

Further Practice: The Normal Distribution

Remember: all questions match the numbered examples on pages 81–87.

- 1** Joel sat for a test and for his mark the z-score was 2.
- Briefly explain the meaning of the above statement.
 - What was Joel's mark if the mean mark was 65% and the standard deviation was 5.5%?

- 2** In a test the mean mark was 71% and the standard deviation was 7%. Victoria's z-score was -2 .
- What information do we get from knowing the z-score?
 - What was Victoria's mark?

- 3** The mean weight of packets of biscuits is 260 g with a standard deviation of 5 g. What is the weight of a packet if the z-score is -1.6 ?

4 Copy and complete the table.

Number				64	73		
z-score	-3	-2	-1	0	1	2	3

- 5** Find the z-score associated with a score of 34 if the mean is 28 and the standard deviation is 2.5.

- 6** The mean mark in an exam was 66 and the standard deviation was 5.6. Brandon scored 80. What is his z-score?

- 7** The mean wage for workers in a particular industry is \$230 per day. The standard deviation is \$15. What z-score will apply to a worker who earns \$206 per day?

- 8** The 24 students in a class all sat for a test. The results, as percentages, are shown below.

73 68 94 67 83 85 79 70 62 56
 90 84 59 47 72 71 83 58 91 85
 75 76 64 78

- Find the mean and standard deviation (σ_n) correct to two decimal places.
- Callum scored 64. What is his z-score? Give the answer correct to one decimal place.

- 9** Find the standard deviation if a score of 83 has a z-score of 2.2 when the mean of the scores is 72.

- 10** Certain packets of sugar have a mean mass of 1.06 kg with a standard deviation of 20 g. What is the mass of a packet of sugar with z-score -1.8 ?

- 11** Molly scored 89% in an exam, which gave her a z-score of 2.7. If the standard deviation was 5%, find the mean.

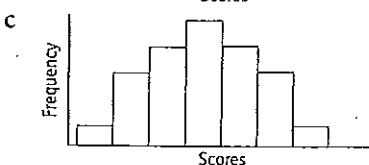
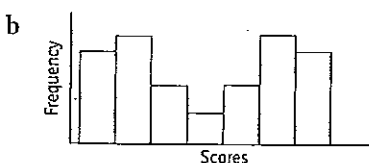
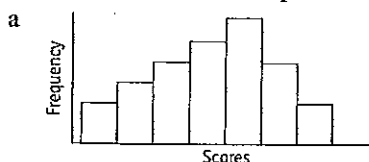
- 12** Corey sat for tests in both geography and history. In geography he scored 62% with a z-score of 2.1 and in history he scored 66% with a z-score of 1.9. Which is the better result?

- Jeremy received a z-score of -1.6 for his algebra test. What does this mean?
- If Jeremy received a z-score of -1.2 for his financial mathematics test, which was the better result?

- 14** Friends Jaelyn and Paige attend different schools and are arguing about who scored better in their yearly science tests.

- Jaelyn scored 74% in her exam. The mean was 68% and the standard deviation was 7.5%. What is Jaelyn's z-score?
- Paige scored 71% in her exam. The mean was 63% and the standard deviation 6%. What was Paige's z-score?
- Who performed better? Justify your answer.

- 15** Determine whether the data could be normally distributed from the shape of the histogram.



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- 16** Consider the scores:
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| 14 | 14 | 15 | 15 | 15 | 15 | 15 | 15 | 15 |
| 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 |
| 17 | 17 | 17 | 17 | 17 | 17 | 17 | 18 | 18 |
| 18 | 18 | 18 | 19 | 19 | 19 | 19 | 20 | 20 |
- Find the mean.
 - What is the mode?
 - What is the median?
 - Draw a frequency histogram.
 - Is the data normally distributed?

- 17** An exam was given to Year 12 students and the results were normally distributed. The table below shows the marks associated with certain z-scores.

Mark	40	49	58	67	76	85	94
z-score	-3	-2	-1	0	1	2	3

- What is the mean?
- What is the standard deviation?
- Approximately what percentage of scores lie between 58 and 76?
- Approximately what percentage of scores lie between 40 and 94?
- Approximately what percentage of scores lie between 49 and 85?

- 18** The number of chocolates in baskets sold at a market is normally distributed with mean 30 and standard deviation 3.
- Between what numbers will the contents of approximately 95% of the baskets lie?
 - Approximately what percentage of baskets will have between 27 and 33 chocolates?

- 19** The marks in an exam were normally distributed with mean 67% and standard deviation 7. Between what marks do 99.7% of the scores lie?

- 20** The marks in an exam are normally distributed with mean 69% and standard deviation 10%.
- Between what marks will a particular score most probably lie?
 - Between what marks will a particular score almost certainly lie?

- 21** The capacities of certain drums of oil are normally distributed with a mean of 11 litres and a standard deviation of 500 mL.
- Draw up a table to show the capacities corresponding to z-scores of -3, -2, -1, 0, 1, 2 and 3.
 - Approximately what percentage of drums have capacities:
 - between 10 L and 12 L?
 - between 11 L and 11.5 L?
 - less than 11 L?
 - less than 10 L?
 - between 11.5 L and 12.5 L?

- 22** Certain packets of dried apricots are labelled as having a mass of 150 g. The contents of the packets are normally distributed with a mean of 160 g and a standard deviation of 5 g. What percentage of packets will hold less than 150 g?


- 23** A survey was conducted of the costs of 3200 plants and the results were found to form a normal distribution. The mean cost was \$18 and the standard deviation was \$2.25. Approximately how many of the plants cost between \$13.50 and \$22.50?


- 24** The manufacturer of certain watches guarantees them for six months. He has had tests conducted on the watches and knows that the results were normally distributed with a mean trouble-free period of 12 months with standard deviation of two months.
- Approximately what percentage of watches would need to be replaced under guarantee?
 - The manufacturer sells 240 000 of the watches this month. How many could he expect to have to replace?

- 25** Hermione runs a coaching college and gives all her students an exam at the end of the course. From past experience Hermione knows that the results are normally distributed with mean 76.5% and standard deviation 7.5%. The latest course has 2000 students and Hermione offers to refund the cost of the course to any student who does not score a certain percentage. What should that percentage be if Hermione is only prepared to refund the cost for three students?

Go to p 286 for Quick Answers
or to pp 315–17 for Worked Solutions


Challenge: The Normal Distribution

-  The mass of certain bags of fertilizer have been found to be normally distributed with mean mass 1.1 t and standard deviation of 50 kg. What is the mass of a bag with z-score 1.6? *Hint 1*

-  80 students did a general knowledge quiz and the results, as marks out of 10, are recorded below.


Mark	1	2	3	4	5	6	7	8	9	10
Number	1	0	5	8	10	12	17	14	9	4


- Find the mean mark.
- Find the standard deviation (σ_n) to two decimal places.
- What percentage of scores lie within one standard deviation of the mean? *Hint 2*

-  Andrew studied a course at college and achieved the following results in weekly tests, with theory and practical components both being marked out of fifty.

Theory	24	30	28	25	34	29	32
Practical	32	40	41	33	36	40	35


- Find the mean and standard deviation (σ_{n-1}) for each set of marks.
- Were Andrew's results more consistent in theory or practical? Justify your answer. *Hint 3*
- In his next test, Andrew scored 35 in theory and 39 in practical. Which would you consider the better result for Andrew? *Hint 4*

-  3600 students sat for an exam, the results of which were normally distributed. The mean mark was 64% and the standard deviation was 7%. How many students would you expect to score 50% or better? *Hint 5*

-  The table has been drawn up to show the z-scores corresponding to certain heights, following a survey of students.


Height (cm)			152			176	
z-score	-3	-2	-1	0	1	2	3

- What is the standard deviation? *Hint 6*
- What is the mean?
- Assuming the data is normally distributed, find the height for which 97.5% of students are taller.

-  A survey was taken of the ages of students in a particular college course. The results are shown in the table below.

Age	Frequency
18	2
19	5
20	7
21	3
22	2
56	1

- What percentage of students are over 50?
- It is known that the ages of the students studying at this college are normally distributed with mean 20 and standard deviation 1.5. The teacher of this course suggests in a staff meeting that 5% of expenditure in the student music library should be on music that appeals to the over 50s. Explain why this is a bad idea. *Hint 7*

-  The five-number summary has been calculated for a set of scores: 37 40 42 44 48. The mean has been calculated to be 42 and the standard deviation is 3.2. What percentage of scores are within two standard deviations of the mean? *Hint 8*

Go to p 287 for Quick Answers
or to pp 317–18 for Worked Solutions

Hint 1: The mean and standard deviation must be in the same units.

Hint 2: The answer is not 68%. (You don't know that the data is normally distributed.) Use the data supplied.

Hint 3: Use the standard deviations from part a.

Hint 4: Consider the mean and standard deviation for each part of the course.

Hint 5: Find the z-score corresponding to 50%.

Hint 6: How many standard deviations are there between the known heights?

Hint 7: Use the mean and standard deviation and the percentages applying to certain z-scores.

Hint 8: What score is two standard deviations above the mean? What score is two standard deviations below the mean? The five-number summary gives the upper and lower extremes, as well as the upper and lower quartiles and the median.

Solutions.

Ch 5: The Normal Distribution

Further Practice p88

1 a Joel's mark is 2 standard deviations above the mean.

$$\begin{aligned} \text{b Mark} &= 65 + 2 \times 5.5 \\ &= 76 \end{aligned}$$

Joel's mark was 76%.

2 a Victoria's mark is 2 standard deviations below the mean.

$$\begin{aligned} \text{b Mark} &= 71 - 2 \times 7 \\ &= 57 \end{aligned}$$

Victoria's mark was 57%.

3 The weight of the packet is 1.6 times the standard deviation below the mean.

$$\begin{aligned} \text{Weight} &= 260 - 1.6 \times 5 \\ &= 252 \end{aligned}$$

The packet will weigh 252 grams.

4 Mean is 64.

$$\begin{aligned} \text{Standard deviation} &= 73 - 64 \\ &= 9 \end{aligned}$$

Number	37	46	55	64	73	82	91
z-score	-3	-2	-1	.0	1	2	3

5 $x = 34$, $\bar{x} = 28$, $s = 2.5$

$$\begin{aligned} z &= \frac{x - \bar{x}}{s} \\ &= \frac{34 - 28}{2.5} \\ &= 2.4 \end{aligned}$$

6 $x = 80$, $\bar{x} = 66$, $s = 5.6$

$$\begin{aligned} z &= \frac{x - \bar{x}}{s} \\ &= \frac{80 - 66}{5.6} \\ &= 2.5 \end{aligned}$$

Brandon has a z-score of 2.5

7 $x = \$206$, $\bar{x} = \$230$, $s = \$15$

$$\begin{aligned} z &= \frac{x - \bar{x}}{s} \\ &= \frac{\$206 - \$230}{\$15} \\ &= -1.6 \end{aligned}$$

The z-score for that worker was -1.6

8 a $\bar{x} = 73.75$

$$\begin{aligned} \sigma_n &= 11.878\ 727\ 49 \dots \\ &= 11.88 \text{ (2 d.p.) [by calculator]} \end{aligned}$$

b $x = 64$, $\bar{x} = 73.75$, $s = 11.88$

$$\begin{aligned} z &= \frac{x - \bar{x}}{s} \\ &= \frac{64 - 73.75}{11.88} \\ &= -0.820\ 707\ 07 \dots \\ &= -0.8 \text{ (1 d.p.)} \end{aligned}$$

Callum's z-score was -0.8

9 $x = 83, \bar{x} = 72, z = 2.2$

$$z = \frac{x - \bar{x}}{s}$$

$$2.2 = \frac{83 - 72}{s}$$

$$= \frac{11}{s}$$

$$2.2s = 11$$

$$s = \frac{11}{2.2}$$

$$= 5$$

The standard deviation is 5.

10 $\bar{x} = 1.06, s = 0.02, z = -1.8$

[The mean and the standard deviation must be in the same units. 20 g = 0.02 kg]

$$z = \frac{x - \bar{x}}{s}$$

$$-1.8 = \frac{x - 1.06}{0.02}$$

$$-0.036 = x - 1.06$$

$$1.024 = x$$

The packet has a mass of 1.024 kilograms.

11 $x = 89, s = 5, z = 2.7$

$$z = \frac{x - \bar{x}}{s}$$

$$2.7 = \frac{89 - \bar{x}}{5}$$

$$13.5 = 89 - \bar{x}$$

$$\bar{x} = 89 - 13.5$$

$$= 75.5$$

The mean mark was 75.5%.

12 Corey did better in geography, because the z-score was higher.

13 a $z = -1.6$

Jeremy's mark was 1.6 times the standard deviation below the mean.

b Financial mathematics was the better result. [It was 1.2 times the standard deviation below the mean.]

14 a $x = 74, \bar{x} = 68, s = 7.5$

$$z = \frac{x - \bar{x}}{s}$$

$$= \frac{74 - 68}{7.5}$$

$$= 0.8$$

Jaclyn's z-score is 0.8

b $x = 71, \bar{x} = 63, s = 6$

$$z = \frac{x - \bar{x}}{s}$$

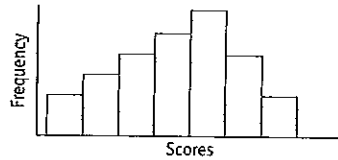
$$= \frac{71 - 63}{6}$$

$$= 1.333 \dots$$

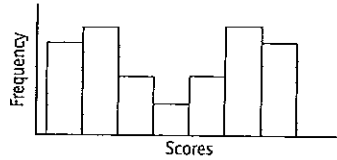
Paige's z-score was 1.3

c Paige performed better. The z-score corresponding to her mark is higher than that corresponding to Jaclyn's mark.

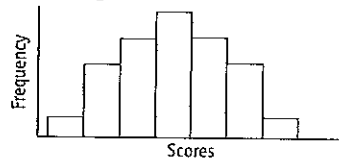
15 a Data is not normally distributed. [Not symmetrical]



b Not normally distributed. [It is symmetrical about the mean but it is not bell-shaped. There are two modes.]



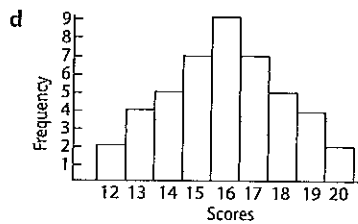
c Normally distributed. [Bell-shaped curve]



16 a Mean = $720 \div 45$
= 16

b Mode = 16

c Median = 16 [The 23rd score.]



e Yes, the data is normally distributed. [The mean, mode and median are all equal to 16. The histogram is 'bell-shaped'.]

17

Mark	40	49	58	67	76	85	94
z-score	-3	-2	-1	0	1	2	3

a Mean = 67 [z = 0]

b $s = 76 - 67$
= 9

c 68% of scores [-1 < z < 1]

d 99.7% of scores [-3 < z < 3]

e 95% of scores [-2 < z < 2]

18 a 95% lie within 2 standard deviations of the mean.

$$\bar{x} + 2s = 30 + 2 \times 3$$

$$= 36$$

$$\bar{x} - 2s = 30 - 2 \times 3$$

$$= 24$$

95% of baskets will have between 24 and 36 chocolates.

b $x = 27, \bar{x} = 30, s = 3$

$$z = \frac{x - \bar{x}}{s}$$

$$= \frac{27 - 30}{3}$$

$$= -1$$

$x = 33, \bar{x} = 30, s = 3$

$$z = \frac{x - \bar{x}}{s}$$

$$= \frac{33 - 30}{3}$$

$$= 1$$

Approximately 68% lie within 1 standard deviation of the mean.
∴ Approximately 68% of baskets will have between 27 and 33 chocolates.

19 99.7% of scores lie within 3 standard deviations of the mean.

$$\bar{x} + 3s = 67 + 3 \times 7$$

$$= 88$$

$$\bar{x} - 3s = 67 - 3 \times 7$$

$$= 46$$

99.7% of marks lie between 46% and 88%

20 a $\bar{x} + 2s = 69 + 2 \times 10$
= 89

$$\bar{x} - 2s = 69 - 2 \times 10$$

$$= 49$$

A score will most probably lie between 49% and 89%

b $\bar{x} + 3s = 69 + 3 \times 10$
= 99

$$\bar{x} - 3s = 69 - 3 \times 10$$

$$= 39$$

A score will almost certainly lie between 39% and 99%

21 a

Capacity (L)	9.5	10	10.5	11	11.5	12	12.5
z-score	-3	-2	-1	0	1	2	3

b i Between 10 L and 12 L is within two standard deviations of the mean. Approximately 95% of drums will be between 10 L and 12 L.

ii 68% of scores lie within 1 standard deviation of the mean. Half will lie between the mean and 1 standard deviation above the mean. Approximately 34% of drums will have capacities between 11 L and 11.5 L.

iii The mean is 11 L. Half of the drums will hold less than the mean. Approximately 50% of drums will have capacities less than 11 L.

iv 95% of drums have capacities between 10 L and 12 L. Half of the remaining drums will have capacity less than 10 L. Approximately 2.5% of drums will have capacities less than 10 litres.

v 99.7% of drums have capacities within 3 standard deviations of the mean and 68% have capacities within 1 standard deviation of the mean.
Difference = 99.7% - 68%
= 31.7%

31.7% of drums have capacities between one and three standard deviations of the mean. Half of these will be between 11.5 L and 12.5 L. Approximately 15.85% of drums will hold between 11.5 L and 12.5 L.

22 $x = 150, \bar{x} = 160, s = 5$

$$z = \frac{x - \bar{x}}{s} = \frac{150 - 160}{5} = -2$$

95% will lie within 2 standard deviations of the mean.
100% - 95% = 5% will lie outside 2 standard deviations.
Half of these will have less than 150 g.
2.5% of packets will have less than 150 g.

23 $\bar{x} = \$18, s = \2.25

$$\bar{x} + 2s = \$18 + 2 \times \$2.25 = \$22.50$$

$$\bar{x} - 2s = \$18 - 2 \times \$2.25 = \$13.50$$

95% of plants cost between \$13.50 and \$22.50.

Number of plants = 95% of 3200
= $0.95 \times 3200 = 3040$

Approximately 3040 plants will cost between \$13.50 and \$22.50

24 a $x = 6, \bar{x} = 12, s = 2$

$$z = \frac{x - \bar{x}}{s} = \frac{6 - 12}{2} = -3$$

Those more than 3 standard deviations below the mean would need to be replaced.
99.7% are within 3 standard deviations of the mean. 0.3% lie outside 3 standard deviations.
Half of these are below the mean.
Required percentage = $0.3\% \div 2 = 0.15\%$

b Number replaced = $0.0015 \times 240\,000 = 360$

The manufacturer would need to replace approximately 360 watches.

25 Percentage to refund = $\frac{3}{2000} \times 100\% = 0.15\%$

99.7% of students score within three standard deviations of the mean.

\therefore 0.3% will score more than three standard deviations from the mean.
0.15% will score more than three standard deviations below the mean.

Percentage = $76.5\% - 3 \times 7.5\% = 54\%$

Hermione should guarantee that students will score 54% or better.

Challenge p90

1 $\bar{x} = 1.1, s = 0.05, z = 1.6$

$$z = \frac{x - \bar{x}}{s}$$

$$1.6 = \frac{x - 1.1}{0.05}$$

$$0.08 = x - 1.1$$

$$x = 1.18$$

The mass of the bag is 1.18 t

2

Mark	1	2	3	4	5	6	7	8	9	10
Number	1	0	5	8	10	12	17	14	9	4

a $\bar{x} = 6.525$

b $\sigma_n = 1.961\,982\,416 \dots = 1.96$ (2 d.p.)

c $\bar{x} + s = 6.525 + 1.96$

$$= 8.485$$

$$\bar{x} - s = 6.525 - 1.96$$

$$= 4.565$$

Scores within 1 standard deviation of the mean are 5, 6, 7 and 8.

Total = $10 + 12 + 17 + 14$

$$= 53$$

Percentage = $\frac{53}{80} \times 100\% = 66.25\%$

3

Theory	24	30	28	25	34	29	32
Practical	32	40	41	33	36	40	35

a Theory: $\bar{x} = 28.857\,1428 \dots$

$$= 28.9$$
 (1 d.p.)

$$\sigma_{n-1} = 3.579\,039\,51 \dots$$

$$= 3.6$$
 (1 d.p.)

Practical: $\bar{x} = 36.714\,2857 \dots$

$$= 36.7$$
 (1 d.p.)

$$\sigma_{n-1} = 3.638\,419\,33 \dots$$

$$= 3.6$$
 (1 d.p.)

b The standard deviation is the same for both so Andrew is equally consistent in both theory and practical.

c The theory mark is more than one standard deviation above the mean. The practical mark is less than one standard deviation above the mean. The theory mark was the better result.

4 $x = 50, \bar{x} = 64, s = 7$

$$z = \frac{x - \bar{x}}{s} = \frac{50 - 64}{7} = -2$$

50% is two standard deviations below the mean. Approximately 2.5% of students should score more than two standard deviations below the mean so 97.5% should score more than 50%.

97.5% of 3600 = 3510

About 3510 students should score 50% or better.

5 a From 152 to 176 is 3 standard deviations.

$$176 - 152 = 24$$

$$\therefore s = 24 \div 3$$

$$= 8$$

Height (cm)			152			176	
z-score	-3	-2	-1	0	1	2	3

b $\bar{x} = 152 + 8$

$$= 160$$

c 95% of students will be within two standard deviations of the mean. Half of the remaining 5% will be taller than 2 standard deviations above the mean. 97.5% of students will be taller than two standard deviations below the mean.

Height = $152 - 8$

$$= 144$$

Approximately 97.5% of students will be taller than 144 cm.

6

Age	Frequency
18	2
19	5
20	7
21	3
22	2
56	1

a 1 student out of 20 is over 50.

Percentage = $\frac{1}{20} \times 100\% = 5\%$

b Approximately 99.7% of students will have ages within 3 standard deviations of the mean. 99.7% are between 15.5 and 24.5 years. Only about 0.15% will be over 24.5 years. It is very unusual to have any students over 50, certainly not 5%.

7 37 40 42 44 48

Two standard deviations above the

$$\text{mean} = 42 + 2 \times 3.2$$

$$= 48.4$$

Two standard deviations below the

$$\text{mean} = 42 - 2 \times 3.2$$

$$= 35.6$$

The lower extreme is 37 and the upper extreme is 48.

100% of scores are within 2 standard deviations of the mean.