Sample Preliminary examination 4

Sample exam 4
Section 1 (Suggested time: 35 minutes)
Multiple-choice questions   25 marks
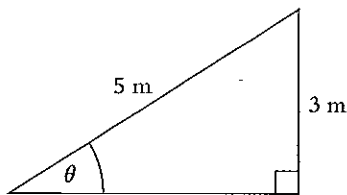
- Attempt ALL questions.
- All questions are of equal value.

- Select the alternative (A, B, C or D) that best answers the question.

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1 Find the value of x if $x^3 + 16 = 8$.

A $x = 2$ B $x = -2$

C $x = 2.88$ D $x = -2.88$

2 The value of $\dfrac{\sqrt[3]{3.16}}{6.23 - 2.36}$ correct to two decimal places is:

A -2.12 B 0.38

C 2.70 D 8.15

3 Which statement is correct?

A $\sin\theta = \dfrac{4}{5}$ B $\sin\theta = \dfrac{3}{5}$

C $\cos\theta = \dfrac{3}{5}$ D $\tan\theta = \dfrac{4}{3}$

4 A school selected 15% of its staff to attend a seminar. One-third of these were unable to attend. What fraction of the school staff went to the seminar?

A $\dfrac{1}{20}$ B $\dfrac{2}{3}$

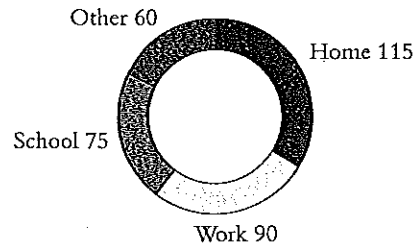C $\dfrac{3}{40}$ D $\dfrac{1}{10}$

5 An antique clock, normally priced at $1450, is sold at a discount of 10%. The selling price is:

A $145   B $14.50

C $1595   D $1305

6 Given that $P = \dfrac{\sqrt[3]{R}}{V^2 S}$ and $R = 107.2$, $V = \dfrac{1}{5.61}$ and $S = 0.001\,701$, the value of P to three significant figures is:

A 8.79×10^4 B 2.543×10^{-1}

C 2.54×10^{-1} D 8.7892×10^4

7 Patrice collected data on where people downloaded music. The results are shown in this sector graph:

The percentage of people who downloaded music from their home is:

A 29.6% B 33.8%
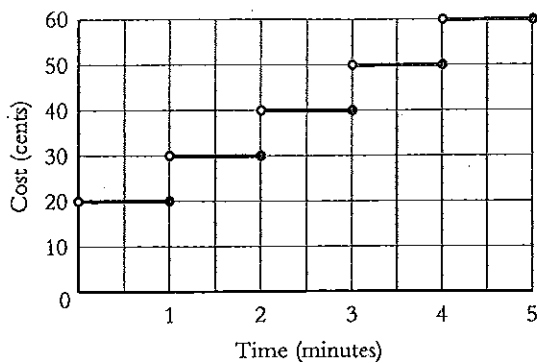
C 51.1% D 115%

8 Express $2^{-2} + 3^{-2}$ as a single fraction.

A $\dfrac{1}{13}$ B $\dfrac{2}{13}$

C $\dfrac{1}{36}$ D $\dfrac{13}{36}$

9 The following formulae refer to simple interest on a loan. They have been re-arranged to make different subjects. Which formula is *incorrect*?

A $I = Prn$ B $P = \dfrac{I}{rn}$

C $n = \dfrac{I}{Pr}$ D $r = \dfrac{In}{P}$

10 The following step graph shows the costs of making a timed phone call:

Which of these statements is *false*?

A A call lasting 120 seconds costs 30 cents.

B The first 1 minute of a phone call costs 20 cents.

C After the first minute, phone call charges increase by 10 cents for each additional minute.

D Two phone calls, each lasting $2\frac{1}{2}$ minutes costs the same as one 5-minute phone call.

11 On a freeway the speed limit is 110 km/h. This is equivalent to:

A $3.0\dot{5}$ m/s   B $30.\dot{5}$ m/s

C $183.\dot{3}$ m/s   D $1833.\dot{3}$ m/s

12 The following illustration shows part of an online conversion for download speed:

Once the 'Calculate' button is clicked, what value should appear in the bottom box?

A 100   B 125

C 800   D 8000

13 As a decimal, $5\frac{1}{2}\%$ is:

A 0.055   B 0.55

C 5.5   D 5.005

14 Ben's wage of $480 per week is increased by $24. The increase as a percentage of the previous wage is:

A 0.5%   B 16%

C 5%   D 5.12%

15 The volume of a sphere is given by the formula $V = \frac{4}{3}\pi r^3$.

A sphere with a radius of 300 mm has a volume of:

A 0.1 m^3 (1 d.p.)   B 1131 mm^3 (4 s.f.)

C 113 cm^3 (3 s.f.)   D 400π mm^3

16 An approximation for $\pi - \dfrac{1.6}{2.3 \times 0.5}$ is:

A 1.32   B 2.79

C 1.34   D 1.75

17 A car has a fuel consumption of 13.4 L/100 km. Timothy drives a distance of 780 km. The amount of fuel used is closest to:

A 85 L   B 95 L

C 105 L   D 115 L

18 Find the difference between the **mean** and the **mode** of these scores: 8, 6, 1, 8, 2

A 0   B 1

C 2   D 3

19 Half of my salary is used to pay the rent and two-fifths of the remainder is saved. The fraction of my salary saved is:

A $\dfrac{1}{5}$ B $\dfrac{9}{10}$

C $\dfrac{2}{7}$ D $\dfrac{2}{5}$

20 Girls comprise 47% of the population of a school that has 636 boys. How many girls are there?

A 1200   B 1353

C 564   D 717

21 When 5 037 000 is written in scientific notation, correct to 3 significant figures, the result is:

A 5.04×10^3 B 5.03×10^6

C 5.03×10^3 D 5.04×10^6

22 $3^6 \times 3^{-2}$ equals:

A 9^4 B 9^8

C 3^4 D 3^8

23 Simplify $\dfrac{a^2b^3}{(ab^2)^2}$.
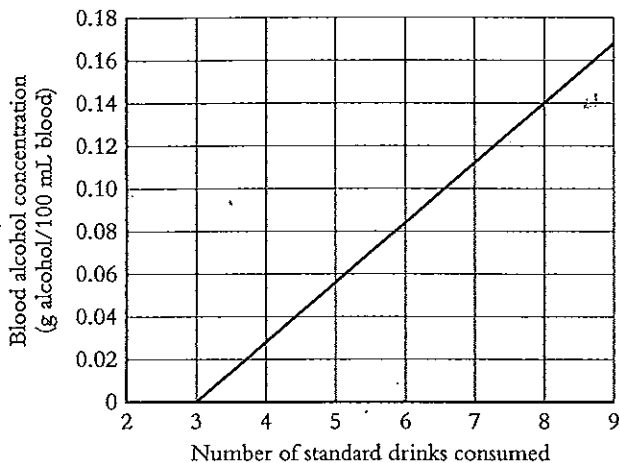
A ab^2 B $\dfrac{1}{b}$

C b D ab

24 Mary is a 65 kg female who is about to go to a party where she anticipates she will be drinking for 4 hours. The blood alcohol concentration (BAC) for Mary can be shown using this graph:

Number of standard drinks consumed

What is the greatest number of standard drinks she can consume in this time period and still remain under the 0.05 BAC limit?

A 3   B 4

C 5   D 6

25 Claire buys a camera for $250, which includes a 10% GST. What is the cost of the camera without the GST?

A $275   B $22.73

C $25   D $227.27

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Sample exam 4
Section 2   (Suggested time: 85 minutes)
Objective-response questions   60 marks

- **Attempt ALL questions.**
- **Each question is worth 15 marks.**

- **Show all working.**

- -

26
a   Given that $N^3 = 6.4 \times 10^{15}$, find N to the nearest whole number.   1 mark

b   It took Colin 7 minutes 45 seconds to download an MP3 file that was 6.2 MB in size.

   i   Calculate the number of kilobits in 6.2 MB.   2 marks

   ii   Calculate the download speed.   2 marks

c   Karin receives a 20% increase in her weekly wage. If the increase is $40, find her new weekly wage.   1 mark

d   A gardening expert recommends 15 litres of water per week for each 1 metre height of gumtree. A garden hose fills a 10-litre bucket in 30 seconds. Calculate the time that should be spent each week watering a 9-metre high gum tree with the hose.   3 marks

e   The number of correct answers given by students in a quiz were:

10	8	5	7	6	8
9	9	10	7	6	6
8	6	7	5	10	10
8	6	5	7	7	9
8	9	7	6	9	5

   i   Construct a dot plot for the data given.   2 marks

   ii   Establish a 5-number summary for the quiz results.   2 marks

   iii   Calculate the range and interquartile range.   2 marks

27 a Eric buys a car priced $75 000 and he is allowed $52 000 trade-in for his old car. The balance is borrowed on time payment over 3 years at 10.5% p.a. flat. He is also charged 5% stamp duty on all money borrowed over $1000. Calculate the:

   i interest to be paid   2 marks

   ii stamp duty   2 marks

   iii monthly instalments if all the debt, excluding the stamp duty, is to be paid by 36 equal monthly repayments.   2 marks

b Ollie owns a car that runs on unleaded petrol (ULP), which has a fuel consumption of 11.5 L/100 km. When converted to run on liquefied petroleum gas (LPG), the car's fuel consumption will be 15.1 L/100 km. Vasili drives an average of 16 500 km each year.

   i Calculate the annual fuel cost of running his car on each type of fuel, given ULP costs 149.9¢/L, and LPG costs 78.7¢/L.   4 marks

   ii How much could Ollie save each year on fuel if he converted to LPG?   1 mark

   iii It costs $3100 to convert the car to run on LPG. How many months would it take to break even if he made the conversion?   2 marks

   iv What distance would he need to travel to break even?   2 marks

28 a A ship is sailing from A to B towards a cliff GT of height 300 m. At A the angle of elevation of the cliff is 80°. At B the angle of elevation is 85°.

[Not to scale] 300 m
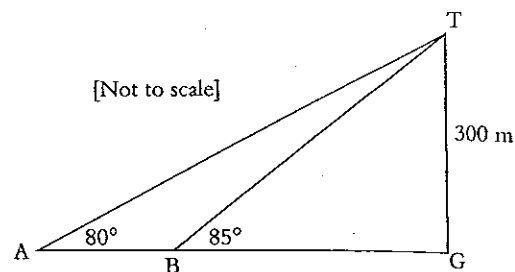
   i Find the distance of the ship from the base of the cliff when it is at A (i.e. the distance AG) to the nearest metre.   1 mark

   ii If in 5 minutes the ship travels to B where the angle of elevation is 85°, find the distance BG.   1 mark

   iii Also find the speed of the ship in kilometres per hour.   2 marks

b When the angle of elevation of the sun is 56°, a vertical pole casts a shadow on horizontal ground that is 2.7 m long.

   i Find the height of the pole.   1 mark

   ii At the same time a tree casts a shadow that is 36 m long. Briefly explain why the two triangles are similar.   1 mark

   iii Use similar triangles to find the height of the tree.   2 marks

c Annemieke runs a business from home making cakes. She is considering expanding her business to include cupcakes and cake pops and decides to conduct a survey to see if people would be interested in buying them and at what price. She sends the survey to every third person on her email and phone contact lists.

   i What type of sampling is this an example of?   1 mark

   ii Briefly describe why this method of surveying is biased.   2 marks

d A survey counted the number of phone calls made in an office over a 24-hour period and the results placed in this bar chart:

Time of day
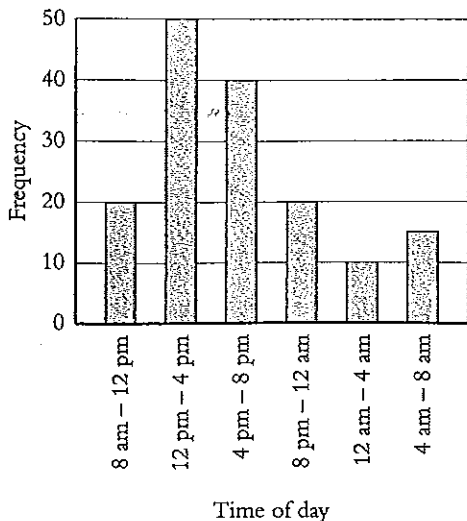
i How many calls were made
altogether? 1 mark

ii What percentage of calls was made
between 8 pm and 12 am? 1 mark

iii Peak calls occur between 8 am and
8 pm If the total cost for these calls
is $37.40, calculate the charge for
each call. 2 marks

29 a There are several formulae for calculating
stopping distance. One of these is:

$$D = \frac{5Vt}{18}(5t + 2.3)$$

where D = Stopping distance (m)

V = Speed (km/h) and

t = Reaction time (s).

Use this formula to calculate the
stopping distance, correct to one
decimal place, of a vehicle travelling
60 km/h, where the driver's reaction
time is 0.7 seconds. 2 marks

b A bicycle track consists of two straight
stretches of 100 m with two semicircles
at the ends with a diameter of 30 m.
The track is 10 m wide.
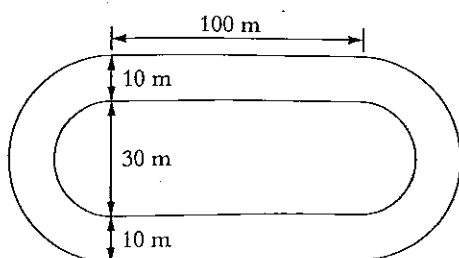
[Not to scale]

i Find the length of fencing required
to fence the outside of the track
(answer to the nearest metre). 3 marks

ii The actual track is to be concreted
with concrete to a depth of 20 cm.
How many cubic metres of concrete
are required? (Answer to one
decimal place.) 2 marks

c In a certain state the stamp duty for a
new private car is listed as:

Car value	Stamp duty
$600 or less	$20
$601 to $35 000	3% ($3 for every $100 or part thereof)
$35 001 to $40 000	$1050 + 11% for the proportion over $35 000
$40 001 and over	4% ($4 for every $100 or part thereof)

i Calculate the stamp duty payable
for a vehicle costing:

A $27 500 2 marks

B $39 950 2 marks

ii How much *more* in stamp duty is
paid for a vehicle costing $44 000
than one costing $22 000? 2 marks

d Lauren prepared a graph to show the
cost of running an art studio:
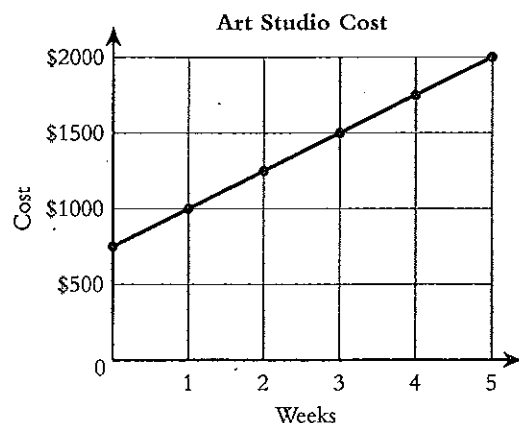
i Calculate the gradient of the line. 1 mark

ii What does the y-intercept represent? 1 mark

Worked Solutions

Sample Preliminary examination 4
Section 1 . . . . pp. 236–238

1 $x^3 + 16 = 8$

$$x^3 = -8$$
$$x = \sqrt[3]{-8}$$
$$x = -2 \quad \checkmark$$

2 $\dfrac{\sqrt[3]{3.16}}{6.23 - 2.36} = \dfrac{\sqrt[3]{3.16}}{3.87}$

$$= 0.379\,185\ldots \text{ [Cal.]}$$
$$= 0.38 \text{ [2 d.p.]} \quad \checkmark$$

3 $\sin\theta = \dfrac{3}{5} \quad \checkmark$

4 $\dfrac{2}{3}$ of the 15% of the staff attended the seminar

$$= \frac{2}{3} \text{ of } 15\%$$
$$= \frac{2}{3} \times 15\%$$
$$= \frac{1}{10} \quad \checkmark$$

5 Normal price = \$1450

Discount = 10%

∴ The clock is sold for 90% of the normal price.

$$= 90\% \times \$1450$$
$$= \$1305 \quad \checkmark$$

6 $R = 107.2, V = \dfrac{1}{5.61}, S = 0.001701$

$$P = \frac{\sqrt[3]{R}}{V^2 S}$$

$$P = \frac{\sqrt[3]{107.2}}{\left(\dfrac{1}{5.61}\right)^2 \times 0.001701}$$

$$P = 87\,892.7401\ldots$$
$$P = 8.79 \times 10^4 \text{ [3 s.f.]} \quad \checkmark$$

7 Total of $115 + 90 + 75 + 60 = 340$ people were surveyed, of which 115 downloaded music from home. This gives

$$\frac{115}{340} \times \frac{100}{1} = 33.8\% \quad \checkmark$$

8 $2^{-2} + 3^{-2} = \dfrac{1}{2^2} + \dfrac{1}{3^2}$

$$= \frac{1}{4} + \frac{1}{9}$$
$$= \frac{13}{36} \quad \checkmark$$

9 The formula given on the formula sheet for simple interest is $I = Prn$. This can be rearranged to make different subjects. Making r the subject gives

$$r = \frac{I}{Pn}, \text{ not } r = \frac{In}{P}. \quad \checkmark$$

10 All statements are correct, except the last. A $2\frac{1}{2}$ minute phone call costs 40 cents, so two of those make 80 cents, while a 5 minute phone call costs 60 cents, so the calls are not the same cost. $\quad \checkmark$

Worked Solutions to Chapter 10

11 Speed limit 110 km/h $= \dfrac{110 \text{ km}}{1 \text{ hour}}$

Speed limit 110 km/h $= \dfrac{110 \times 1000}{1 \times 60 \times 60}$

$= 30.\dot{5}$ m/s ✓

12 There are 8 bits to a byte. Thus $1000 \div 8$ means 125 will appear on the calculator. ✓

13 $5\dfrac{1}{2}\% = \dfrac{5.5}{100}$

$= 0.055$ ✓

14 Previous wage = $480 per week
Increase = $24
Increase as a percentage of the previous wage

$= \dfrac{24}{480} \times 100\%$

$= 5\%$ ✓

15 $r = 300$ mm $= 0.300$ m

$V = \dfrac{4}{3}\pi r^3$

$V = \dfrac{4}{3} \times \pi \times (0.300)^3$

$V = 0.113097\ldots$ m^3 [Cal.]
$V = 0.1$ m^3 [1 d.p.] ✓

16 $\pi - \dfrac{1.6}{2.3 \times 0.5} = \pi - \dfrac{1.6}{1.15}$

$= 1.750288\ldots$ [Cal.]

$= 1.75$ [approximately] ✓

17 Each 100 km driven uses 13.4 L. So driving 7.8×100 kilometres uses $7.8 \times 13.4 = 104.52$ L, which is closest to 105 L. ✓

18 Mean $= \dfrac{8 + 6 + 1 + 8 + 2}{5}$

$= \dfrac{25}{5}$

$= 5$

Mode $= 8$
Difference $= 8 - 5$

$= 3$ ✓

19 Salary saved $= \dfrac{2}{5}$ of half

Salary saved $= \dfrac{2}{5} \times \dfrac{1}{2}$

Salary saved $= \dfrac{1}{5}$ ✓

20 $100\% - 47\% = 53\%$
53% of the population are boys
$\therefore$ 53% of the population $= 636$

1% of the population $= \dfrac{636}{53}$

100% of the population $= \dfrac{636}{53} \times 100$

$= 1200$

Number of girls $= 1200 - 636$

$= 564$ ✓

21 $5\,037\,000 = 5.037 \times 10^6$

$= 5.04 \times 10^6$ ✓

22 $3^6 \times 3^{-2} = 3^{6 + -2}$

$= 3^4$ ✓

23 $\dfrac{a^2 b^3}{(ab^2)^2} = \dfrac{a^2 b^3}{a^2 b^4}$

$= \dfrac{1}{b}$ ✓

24 From the graph, four standard drinks give her a BAC of around 0.028, but five standard drinks has her at 0.056, which is over the 0.05 limit. Thus to stay under the 0.05 limit after 4 hours, Mary must consume no more than four standard drinks. ✓

25 $110\% = \$250$

$1\% = \dfrac{\$250}{110}$

$100\% = \dfrac{\$250}{110} \times 100$

$= \$227.27$ ✓

Sample Preliminary examination 4
Section 2 . . . . pp. 238–240

26 a $N^3 = 6.4 \times 10^{15}$

$N = \sqrt[3]{6.4 \times 10^{15}}$

$N = 185\,663.5533\ldots$ [Cal.]

$N = 185\,664$ [to nearest whole number] ✓

b i Number of kilobits $= 6.2 \times 8 \times 1024$ ✓

$= 50\,790.4$ kilobits ✓

ii 7 min 45 s $= 7 \times 60 + 45 = 465$ s ✓

Download speed $= \dfrac{50\,790.4}{465}$

$= 109.2$ kilobits/second ✓

c   20% of the amount = $40

$$1\% \text{ of the amount} = \frac{\$40}{20}$$

$$100\% \text{ of the amount} = \frac{\$40}{20} \times 100$$

$$= \$200$$

Karin's new weekly wage = $200 + $40

$$= \$240 \quad \checkmark$$

d   Amount of water needed per week for 1-metre high gum tree = 15 L

Amount of water needed per week for 9-metre high gum tree = (15 × 9) L

$$= 135 \text{ L} \quad \checkmark$$

A garden hose fills a 10-litre bucket in 30 seconds.

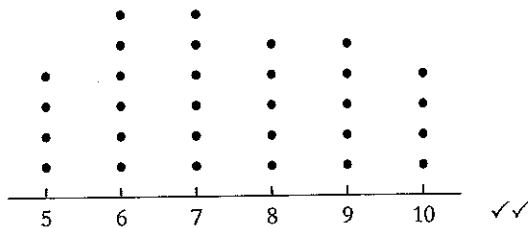$$\therefore \text{ Time spent on 1 litre} = \frac{30}{10} \text{ seconds}$$

$$= 3 \text{ seconds} \quad \checkmark$$

Time spent on 135 L = (3 × 135) seconds

$$= 405 \text{ seconds}$$

$$= 6 \text{ minutes and 45 seconds} \quad \checkmark$$

e   i

5   6   7   8   9   10   ✓✓

ii   Lowest score = 5; Lower quartile = 6;
Median = 7; Upper quartile = 9;
Highest score = 10   ✓✓

iii   Range = Highest score – Lowest score

$$= 10 - 5$$

$$= 5 \quad \checkmark$$

Interquartile range
= Upper quartile – Lower quartile

$$= 9 - 6$$

$$= 3 \quad \checkmark$$

27   a   i   Amount borrowed = $75 000 – $52 000

$$= \$23 000$$

P = $23 000, r = 10.5% p.a., n = 3 years   ✓

Interest to be paid

I = Prn

I = 23 000 × 10.5% × 3

I = $7245   ✓

ii   Excess over $1000 = $23 000 – $1000

$$= \$22 000 \quad \checkmark$$

Stamp duty to be paid = 5% of $22 000

$$= 5\% \times \$22 000$$

$$= \$1100 \quad \checkmark$$

iii   All the debt = $23 000 + $7245

$$= \$30 245 \quad \checkmark$$

$$\therefore \text{ Monthly repayment} = \frac{\$30 245}{36}$$

$$= \$840.138 88... \text{ [Cal.]}$$

$$= \$840.14$$

$$\text{[to nearest cent]} \quad \checkmark$$

b   i   ULP: He uses 11.5 × 165 = 1897.5 L annually   ✓

Cost is 1897.5 × 149.9¢ = 284 435.25 cents

$$= \$2844.35 \quad \checkmark$$

LPG: He uses 15.1 × 165 = 2491.5 L annually   ✓

Cost is 2491.5 × 78.7¢ = 196 081.05 cents

$$= \$1960.81 \quad \checkmark$$

ii   Saving is $2844.35 – $1960.81 = $882.54   ✓

iii $$\frac{\$3100}{\$882.54} = 3.51 \text{ years,} \quad \checkmark$$

which is around 42 months   ✓

iv   He drives $$\frac{16 500}{12} = 1375 \text{ km per month} \quad \checkmark$$

So in 42 months he covers 1375 × 42

$$= 57 750 \text{ km} \quad \checkmark$$

28   a   i $$\tan 80° = \frac{300}{AG}$$

$$AG \tan 80° = 300$$

$$AG = \frac{300}{\tan 80°}$$

$$AG = 52.898 09... \text{ m}$$

$$AG = 53 \text{ m [to nearest metre]} \quad \checkmark$$

ii $$\tan 85° = \frac{300}{BG}$$

$$BG \tan 85° = 300$$

$$BG = \frac{300}{\tan 85°}$$

$$BG = 26.246 59... \text{ [Cal.]}$$

$$BG = 26 \text{ m [to nearest metre]} \quad \checkmark$$

iii $$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

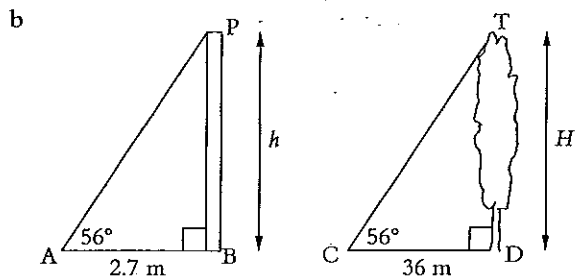$$\text{Speed} = \frac{AG - BG}{5 \text{ minutes}}$$

$$\text{Speed} = \frac{(52.898 - 26.25) \text{ m}}{5 \text{ minutes}}$$

$$S = \frac{26.648 \text{ m}}{\frac{5}{60} \text{ hours}} \quad \checkmark$$

$$S = 319.776 \text{ m/h}$$

$$S = 320 \text{ m/h} \quad \checkmark$$

Worked Solutions to Chapter 10

b

i $\dfrac{h}{2.7} = \tan 56°$

$h = 2.7 \tan 56°$

$h = 4.002914615\dots$ [Cal.]

$h = 4$ m [to nearest metre] ✓

ii Three pairs of corresponding angles are equal and hence the triangles are similar. ✓

iii Since the triangles are similar, their matching sides should be in the same ratio. ✓

$\therefore \dfrac{H}{4} = \dfrac{36}{2.7}$

$H = \dfrac{36}{2.7} \times 4$

$H = 53.33333\dots$ [Cal.]

$H = 53.3$ m [correct to 1 d.p.]

$\therefore$ The height of the tree is 53.3 metres. ✓

c i This is an example of systematic sampling. ✓

ii This method of sampling is biased as she is only asking people she knows. ✓

They may not even be potential customers, so their answers would not represent her target population. ✓

d i Total calls $= 20 + 50 + 40 + 20 + 10 + 15$

$= 155$ ✓

ii Calls between these times $= 20$

$\therefore$ Percentage $= \dfrac{20}{155} \times \dfrac{100}{1} = 12.9\%$ ✓

iii A total of $20 + 50 + 40 = 110$ calls were made in this time. ✓

Each call costs $\dfrac{\$37.40}{110} = \$0.34$ ✓

29 a $D = 5 \times 60 \times \dfrac{0.7}{18} \times (5 \times 0.7 + 2.3) = 67.7$ m ✓

[1 m for correct number of decimal places] ✓

b i Length of fencing required

$= 100 + 100 + 2\pi \times \left(\dfrac{10 + 30 + 10}{2}\right)$ ✓

$= 200 + 2\pi(25)$

$= 200 + 157.079\dots$ [Cal.] ✓

$= 357.08$ m ✓

ii Area $= 2(100 \times 10) + (\pi R^2 - \pi r^2)$

$= 2000 + \pi(R^2 - r^2)$

$= 2000 + \pi(25^2 - 15^2)$

$= 2000 + \pi(400)$

$= 2000 + 1256.637\dots$ [Cal.]

$= 3256.64$ ✓

Volume $= Ah$

Volume $= (3256.64 \times 0.2)$ m^3

Volume $= 651.328$ m^3

Amount of concrete needed is 651.3 m^3. ✓

c i A There are $\dfrac{27500}{100} = 275$ hundreds in \$27 500 ✓

So the stamp duty is $275 \times 3 = \$825$ ✓

B Stamp duty $= \$1050 + \dfrac{11}{100} \times (\$39 950 - \$35 000),$ ✓ which equals \$1594.50 ✓

ii For the \$22 000 vehicle, stamp duty $= 220 \times 3$

$= \$660.$

For the \$44 000 vehicle, stamp duty

$= 440 \times 4 = \$1760.$ ✓

Difference $= \$1760 - \$660$

$= \$1100$ ✓

d i Gradient $= \dfrac{\text{Rise}}{\text{Run}}$

$= \dfrac{2000 - 750}{5 - 0}$

$= 250$ ✓

ii The y-intercept represents the fixed cost of opening the art studio. ✓