

Topic Test: The Normal Distribution

Remember: these are HSC-type questions.

Time allowed: 40 minutes Total marks: 25

Part A (Suggested time: 15 minutes)

Choose the correct answer (A, B, C or D) for each question. One mark each

- 1 Cathy sat for a test and for her result the z-score was -2 . If the mean was 61% and the standard deviation 7%, what was Cathy's mark?
 A 47% B 52%
 C 54% D 59%

- 2 The mean number of people attending a show is 565 with a standard deviation of 37.5. What z-score corresponds to an attendance of 655 people at the show?
 A 1.8 B 2
 C 2.4 D 2.5

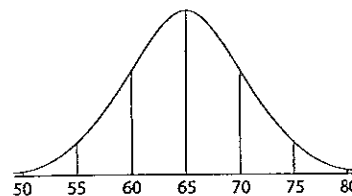
- 3 Tara sat for exams in four subjects. Her results (as percentages) and the mean and standard deviation for each subject are shown in the table.

Subject	Tara's mark	Mean	Standard deviation
English	58	55	4.8
Maths	73	69	5.4
Science	68	61	11.2
History	81	72	14.4

Which was Tara's best result?

- A English B Maths
 C Science D History
- 4 In a normal distribution the mean number was 240 and standard deviation 18. What percentage of numbers lie between 258 and 276?
 A 34% B 27%
 C 47.5% D 13.5%
- 5 Ivan scored 73% in a test and achieved a z-score of 1.8. If the standard deviation was 5.5%, what was the mean?
 A 63.1% B 65.7%
 C 80.3% D 82.9%

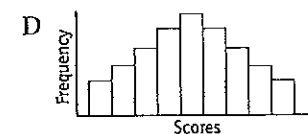
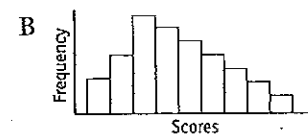
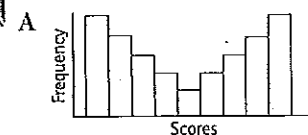
- 6 The results of a national examination were normally distributed with mean 65 and standard deviation 5. The distribution is sketched below.



Holly received a z-score of 1.2. Between what marks did Holly's mark lie?

- A 60 and 65 B 65 and 70
 C 70 and 75 D 75 and 80
- 7 Referring to question 6, what percentage of students would you expect to have scored less than 50?
 A 0.3% B 0.15%
 C 2.5% D 3%

- 8 Which set of data could be normally distributed?



9 In an exam for which 300 students sat, the results were normally distributed. Matthew received a z -score of -0.98 . He made the following two statements:

I: My mark is less than one standard deviation below the mean.

II: I beat about 50 other students.

Which statement(s) are correct?

- A both B neither
C I but not II D II but not I

10 A survey was taken of the weight of certain muesli bars and the mean was found to be 31 g. A bar with a weight of 29.8 g has a z -score of -1.6 . What is the standard deviation?

- A 1.333 g B 1.2 g
C 1.92 g D 0.75 g

Part B

(Suggested time: 25 minutes)

Show all working.

15 marks

11 a Briefly explain what it means to have a z -score of 1. 1 mark

b Stella sat for a physics exam and received a z -score of 1. If Stella's raw mark was 85% and the standard deviation was 8.5%, what was the mean? 1 mark

c David sat for the same exam. His raw mark was 51%. What was his z -score? 1 mark

d In a chemistry test, David scored 116. The mean mark was 132 and the standard deviation was 6. Which was David's better result? Justify your answer. 2 marks

12 The ages of a sample of people at a conference were recorded. The results were:

29 49 38 24 27 32 34 25
44 37 27 36

a What is the mean age? 1 mark

b What is the standard deviation to two decimal places? 1 mark

c Jo is 29. What is her z -score? 1 mark

d Find the age of the person whose z -score is 2. 1 mark

e How many people are older than the person whose z -score is 2. Is this what you would expect? Justify your answer. 2 marks

13 Tests were conducted on the length of time certain fuses could work before failing. The results were found to be normally distributed, with mean eighteen months and standard deviation three months.

a Draw up a table showing the length of time associated with z -scores of $-3, -2, -1, 0, 1, 2$ and 3 . 1 mark

b Approximately what percentage of fuses should last between 15 and 21 months? 1 mark

c What percentage of fuses would you expect to last longer than 27 months? 1 mark

d The manufacturers wish to guarantee the fuses for a certain length of time. If they are willing to replace up to 2.5% of fuses, for what period of time should they guarantee them? 1 mark

Solutions

Topic Test p93

1 Cathy's mark was 2 standard deviations below the mean.
 Mark = $(61 - 2 \times 7)\%$
 = 47%

A

2 $x = 655, \bar{x} = 565, s = 37.5$
 $z = \frac{x - \bar{x}}{s}$
 = $\frac{655 - 565}{37.5}$
 = 2.4

C

3 English: $z = \frac{58 - 55}{4.8}$
 = 0.625

maths: $z = \frac{73 - 69}{5.4}$
 = 0.740 74 ...

science: $z = \frac{68 - 61}{11.2}$
 = 0.625

history: $z = \frac{81 - 72}{14.4}$
 = 0.625

Maths is the best result.

B

4 258 is one standard deviation above the mean. 276 is two standard deviations above the mean.
 Percentage between one and two standard deviations off the mean
 = 95% - 68%
 = 27%
 Half of these are above the mean.
 Required percentage
 = $(27 \div 2)\%$
 = 13.5%

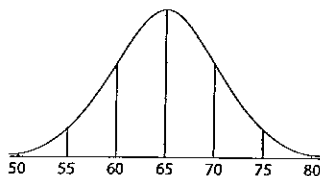
D

5 $x = 73, z = 1.8, s = 5.5$
 $z = \frac{x - \bar{x}}{s}$
 $1.8 = \frac{73 - \bar{x}}{5.5}$
 $9.9 = 73 - \bar{x}$
 $\bar{x} = 63.1$
 The mean is 63.1%

A

6 Holly's score is between 1 and 2 standard deviations above the mean. Her mark is between 70 and 75.

C

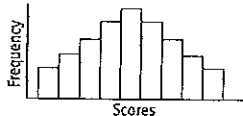


7 50 is three standard deviations below the mean.
 99.7% lie within 3 standard deviations.
 Half of 0.3% lie below 50.
 0.15%

B

8 Only D is a bell-shaped curve.

D



9 Statement I is correct.
 [Matthew's score is 0.98 times the standard deviation below the mean.]
 Approximately 16% had a score that was more than one standard deviation below the mean.
 16% of 300 = 48
 Statement II is correct.
 Both statements are correct.

A

10 $x = 29.8, \bar{x} = 31, z = -1.6$
 $z = \frac{x - \bar{x}}{s}$
 $-1.6 = \frac{29.8 - 31}{s}$
 $-1.6s = -1.2$
 $s = 0.75$

The standard deviation is 0.75 g.

D

11 a A z-score of 1 means the score is one standard deviation above the mean.

✓

b Mean = $85 - 8.5$
 = 76.5

The mean was 76.5%

✓

c $x = 51, \bar{x} = 76.5, s = 8.5$
 $z = \frac{x - \bar{x}}{s}$
 = $\frac{51 - 76.5}{8.5}$
 = -3

✓

d $x = 116, \bar{x} = 132, s = 6$
 $z = \frac{x - \bar{x}}{s}$
 = $\frac{116 - 132}{6}$
 = -2.666 666 ...

✓

Chemistry is the better result.

✓

The z-score is higher.

✓

12 29 49 38 24 27 32
 34 25 44 37 27 36

a Mean = 33.5 [by calculator] ✓

b $\sigma_{n-1} = 7.740\ 096\ 311 \dots$
 = 7.74 (2 d.p.) ✓

c $x = 29, \bar{x} = 33.5, s = 7.74$
 $z = \frac{x - \bar{x}}{s}$
 = $\frac{29 - 33.5}{7.74}$
 = -0.581 395 348 ...
 = -0.58 (2 d.p.) ✓

d Age = $33.5 + 2 \times 7.74$
 = 48.98
 The person with a z-score of 2 is 49 years old. ✓

e There are no people older than the person whose z-score is 2. ✓
 Because the sample is only small it would be highly likely that there would be no one older than a person with a z-score of 2. ✓

13 a

Time (months)	9	12	15	18	21	24	27
z-score	-3	-2	-1	0	1	2	3

✓

b 15 and 21 months are one standard deviation each side of the mean.
 Approximately 68% of fuses should last between 15 and 21 months. ✓

c 99.7% are within 3 standard deviations of the mean.
 0.3% lie outside this range.
 Half of these are longer than 27 months.
 0.15% should last longer than 27 months. ✓

d 95% lie within 2 standard deviations of the mean. 5% lie outside this range. Half of 5% is 2.5%.
 Two standard deviations below the mean is 12 months.
 The manufacturer should guarantee the fuses for 12 months. ✓