear to be symmetrical. At the lower end, Terry's median (69) is equal to Kim's lower quartile, Kim scored fewer marks than Terry in the 67 to 69 range. It is possible that 75% of Kim's marks are above 69 but only 50% of Terry's are above 69.

Based on the available evidence, it appears that Kim's results are slightly better than Terry's, although Terry did score one or more result of

76, which was higher than Kim's highest result of 75. More evidence is required to make a definite judgement. It would be helpful to know the results Kim and Terry scored on every test.

Note: It is possible that Terry and Kim scored the same total number of marks (1420) and the same result on 13 of the 20 tests.

It is also possible that Terry scored 1426 marks and Kim scored 1408. It is also possible that Terry scored 1399 and Kim scored 1432.

Question 30

(a) *Method 1:*
Formula for compund interest from the Formulae Sheet,

$$A = P(1 + r)^n,$$

where $A = 20000$

and $r = 3 \div 100 \div 12 = 0.0025$

and $n = 15 \times 12 = 180$

Thus $20000 = P(1 + 0.0025)^{180}$

$$P(1 + 0.0025)^{180} = 20000$$

$$P = \frac{20000}{(1.0025)^{180}}$$

$$P = 12759.726......$$

Therefore they should invest $12 759.73 now.

Method 2:
Formula for present value From the Formulae Sheet,

present value is given by:

$$PV = \frac{FV}{(1 + r)^n},$$

where $FV = 20000$

and $r = 3 \div 100 \div 12 = 0.0025$

and $n = 15 \times 12 = 180$

$$PV = \frac{20000}{(1.0025)^{180}}$$

$$= 12759.726......$$

They should invest $12 759.73 now

(b) (i) The number 0.83 is positive, which indicates that the countries with higher expenditure tend to have higher life expectancy.

Also, on a scale from 0 to 1, the number 0.83 is close to 1, which indicates that most of the markers on the scatterplot will be quite close to the least-squares line of best fit.

(ii) $IQR = Q_U - Q_L$
$= 22.5 - 8.4$
$= 14.1$.

(iii) From the Formulae Sheet, outliers are scores greater than
$Q_U + 1.5 \times IQR$
$22.5 + 1.5 \times 14.1 = 43.65$
Since 47.6 is greater than 43.65, this country is an outlier.

(iv) By calculator,
$\bar{x} = 16.14$
$\sigma_x = 7.0274......$
$= 7.03 \text{ (2 dp)}$.

(v) From the Formulae Sheet, the gradient of the least squares line of best fit is given by:
$$\text{gradient} = r \times \frac{\text{standard dev. of } y \text{ scores}}{\text{standard dev. of } x \text{ scores}}$$
$$= 0.83 \times \frac{10.94}{7.03}$$
$$= 1.2916......$$
$$= 1.29 \text{ (2 dp)}$$

From the Formulae Sheet, the y-intercept of the least squares line of best fit is given by:
y-int. $= \bar{y} - (\text{gradient} \times \bar{x})$
$= 70.73 - (1.2916.... \times 16.14)$
$= 49.8829......$
$= 49.9 \text{ (2 dp)}$

From the Formulae Sheet, the gradient-intercept form of a straight line is:

$y = mx + b$
where:
the gradient is $m = 1.29$
the y-intercept is $b = 49.9$
$\therefore y = 1.29x + 49.9$, as required.

(vi) See the graph at the end.
Instructions for students:
- Mark the point $(0, 49.9)$
- Choose a value for x such as $x = 20$ and substitute into $y = 1.29x + 49.9$, which gives
$y = 1.29 \times 20 + 49.9$
$y = 75.7$
- Plot the point $(20, 75.7)$
- Draw a line from $(0, 49.9)$ through $(20, 75.7)$.

(vii) *Method 1: Using the graph*
The arrows on the graph indicate the life expectency would be approximately 74 years.

Method 2: Using the equation
Substituting $x = 18$ into
$y = 1.29x + 49.9$ gives
$y = 1.29 \times 18 + 49.9$
$y = 73.12$
so the life expectency would be approximately 73 years.

(viii) Substituting $x = 60$ into
$y = 1.29x + 49.9$ gives
$y = 1.29 \times 60 + 49.9$
$y = 127.3$
so the life expectency is estimated to be 127 years, which is not realistic.

Graph for Question 30(b):

Expenditure per primary school student and life expectancy
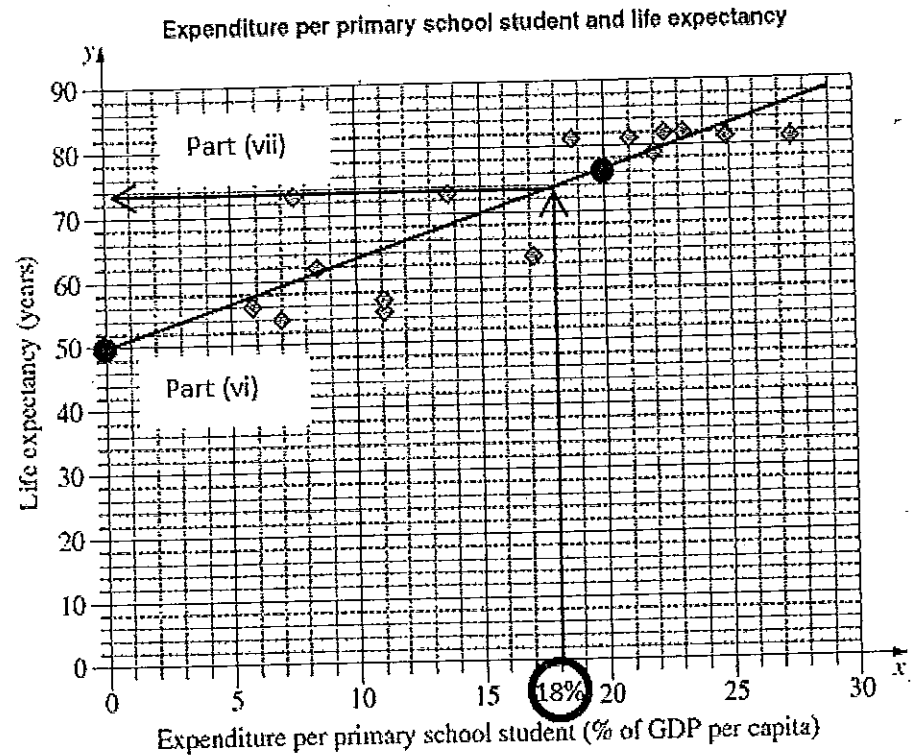
End of Mathematics General 2 solutions