

2006 HIGHER SCHOOL CERTIFICATE EXAMINATION PAPER GENERAL MATHEMATICS

Section I

22 marks

Attempt Questions 1 – 22

Allow about 30 minutes for this section

- 1 The probability of an event occurring is $\frac{9}{10}$.

Which statement best describes the probability of this event occurring?

- (A) The event is likely to occur. (B) The event is certain to occur.
(C) The event is unlikely to occur. (D) The event has an even chance of occurring.

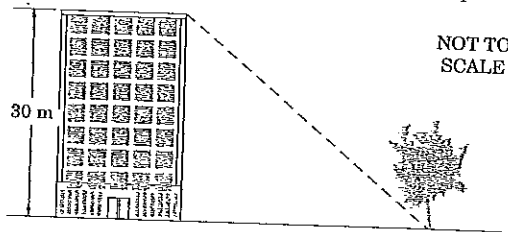
- 2 If $V = \frac{4}{3}\pi r^3$, what is the value of V when $r = 2$, correct to two decimal places?

- (A) 8.38 (B) 12.57 (C) 25.13 (D) 33.51

- 3 The angle of depression of the base of the tree from the top of the building is 65° . The height of the building is 30 m.

How far away is the base of the tree from the building, correct to one decimal place?

- (A) 12.7 m
(B) 14.0 m
(C) 33.1 m
(D) 64.3 m



- 4 A set of scores is displayed in a stem-and-leaf plot.

What is the median of this set of scores?

- (A) 28 (B) 30
(C) 33 (D) 47

1	2	2	3
2	5	8	
3	8	9	
4	1	3	9

- 5 A salesman earns \$200 per week plus \$40 commission for each item he sells.

How many items does he need to sell to earn a total of \$2640 in two weeks?

- (A) 33 (B) 56 (C) 61 (D) 66

- 6 Marcella is planning to roll a standard six-sided die 60 times.

How many times would she expect to roll the number 4?

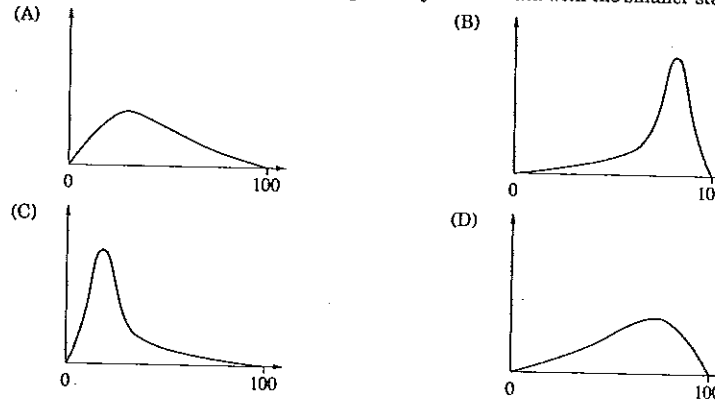
- (A) 6 (B) 10 (C) 15 (D) 20

- 7 Which equation represents the relationship between x and y in this table?

x	0	2	4	6	8
y	1	2	3	4	5

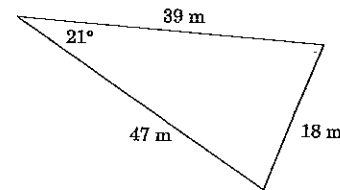
- (A) $y = 2x + 1$ (B) $y = 2x - 2$ (C) $y = \frac{x}{2} - 2$ (D) $y = \frac{x}{2} + 1$

- 8 Which of these graphs best represents positively skewed data with the smaller standard deviation?



- 9 What is the area of this triangle, to the nearest square metre?

- (A) 152 m²
(B) 283 m²
(C) 328 m²
(D) 351 m²



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- 10 Kay randomly selected a marble from a bag of marbles, recorded its colour and returned it to the bag. She repeated this process a number of times.

Colour	Tally	Frequency
Red		7
Blue		3
Yellow		2
Green		4
Purple		8

Based on these results, what is the best estimate of the probability that Kay will choose a green marble on her next selection?

- (A) $\frac{5}{24}$ (B) $\frac{1}{24}$ (C) $\frac{1}{6}$ (D) $\frac{1}{5}$

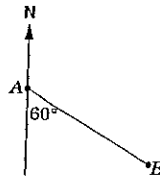
- 11 Peter rides his bike at a speed of 27 km/h. What is this speed in m/s?

- (A) 7.5 (B) 18.75 (C) 97.2 (D) 450

- 12 The mean of a set of 5 scores is 62.
What is the new mean of the set of scores after a score of 14 is added?
(A) 38 (B) 54 (C) 62 (D) 76

- 13 What is the bearing of A from B?

- (A) 060°
(B) 120°
(C) 150°
(D) 300°



- 14 In 2004 there were 13.5 million registered motor vehicles in Australia. The number of registered motor vehicles is increasing at a rate of 2.3% per year.
Which expression represents the number (in millions) of registered motor vehicles, if y represents the number of years after 2004?

- (A) $13.5 \times (1.023)^y$ (B) $13.5 \times (0.023)^y$ (C) $13.5 \times (1.023) \times y$ (D) $13.5 \times (0.023) \times y$

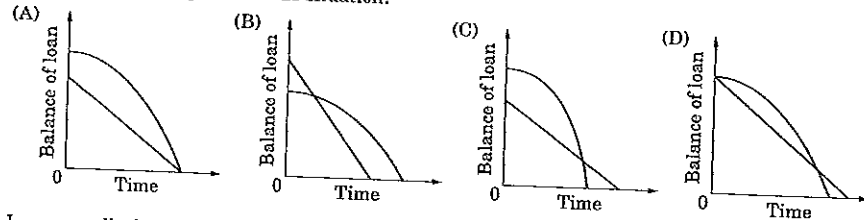
- 15 Two people are to be selected from a group of six people to form a team.
How many different teams can be formed?

- (A) 15 (B) 18 (C) 30 (D) 36

- 16 Two families borrow different amounts of money on the same day.

The Wang family has a flat rate loan. The Salama family has a reducing balance loan and repays the loan earlier than the Wang family.

Which graph best represents this situation?



- 17 In a normally distributed set of scores, the mean is 23 and the standard deviation is 5.
Approximately what percentage of the scores will lie between 18 and 33?

- (A) 34% (B) 47.5% (C) 68% (D) 81.5%

- 18 What is the formula for q as the subject of $4p = 5t + 2q^2$?

- (A) $q = \pm \frac{\sqrt{5t - 4p}}{2}$ (B) $q = \pm \frac{\sqrt{4p - 5t}}{2}$ (C) $q = \pm \frac{\sqrt{5t - 4p}}{2}$ (D) $q = \pm \frac{\sqrt{4p - 5t}}{2}$

- 19 Makoua and Macapá are two towns on the equator. The longitude of Makoua is 16°E and the longitude of Macapá is 52°W .

How far apart are these two towns if the radius of Earth is approximately 6400 km?

- (A) 4000 km (B) 7600 km (C) 1 447 600 km (D) 2 734 400 km

- 20 The radius of a sphere is increased by 10%.

What is the percentage increase in its surface area?

- (A) 10% (B) 20% (C) 21% (D) 33%

- 21 Bill borrows \$420 000 to buy a house. Interest is charged at 7.2% per annum, compounded monthly.

How much does he owe at the end of the first month, after he has made a \$4000 repayment?

- (A) \$418 496 (B) \$418 520 (C) \$445 952 (D) \$446 240

- 22 This income tax table is used to calculate Evelyn's tax payable.

Taxable income	Tax payable
\$0 – \$20 000	Nil
\$20 001 – \$45 000	Nil plus 10 cents for each \$1 over \$20 000
\$45 001 – \$70 000	\$2500 plus 35 cents for each \$1 over \$45 000
\$70 001 and above	\$11 250 plus 52 cents for each \$1 over \$70 000

Evelyn's taxable income increases from \$50 000 to \$80 000.

What percentage of her increase will she pay in additional tax?

- (A) 15.25% (B) 40.7% (C) 43.5% (D) 52%

Section II

78 marks

Attempt Questions 23 – 28

Allow about 2 hours for this section

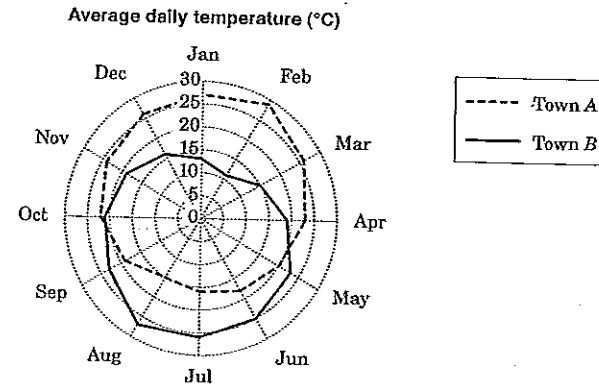
Question 23 (13 marks)

Marks

(a) Simplify $\frac{ab^2}{w} \times \frac{4w}{3b}$.

2

- (b) This radar chart was used to display the average daily temperatures each month for two different towns.



- (i) What is the average daily temperature of Town B for April? 1
(ii) In which month do the average daily temperatures of the two towns have the greatest difference? 1
(iii) In which months is the average daily temperature in Town B higher than in Town A? 1

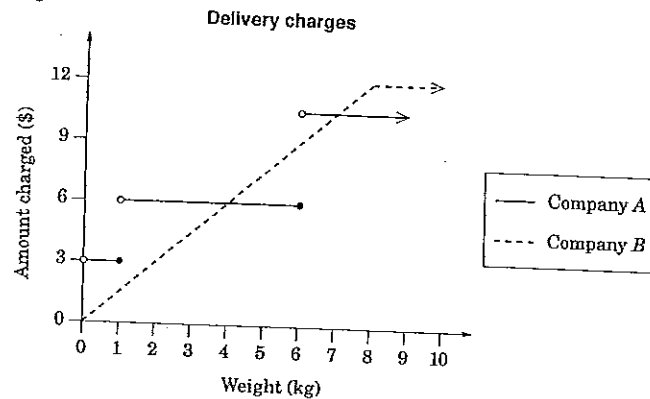
- (c) Vicki wants to investigate the number of hours spent on homework by students at her high school.
(i) Briefly describe a valid method of randomly selecting 200 students for a sample. 1

- (ii) Vicki chooses her sample and asks each student how many hours (to the nearest hour) they usually spend on homework during one week. The responses are shown in the frequency table.

Number of hours spent on homework in a week	Frequency
0 to 4	69
5 to 9	72
10 to 14	38
15 to 19	21

What is the mean amount of time spent on homework?

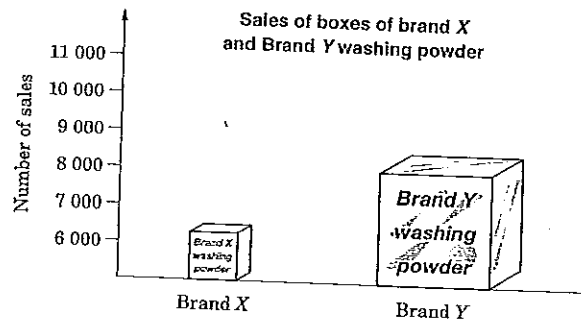
- (d) The graph shows the amounts charged by Company A and Company B to deliver parcels of various weights.



- (i) How much does Company A charge to deliver a 3 kg parcel?
 (ii) Give an example of the weight of a parcel for which both Company A and Company B charge the same amount.
 (iii) For what weight(s) is it cheaper to use Company A?
 (iv) What is the rate per kilogram charged by Company B for parcels up to 8 kg?

Question 24 (13 marks)

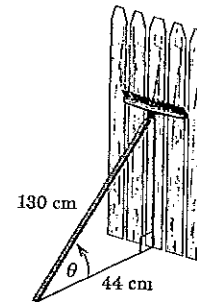
(a)



List TWO ways in which this graph is misleading.

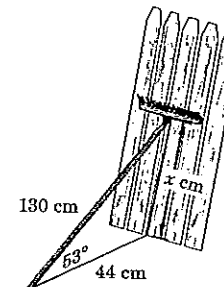
- (b) A 130 cm long garden rake leans against a fence. The end of the rake is 44 cm from the base of the fence.

(i) If the fence is vertical, find the value of θ to the nearest degree.



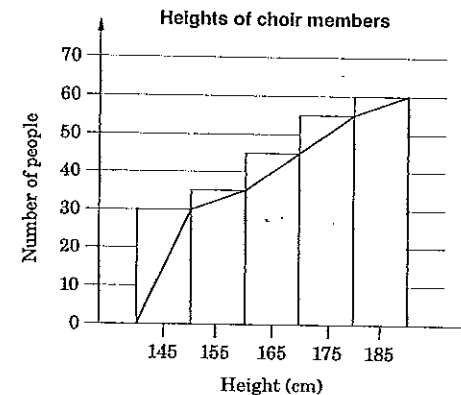
NOT TO SCALE

- (ii) The fence develops a lean and the rake is now at an angle of 53° to the ground. Calculate the new distance (x cm) from the base of the fence to the head of the rake. Give your answer to the nearest centimetre.



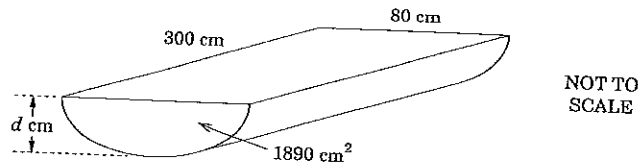
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- (c) The heights of the 60 members of the choir were recorded. These results were grouped and then displayed as a cumulative frequency histogram and polygon. The shortest person in the choir is 140 cm and the tallest is 190 cm.



Draw an accurate box-and-whisker plot to represent the data.

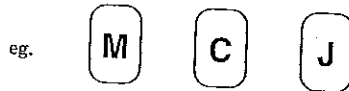
- (d) A water trough has a uniform cross-section in the shape of half an ellipse. The width of the trough is 80 cm and the area of the shaded cross-section is 1890 cm².



- (i) The trough is 300 cm long. Calculate the number of litres of water the trough will hold when it is full. 2
- (ii) Calculate the depth of the trough (d cm). Give your answer to the nearest centimetre. 2

Question 25 (13 marks)

- (a) Three cards labelled C , J and M can be arranged in any order.

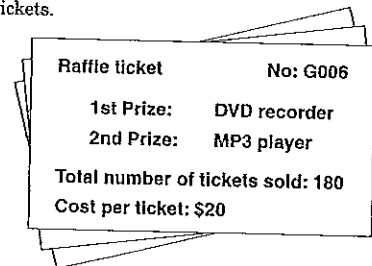


- (i) In how many different ways can the cards be arranged? 1
- (ii) What is the probability that the second card in an arrangement is a J ? 1
- (iii) What is the probability that the last card in an arrangement is not a C ? 1
- (b) In June, Ms Bigspender received a statement for her credit card account. The account has no interest-free period. Simple interest is calculated and charged to her account on the statement date.

Ms Ima Bigspender		Sum Bank	
Credit limit: \$2000		Credit Card Statement	
Statement date: 20 June 2006			
Previous balance	Payments	Purchases	Interest charged
\$263.83	\$263.83	\$617.72	
Date	Purchases	Amount	
23 May	Concert tickets	\$617.72	
Annual percentage rate: 18.2%			
Daily percentage rate: 0.0498%			
Note: Interest is charged on amounts from (and including) the date of purchase up to (and including) the statement date.			

- (i) For how many days is she charged interest on her purchase? 1
- (ii) Calculate the interest charged to her account. 2

- (c) Sonia buys three raffle tickets.



- (i) What is the probability that Sonia wins first prize? 1
- (ii) What is the probability that she wins both prizes? 2
- (d) Paul invested money in a bank for 4 years. The stated interest rate on the account was 6.1% per annum *compounded annually*. This is equivalent to an effective simple interest rate of 6.68% per annum. 4

The formula Paul used to calculate the effective simple interest rate was:

$$E = \frac{(1+r)^n - 1}{n}$$

where r is the stated interest rate per period (expressed as a decimal)

E is the effective simple interest rate per period (expressed as a decimal)

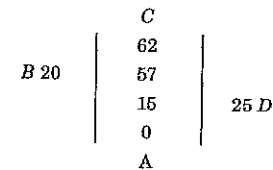
n is the number of periods

Martha invested money in a different bank for 4 years. The stated interest rate on her investment was 6% per annum *compounded monthly*.

Martha thinks that she has a better deal than Paul. Do you agree? Justify your answer by comparing their effective simple interest rates.

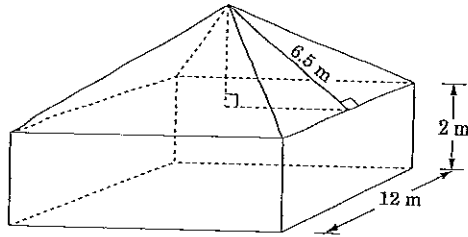
Question 26 (13 marks)

- (a) Daniel conducts an offset survey to sketch a diagram, $ABCD$, of a block of land. Daniel walks from A to C , a distance of 62 m. When he is 15 m from A , he notes that point D is 25 m to his right. When he is 57 m from A , he notes that point B is 20 m to his left. This is his notebook entry.



- (i) Draw a neat sketch of the block of land. Label A , B , C and D on your diagram. 1
- (ii) Calculate the distance from C to D . (Give your answer to the nearest metre.) 2

- (b) The roof of this greenhouse is a square pyramid with identical triangular faces. The sides of the greenhouse are rectangles and there is no floor. The dimensions of the greenhouse are shown on the diagram.



- (i) Sketch a possible net of the greenhouse. 1
 (ii) Calculate the surface area of the greenhouse. 2
- (c) A new test has been developed for determining whether or not people are carriers of the Gaussian virus. Two hundred people are tested. A two-way table is being used to record the results.

	Positive	Negative
Carrier	74	12
Not a carrier	16	A

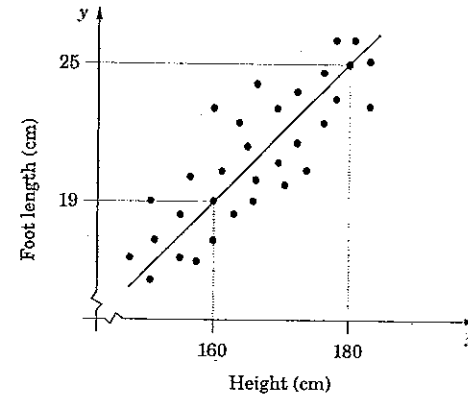
- (i) What is the value of A? 1
 (ii) A person selected from the tested group is a carrier of the virus. What is the probability that the test results would show this? 2
 (iii) For how many of the people tested were their test results inaccurate? 1
- (d) Cassie flew from London (52°N, 0°E) to Manila (15°N, 120°E). Her plane left London at 9.30 am Monday (London time), stopped for 5 hours in Singapore and arrived in Manila at 4.00 pm Tuesday (Manila time). What was the total flying time? (Ignore time zones.) 3

Question 27 (13 marks)

- (a) Liliansa wants to borrow money to buy a house. The bank sent her an email with the following table attached.

Amount borrowed	Monthly repayments				
	Term of loan				
	10 years 120 months	15 years 180 months	20 years 240 months	25 years 300 months	30 years 360 months
\$80 000	\$970.62	\$764.52	\$669.15	\$617.45	\$587.01
\$90 000	\$1091.95	\$860.09	\$752.80	\$694.63	\$660.39
\$100 000	\$1213.28	\$955.65	\$836.44	\$771.82	\$733.76
\$110 000	\$1334.60	\$1051.22	\$920.08	\$849.00	\$807.14
\$120 000	\$1455.93	\$1146.78	\$1003.73	\$926.18	\$880.52
\$130 000	\$1577.26	\$1242.35	\$1087.37	\$1003.36	\$953.89
\$140 000	\$1698.59	\$1337.91	\$1171.02	\$1080.54	\$1027.27
\$150 000	\$1819.91	\$1433.48	\$1254.66	\$1157.72	\$1100.65
\$160 000	\$1941.24	\$1529.04	\$1338.30	\$1234.91	\$1174.02

- (i) Liliansa decides that she can afford \$1000 per month on repayments. What is the maximum amount she can borrow, and how many years will she have to repay the loan? 1
 (ii) Zali intends to borrow \$160 000 over 15 years from the same bank. If she chooses to borrow \$160 000 over 20 years instead, how much more interest will she pay? 2
- (b) Each member of a group of males had his height and foot length measured and recorded. The results were graphed and a line of fit drawn.



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- (i) Why does the value of the y-intercept have no meaning in this situation? 1
 (ii) George is 10 cm taller than his brother Harry. Use the line of fit to estimate the difference in their foot lengths. 1
 (iii) Sam calculated a correlation coefficient of -1.2 for the data. Give TWO reasons why Sam must be incorrect. 2
- (c) Kai purchased a new car for \$30 000. It depreciated in value by \$2000 per year for the first three years. After the end of the third year, Kai changed the method of depreciation to the declining balance method at the rate of 25% per annum.

- (i) Calculate the value of the car at the end of the third year. 1
 (ii) Calculate the value of the car seven years after it was purchased. 2
 (iii) Without further calculations, sketch a graph to show the value of the car over the seven years. Use the horizontal axis to represent time and the vertical axis to represent the value of the car. 3

Question 28 (13 marks)

(a) On a bridge, the toll of \$2.50 is paid in coins collected by a machine. The machine only accepts two-dollar coins, one-dollar coins and fifty-cent coins.

- (i) List the different combinations of coins that could be used to pay the \$2.50 toll.
- (ii) Jill has three two-dollar coins, six one-dollar coins and two fifty-cent coins. She selects two coins at random. What is the probability that she selects exactly \$2.50?

(iii) At the end of a day, the machine contains x two-dollar coins, y one-dollar coins and w fifty-cent coins.

Write an expression for the total value of coins in dollars in the machine.

(b) A new tunnel is built. When there is no toll to use the tunnel, 6000 vehicles use it each day. For each dollar increase in the toll, 500 fewer vehicles use the tunnel.

- (i) Find the lowest toll for which no vehicles will use the tunnel.
- (ii) For a toll of \$5.00, how many vehicles use the tunnel each day and what is the total daily income from tolls?
- (iii) If d (dollars) represents the value of the toll, find an equation for the number of vehicles using the tunnel each day in terms of d .
- (iv) Anne says 'A higher toll always means a higher total daily income'. Show that Anne is incorrect and find the maximum daily income from tolls. (Use a table of values, or a graph, or suitable calculations.)

End of paper

**2006 HIGHER SCHOOL CERTIFICATE
SOLUTIONS
GENERAL MATHEMATICS**

SECTION I

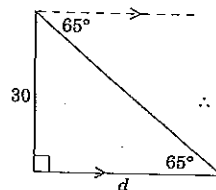
SUMMARY

- | | | | |
|------|-------|-------|-------|
| 1. A | 7. D | 13. D | 18. D |
| 2. D | 8. C | 14. A | 19. B |
| 3. B | 9. C | 15. A | 20. C |
| 4. C | 10. C | 16. C | 21. B |
| 5. B | 11. A | 17. D | 22. B |
| 6. B | 12. B | | |

1. (A) $P(\text{impossible event}) = 0$.
A likely event has a probability between $\frac{1}{2}$ and 1.
 $P(\text{certain event}) = 1$.
So $P(\text{event}) = \frac{9}{10}$ would be that the event is likely to occur.

2. (D) $V = \frac{4}{3}\pi r^3$.
Given $r = 2$:
 $V = \frac{4}{3} \times \pi \times 2^3$
 $= 33.510\ 321\ 64$.
 $\therefore V \doteq 33.51$ (2 dec. places).

3. (B) $\tan 65^\circ = \frac{30}{d}$
 $d = \frac{30}{\tan 65^\circ}$
 $= 13.989\ 229\ 74 \dots$
 $\therefore \text{Distance} \doteq 14.0$ m
(1 dec. place).



4. (C) For a set of 10 scores, the median is the average of the 5th and 6th scores:
5th score = 28
6th score = 38
Median = $\frac{28 + 38}{2}$
 $= 33$.

1	2	2	3
2	5	8	
3	8	9	
4	1	3	9

5. (B) **METHOD 1**

For 2 weeks, the salesman earns
 $2 \times \$200 = \400 plus \$40 per item sold.
Total commission = $\$2640 - \400
 $= \$2240$.
Number of items sold = $\$2640 \div 40$
 $= 56$.

METHOD 2

Let x = number of items sold.
 $2 \times 200 + 40x = 2640$
 $400 + 40x = 2640$
 $40x = 2240$
 $x = \frac{2240}{40}$

\therefore Number of items sold in 2 weeks is 56.

6. (B) $P(\text{rolling } 4) = \frac{1}{6}$.
Expected number of 4's = $P(\text{rolling } 4) \times 60$
 $= \frac{1}{6} \times 60$
 $= 10$.

7. (D) **METHOD 1**

x	0	2	4	6	8
y	1	2	3	4	5

Equation is $y = \frac{x}{2} + 1$.

Check by substituting values from the table.

- $x = 0$: $y = \frac{0}{2} + 1 = 1$
- $x = 2$: $y = \frac{2}{2} + 1 = 2$
- $x = 4$: $y = \frac{4}{2} + 1 = 3$
- $x = 6$: $y = \frac{6}{2} + 1 = 4$
- $x = 8$: $y = \frac{8}{2} + 1 = 5$

METHOD 2

Equation is in the form $y = mx + b$.

$$m = \frac{\text{change in } y}{\text{change in } x}$$

$$= \frac{2-1}{2-0}, \text{ using } (0, 1) \text{ and } (2, 2)$$

$$= \frac{1}{2}$$

$$\therefore y = \frac{1}{2}x + b$$

To find the value of b (the y -intercept), substitute $(0, 1)$ from the table, or notice that the point $(0, 1)$ from the table tells us that the y -intercept is 1.

$$y = \frac{1}{2}x + b$$

$$1 = \frac{1}{2} \times 0 + b$$

$$1 = b$$

$$\therefore b = 1$$

$$\therefore \text{Equation is } y = \frac{1}{2}x + 1 \text{ or } y = \frac{x}{2} + 1$$

8. (C) The correct graph is (C) because it shows a distribution which is positively skewed with a smaller standard deviation than A. (Note: A smaller standard deviation means a smaller spread of scores so the data on the graph will be packed more closely.)
- (A) shows positively skewed with a larger standard deviation than C.
- (B) shows negatively skewed with a smaller standard deviation than D.
- (D) shows negatively skewed with a larger standard deviation than B.

9. (C) Area = $\frac{1}{2}ab \sin C$

$$= \frac{1}{2} \times 39 \times 47 \times \sin 21^\circ$$

$$= 328.444 \ 2258 \dots$$

$$\therefore \text{Area} \doteq 328 \text{ m}^2 \text{ (nearest m}^2\text{)}$$

10. (C) Total frequency = $7 + 3 + 2 + 4 + 8 = 24$.

$$\therefore \text{The best estimate for } P(\text{green marble}) = \frac{4}{24} = \frac{1}{6}$$

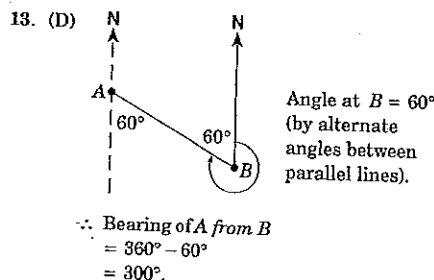
11. (A) $27 \text{ km/h} = 27\ 000 \text{ m/h}$

$$= (27\ 000 \div 60) \text{ m/min}$$

$$= (27\ 000 \div 60) \div 60 \text{ m/s}$$

$$= 7.5 \text{ m/s}$$

12. (B) Mean of 5 scores = 62.
- $$\therefore \text{Sum of the 5 scores} = 62 \times 5 = 310$$
- $$\therefore \text{Sum of the 6 scores} = 310 + 14 = 324$$
- $$\therefore \text{Mean of the 6 scores} = \frac{324}{6} = 54$$

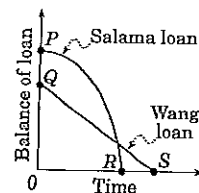


14. (A) $2.3\% = 2.3 \div 100 = 0.023$
- Increase of 2.3% means multiplying by $(1 + 0.023) = 1.023$.
- After 1 year, the number of vehicles (in millions) = 13.5×1.023 .
- After 2 years, the number = $(13.5 \times 1.023) \times 1.023 = 13.5 \times 1.023^2$.
- After 3 years, the number = $(13.5 \times 1.023^2) \times 1.023 = 13.5 \times 1.023^3$.
- After y years, the number = 13.5×1.023^y .

15. (A) **METHOD 1**
- Require *unordered* selection.
- Number of ways of selecting 2 people from a total of 6 people = $\frac{6 \times 5}{2} = 15$.

- METHOD 2**
- As selection is *unordered*, require a combination $({}^nC_r)$.
- Number of ways of selecting 2 people from a total of 6 people = ${}^6C_2 = 15$.

16. (C)



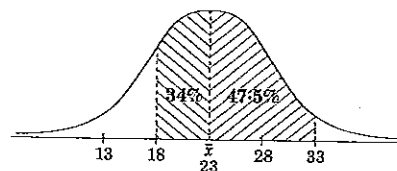
The correct graph is (C), as this shows:

- different amounts of money borrowed by the Salama and Wang families (indicated by points P and Q on the graph shown above); and
- the reducing balance loan for the Salama family (represented by the curve) is repaid before the flat rate loan for the Wang family (represented by the straight line) since the curve cuts the horizontal axis (at point R) before the line cuts the horizontal axis (at point S).

17. (D) **METHOD 1**

For a normal distribution:

- 68% of scores lie within 1 standard deviation of the mean, ie. 34% of scores lie between 1 standard deviation below the mean (in this case 18) and the mean (in this case 23).
- 95% of scores lie within 1 standard deviation of the mean, ie. 47.5% of scores lie between the mean (28) and 2 standard deviations above the mean (33).



So the required percentage of scores = $34\% + 47.5\% = 81.5\%$.

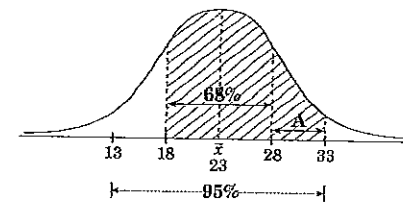
METHOD 2

In this case, mean $(\bar{x}) = 23$
standard deviation = 5.

For a normal distribution:

- 68% of scores lie within 1 standard deviation of the mean (in this case between 18 and 28),
- 95% of scores lie within 2 standard

deviations of the mean (in this case between 13 and 33).



Percentage represented by Region A = $\frac{95\% - 68\%}{2} = 13.5\%$.

So the required percentage of scores = $68\% + 13.5\% = 81.5\%$.

18. (D) $4p = 5t + 2q^2$

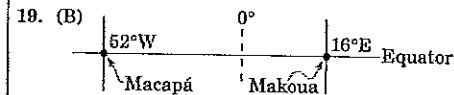
$$4p - 5t = 2q^2$$

Rearranging, $2q^2 = 4p - 5t$

$$q^2 = \frac{4p - 5t}{2}$$

Taking the square root of both sides,

$$\therefore q = \pm \sqrt{\frac{4p - 5t}{2}}$$



Difference in longitude = $52^\circ + 16^\circ = 68^\circ$.

METHOD 1

Use the arc length of a circle formula.

In this case:

$$\theta = 68^\circ$$

$$r = 6400 \text{ (given radius of Earth is approximately 6400 km)}$$

$$l = \frac{\theta}{360} \times 2\pi r$$

$$= \frac{68}{360} \times 2 \times \pi \times 6400$$

$$= 7595.672 \ 905 \dots \text{ km}$$

$$\doteq 7600 \text{ km (to nearest 100 km)}$$

METHOD 2

The 2 towns are on the Equator, which is a great circle.

An angular distance of 1° on a great circle equals a distance of 60 nautical miles.

In this case the angular distance between the 2 towns is 68° .

So the distance between the 2 towns
 = (68×60) nautical miles
 = 4080 nautical miles
 = 4080×1.852 km (since 1 nautical mile = 1.852 km)
 = 7556.16 km
 ≈ 7600 km (to nearest 100 km).

20. (C) **METHOD 1**

Let the radius of the original sphere be 100 cm. Then if the radius is increased by 10%, the radius of the large sphere is $100 \text{ cm} + 10 \text{ cm} = 110 \text{ cm}$.

Surface area of the original sphere
 = $4\pi r^2$
 = $4 \times \pi \times (100)^2$
 = $4 \times \pi \times 10\,000$
 = $40\,000\pi \text{ cm}^2$.

Surface area of the large sphere
 = $4\pi r^2$
 = $4 \times \pi \times (110)^2$
 = $4 \times \pi \times 12\,100$
 = $48\,400\pi \text{ cm}^2$.

So, increase in surface area
 = $48\,400\pi - 40\,000\pi$
 = 8400π .

So the percentage increase
 = $\frac{8400\pi}{40\,000\pi} \times 100\%$
 = $\frac{21}{100} \times 100\%$
 = 21%.

METHOD 2

For a sphere with radius r , the surface area = $4\pi r^2$. A sphere with the radius increased by 10% has radius:

$$r + 10\% \times r = 1r + 0.1r = 1.1r.$$

Surface area = $4\pi(1.1r)^2$
 = $4\pi(1.21r^2)$
 = $1.21(4\pi r^2)$.

\therefore The surface area has increased by 0.21, ie. by 21%.

21. (B) Interest rate = 7.2% annually
 = $\frac{7.2\%}{12}$ monthly
 = $\frac{0.072}{12}$ per month.
 \therefore Interest = $\$420\,000 \times \frac{0.072}{12}$
 = $\$2520$.

Balance owing
 = $\$420\,000 + \$2520 - \$4000$
 = $\$418\,520$.

22. (B) Tax payable on a taxable income of $\$50\,000$
 = $\$2500 + 0.35 \times (\$50\,000 - \$45\,000)$
 = $\$4250$.

Tax payable on a taxable income of $\$80\,000$
 = $\$11\,250 + 0.52 \times (\$80\,000 - \$70\,000)$
 = $\$16\,450$.

Additional tax = $\$16\,450 - \4250
 = $\$12\,200$.

Pay increase = $\$80\,000 - \$50\,000$
 = $\$30\,000$.

Percentage of increase Evelyn will pay in additional tax = $\frac{12\,200}{30\,000} \times 100\%$
 = 40.666 666 67%
 $\approx 40.7\%$ (to 1 dec. place).

SECTION II

Question 23

(a) $\frac{ab^2}{1} \times \frac{4a}{3b} = \frac{ab}{1} \times \frac{4}{3}$
 = $\frac{4ab}{3}$.

(b) At Town B in April, the average daily temperature is

(i) approximately 19°C .

(ii) Looking along the rays (radii) for each month, the solid line and the dotted line are furthest apart in February.

(iii) Average daily temperature of Town B is greater than Town A from May to September.

(c) (i) One possible method: assign each student a number and draw numbers out of a box or have a computer randomly generate numbers until 200 students are selected.

(ii)

Class	Frequency (f)	Class Centre (CC)
0 - 4	69	2
5 - 9	72	7
10 - 14	38	12
15 - 19	21	17

$$\bar{x} = \frac{1455}{200}$$

$$= 7.275 \text{ hours.}$$

(d) (i) The charge for a 3 kg parcel by Company A is \$6.

(ii) The same amount is charged by both companies where the two graphs meet, ie. 4 kg or 7 kg.

(iii) Company A is cheaper for parcels greater than 4 kg up to and including 6 kg, and for parcels greater than 7 kg.

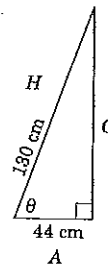
(iv) Gradient = $\frac{\text{rise}}{\text{run}} = \frac{\$12}{8 \text{ kg}}$
 = $\$1.50/\text{kg}$.

Question 24

(a) Any 2 of the following:

- The vertical scale (number of sales) does not begin at zero.
- The box for Brand Y should only be taller in 1 dimension, rather than bigger in 3 dimensions.
- It is difficult to determine the heights of the two boxes.

(b) (i)



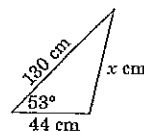
SOH CAH TOA

$$\cos \theta = \frac{44}{130}$$

$$\theta = 70.2168 \dots$$

$$\approx 70^\circ \text{ (to nearest degree).}$$

(ii) Using the cosine rule:



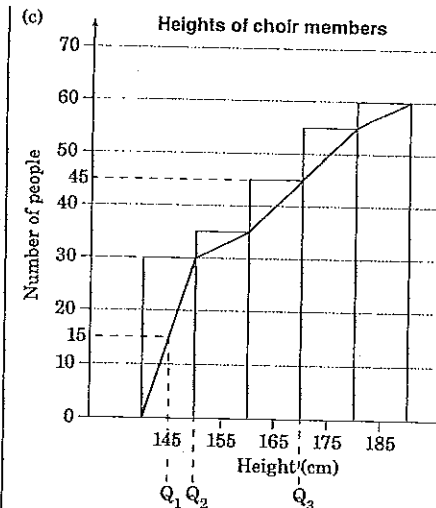
$$x^2 = 130^2 + 44^2 - 2 \times 130 \times 44 \cos 53^\circ$$

$$= 11\,951.236\,14 \dots$$

$$x = \sqrt{11\,951.236\,14 \dots}$$

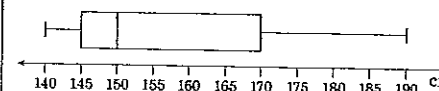
$$= 109.321\,7093 \dots$$

$$\approx 109 \text{ cm (to the nearest cm).}$$



Five number summary:

- Lowest score = 140 cm
- Lowest quartile, Q_1 = 145 cm
- Median, Q_2 = 150 cm
- Upper quartile, Q_3 = 170 cm
- Highest score = 190 cm



(d) $V = Ah$

(i) = $1890 \text{ cm}^2 \times 300 \text{ cm}$
 = $567\,000 \text{ cm}^3$
 = $567\,000 \text{ mL}$
 = 567 L .

(ii) Cross-sectional area:

$$\frac{1}{2} \pi ab = 1890$$

$$\frac{1}{2} \pi 40d = 1890$$

$$20\pi d = 1890$$

$$d = \frac{1890}{20\pi}$$

$$= 30.080\,28 \dots$$

$$= 30 \text{ cm (to the nearest cm).}$$

Question 25

(a) (i) **METHOD 1**

Using a list:

- | | | | |
|-----|-----|-----|-----------|
| MCJ | CMJ | JMC | } 6 ways. |
| MJC | CJM | JCM | |

METHOD 2

Multiplying the number of choices:

$$3 \times 2 \times 1 = 6 \text{ ways.}$$

(ii) 2 possibilities: *CJM* or *MJC*

$$\therefore P(\text{2nd card is } J) = \frac{2}{6} = \frac{1}{3}.$$

(iii) **METHOD 1**

4 possibilities: *CJM*, *CMJ*, *JCM*, *MCJ*

$$\therefore P(\text{last card not } C) = \frac{4}{6} = \frac{2}{3}.$$

METHOD 2

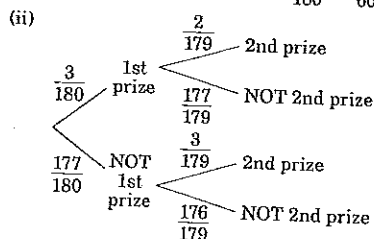
$$\begin{aligned} P(\text{last card not } C) &= 1 - P(\text{last card is } C) \\ &= 1 - \frac{2}{6} \\ &= \frac{4}{6} \\ &= \frac{2}{3}. \end{aligned}$$

(b) (i) 9 days in May + 20 days in June = 29 days.

$$\begin{aligned} \text{(ii) } I &= Prn \quad P = \$617.72 \\ r &= 0.0498 + 100 = 0.000498 \\ n &= 29 \end{aligned}$$

$$\begin{aligned} I &= 617.72 \times 0.000498 \times 29 \\ &= 8.92111224 \dots \\ &\approx \$8.92. \end{aligned}$$

(c) (i) $P(\text{winning the first prize}) = \frac{3}{180} = \frac{1}{60}$.



$$\begin{aligned} P(\text{wins both prizes}) &= P(\text{wins 1st prize and the 2nd prize}) \\ &= \frac{3}{180} \times \frac{2}{179} \\ &= \frac{1}{5370}. \end{aligned}$$

$$\text{(d) } E = \frac{(1+r)^n - 1}{n}$$

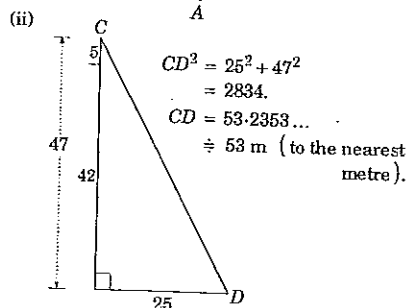
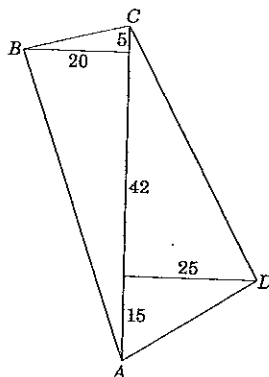
$$\begin{aligned} r &= 6\% \text{ p.a.} \\ &= 6 \div 100 + 12 \text{ per month} \\ &= 0.005. \\ n &= 4 \times 12 = 48 \text{ months.} \end{aligned}$$

$$\begin{aligned} E &= \frac{(1+0.005)^{48} - 1}{0.005} \\ &= 0.0056351 \dots \text{ per month} \\ &= 0.0056351 \dots \times 12 \text{ per year} \\ &= 0.0676 \dots \text{ per annum} \\ &= 6.76\% \text{ per annum.} \end{aligned}$$

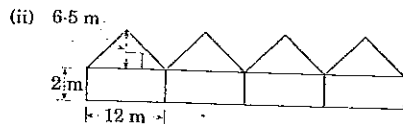
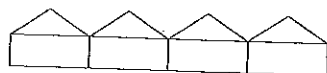
Martha's effective interest rate is higher, so she earns more interest. Therefore her deal is better.

Question 26

(a) (i)



(b) (i)



$$\begin{aligned} \text{Area of roof} &= 4 \times \frac{1}{2} \times 12 \times 6.5 \\ &= 156 \text{ m}^2. \\ \text{Area of sides} &= 4 \times 12 \times 2 \\ &= 96 \text{ m}^2. \\ \text{Total area} &= 252 \text{ m}^2. \end{aligned}$$

(c) (i) $A = 200 - (74 + 16 + 12) = 98$.

(ii) Total number of carriers = $74 + 12 = 86$.

$$\begin{aligned} P(\text{the test is positive to a carrier}) &= \frac{74}{86} \\ &= \frac{37}{43}. \end{aligned}$$

(iii) An inaccurate result is shown when a carrier is tested negative or when a non-carrier is tested positive.

So inaccurate results = $12 + 16 = 28$.

(d) Time difference between London and Manila = $120 \times 4 \text{ min}$

= 480 min

= 8 hours.

\therefore Leaves London at 9:30 am + 8 hours

= 5:30 pm.

Time between Monday 5:30 pm and

Tuesday 4:00 pm = $22\frac{1}{2}$ hours.

$$\begin{aligned} \therefore \text{Flight time} &= 22\frac{1}{2} \text{ h} - 5 \text{ h (stopping time)} \\ &= 17\frac{1}{2} \text{ h.} \end{aligned}$$

Question 27

(a) (i) From the table, the largest amount she can borrow with a repayment under \$1000 is \$130 000 over 30 years.

(ii) **METHOD 1**

$$\begin{aligned} \text{Interest paid over 20 years} &= \$1338.30 \times 240 - 160\,000 \\ &= \$161\,192.00. \end{aligned}$$

$$\begin{aligned} \text{Interest paid over 15 years} &= \$1529.04 \times 180 - 160\,000 \\ &= \$115\,227.20. \end{aligned}$$

$$\begin{aligned} \text{Extra interest she will pay} &= \$161\,192.00 - \$115\,227.20 \\ &= \$45\,964.80. \end{aligned}$$

METHOD 2

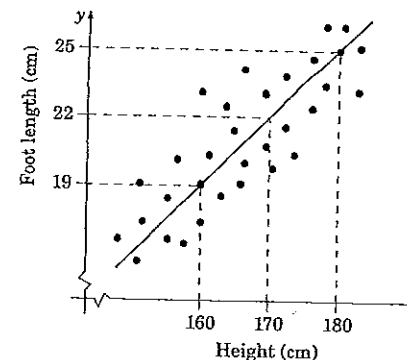
$$\begin{aligned} \text{Extra interest} &= 240 \times \$1338.30 - 180 \times \$1529.04 \\ &= \$45\,964.80. \end{aligned}$$

(b) (i) The y-intercept is at $x = 0$, so a height of 0 cm has no meaning.

(ii) **METHOD 1**

From the line of best fit:

$$\left. \begin{aligned} \text{If George} &= 160 \text{ cm height} \\ &\rightarrow 19 \text{ cm foot length} \\ \text{Brother} &= 170 \text{ cm height} \\ &\rightarrow 22 \text{ cm foot length} \end{aligned} \right\} \text{difference} = 3 \text{ cm.}$$



METHOD 2

A $180 - 160 = 20$ cm increase in height is related to a $25 - 19 = 6$ cm increase in foot length.

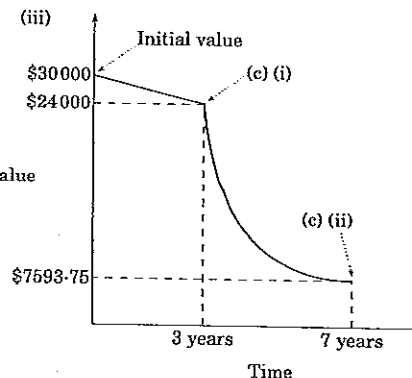
\therefore A 10 cm increase in height is related to a 3 cm increase in foot length.

- (b) (iii)
- The correlation coefficient should be positive because the gradient of the line of best fit is positive.
 - The magnitude of the correlation should be between -1 and 1 .

(c) (i) $\$30\,000 - 3 \times \$2000 = \$24\,000$.

$$\begin{aligned} \text{(ii) } S &= V_0(1-r)^n & V_0 &= \$24\,000 \\ r &= 25 \div 100 = 0.25 \\ n &= 7 - 3 = 4 \end{aligned}$$

$$\begin{aligned} S &= 24\,000(1-0.25)^4 \\ &= 24\,000(0.75)^4 \\ &= \$7593.75. \end{aligned}$$



Question 28

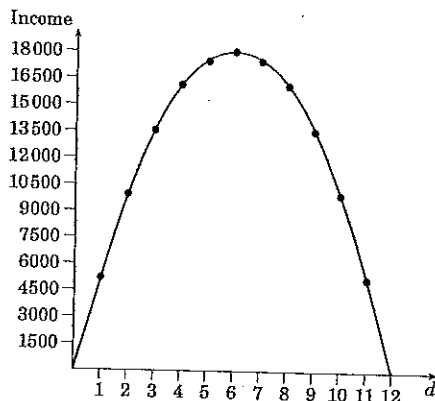
- (a) (i) $\$2.50 = \$2.00 + 50c$
 $= \$1 + \$1 + 50c$
 $= \$1 + 50c + 50c + 50c$
 $= 50c + 50c + 50c + 50c + 50c.$
- (ii) $P(\$2.50) = P(\$2 \text{ in the 1st draw and } 50c \text{ in the 2nd draw})$
 $+ P(50c \text{ in the 1st draw and } \$2 \text{ in the 2nd draw})$
 $= \frac{3}{11} \times \frac{2}{10} + \frac{2}{11} \times \frac{3}{10}$
 $= \frac{6}{55}$
- (iii) Total value = $\$(2x + 1y + 0.5w)$
 $= \$(2x + y + 0.5w).$
- (b) (i) Since a \$1 increase in toll will lead to 500 fewer vehicles using the tunnel, then a \$12 increase in the toll will lead to $12 \times 500 = 6000$ fewer vehicles using the tunnel. Hence the lowest toll for which no vehicles will use the tunnel is \$12.
- (ii) The number of vehicles that will use the tunnel = $6000 - 5 \times 500$
 $= 3500$ vehicles.
 Income = $3500 \times \$5 = \$17\,500.$
- (iii) If the toll is \$1 then $6000 - 1 \times 500$ will use the tunnel.
 If the toll is \$2 then $6000 - 2 \times 500$ will use the tunnel.
 Hence if the toll is d (dollars), then $6000 - d \times 500$ will use the tunnel.
 So, $V = 6000 - 500d.$

(b) (iv) METHOD 1

Toll (\$ d)	No. vehicles (v)	Income ($d \times v$)
0	6000	0
1	5500	5500
2	5000	10 000
3	4500	13 500
4	4000	16 000
5	3500	17 500
6	3000	18 000
7	2500	17 500
8	2000	16 000
9	1500	13 500
10	1000	10 000
11	500	5500
12	0	0

From the table, the maximum daily income is \$18 000.
 Anne is incorrect because a higher toll also means fewer vehicles, so lower income.

METHOD 2



From the graph, the maximum income is \$180 000. Anne is incorrect for tolls bigger than 6 because more tolls leads to less income.

END OF GENERAL MATHEMATICS SOLUTIONS

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Section I

22 marks

Attempt Questions 1 – 22

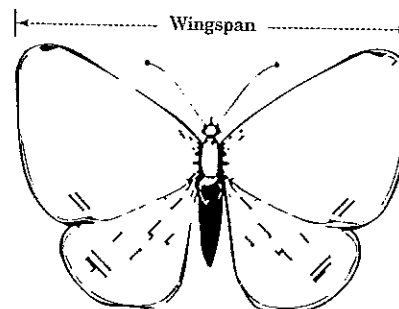
Allow about 30 minutes for this section

- 1 What is the mean of the set of scores?
 3, 4, 5, 6, 6, 8, 8, 8, 15
 (A) 6 (B) 7 (C) 8 (D) 9
- 2 What is the value of $\frac{a-b}{4}$, if $a = 240$ and $b = 56$?
 (A) 4 (B) 46 (C) 226 (D) 736

- 3 Four radio stations reported the probability of rain as shown in the table.
 Which radio station reported the highest probability of rain?
 (A) 2AT (B) 2BW
 (C) 2CZ (D) 2DL

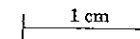
Radio station	Probability of rain
2AT	0.53
2BW	17%
2CZ	$\frac{13}{25}$
2DL	0.6

- 4 The diagram is a scale drawing of a butterfly.



What is the actual wingspan of the butterfly?

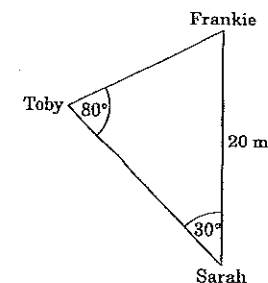
- (A) 2.5 cm (B) 3 cm
 (C) 15 cm (D) 18.75 cm



SCALE

- 5 Which formula should be used to calculate the distance between Toby and Frankie?

- (A) $\frac{a}{\sin A} = \frac{b}{\sin B}$ (B) $c^2 = a^2 + b^2$
 (C) $A = \frac{1}{2}ab \sin C$ (D) $c^2 = a^2 + b^2 - 2ab \cos C$



NOT TO SCALE