

# Topic Test: Algebra and Modelling

Total time allowed: 45 minutes Total marks: 35

1 Solve the equation  $\frac{x-4}{4} = 5$ .

- A 16                      B 24  
C 9                         D 12

2 Simplify  $\left(\frac{x^6}{x^3}\right)^2$ .

- A  $x^3$                       B  $x^6$   
C  $x^9$                       D  $x^{12}$

3 Using the formula  $h = 8 + \frac{18-a}{2}$ , what is  $h$  if  $a = 6$ ?

- A 6                         B 14  
C 17                        D 20

4 The equal sides of an isosceles triangle are each 3 cm longer than the third side. The third side has a length of  $x$  cm. What is the perimeter of the triangle?

- A  $(3x + 6)$  cm            B  $(3x - 3)$  cm  
C  $(3x + 3)$  cm            D  $6x$  cm

5  $\frac{24x^2y}{15} \div \frac{8xy^3}{5a} = ?$

- A  $\frac{ax}{y^2}$                       B  $\frac{64x^3y^4}{25a}$   
C  $\frac{x^3y^4}{a}$                       D  $\frac{y^2}{ax}$

6 What is the gradient of the line  $18x + 6y - 4 = 0$ ?

- A 3                         B  $\frac{1}{3}$   
C -3                        D 18

7 The base length  $l$  of a square pyramid of volume  $V$  and perpendicular height  $h$  is given by  $l = \sqrt{\frac{3V}{h}}$ .

Find  $l$  correct to one decimal place if  $V = 835$  and  $h = 10.5$ .

- A 5.6                      B 15.4  
C 360.7                    D 650.2

8 A number is decreased by 5 and then this amount is doubled. The result is 62. Which of these equations represents this information?

- A  $5 - 2x = 62$             B  $2x - 5 = 62$   
C  $2(5 - x) = 62$         D  $2(x - 5) = 62$

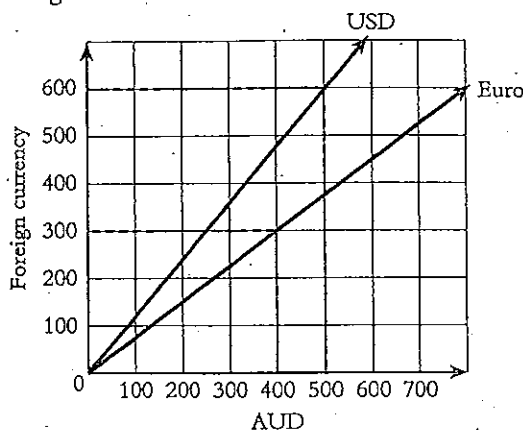
9 Simplify  $3(2x - 1) - 2(x - 2)$ .

- A  $4x + 1$                       B  $4x + 2$   
C  $4x + 3$                       D  $4x + 4$

10 Solve  $\sqrt{9m} = 9$ .

- A  $m = \frac{1}{3}$                       B  $m = 1$   
C  $m = 3$                       D  $m = 9$

11 Using the graph, how many USD can be bought with 300 Euro?



- A 250                        B 400  
C 480                        D 600

12 What is the gradient of the line that passes through the points  $(-2, 0)$  and  $(0, 4)$ ?

- A  $\frac{1}{2}$                          B 2  
C  $-\frac{1}{2}$                         D -2

13 Which of the following is a linear equation?

- A  $y = x^2 + 7$                 B  $y = 5 - 7$   
C  $y = 3x - 2$                 D  $y = \sqrt{x} - 5$

14 The graph  $3x + y = 9$  cuts the  $x$ -axis at which point?

- A  $(3, 0)$                       B  $(0, 3)$   
C  $(0, 9)$                       D  $(9, 0)$

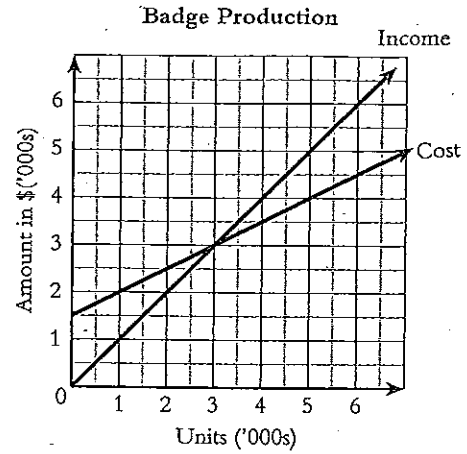
15 What is the gradient of the line represented by the equation  $3x - 5y = 5$ ?

- A  $\frac{3}{5}$                          B  $\frac{5}{3}$   
C 3                         D -5

16 a If  $x^2 = 4ay$ , find  $x$  if  $a = \frac{1}{2}$ ,  
 $y = 8$  and  $x > 0$ .

1 mark

19 Ian produces novelty badges. Use the graph of his costs and income to answer the following questions:



b. The formula,  $t = \frac{m_1 - m_2}{1 + m_1 m_2}$  is used to find the angle between two straight lines with gradients  $m_1$  and  $m_2$ . Find the value of  $t$  if

$$m_1 = \frac{1}{2} \text{ and } m_2 = -\frac{1}{3}$$

1 mark

a What does the  $y$ -intercept for the Cost graph represent?

1 mark

c  $\frac{8x^2y^2}{5xyz} \times \frac{15xz^2}{48y^4}$

1 mark

b What is the gradient of the Income graph?

1 mark

17 Simplify  $\frac{3x}{7} + \frac{x}{5}$ .

1 mark

c What does the gradient represent?

1 mark

18 If  $T^5 = 1.85 \times 10^6$ , find  $T$ , giving your answer correct to two decimal places.

1 mark

d How many badges does Ian need to sell to break even?

1 mark

e What is the variable cost of producing each badge?

1 mark

b For the equation  $3x - y + 4 = 0$

i Make  $y$  the subject.

1 mark

20

a Complete the table of values for  $y = 3x + 2$ :

$x$	-3	-2	-1	0	1	2	3
$y$							

1 mark

b Draw the graph of  $y = 3x + 2$  for values of  $x$  between  $-3$  and  $3$ .

2 marks

ii Draw a sketch graph of the line using only the  $y$ -intercept and the gradient.

2 marks

c Where does the line  $y = 3x + 2$  cut the  $x$ -axis?

1 mark

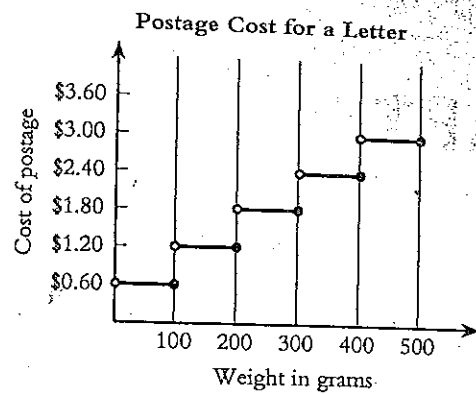
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a Find the equation of a line that has a gradient of 2 and  $y$ -intercept of  $-3$ .

1 mark

22

The step graph below shows the cost of posting letters of various weights:



a What is the cost of sending three letters of weights 125 g, 215 g and 370 g?

1 mark

b What is the cost of sending a letter  
that weighs 400 g?

1 mark

$$\begin{aligned} \frac{x-4}{4} &= 5 \\ x-4 &= 20 \\ x &= 24 \quad \checkmark \end{aligned}$$

$$\begin{aligned} \left(\frac{x^6}{x^3}\right)^2 &= \frac{x^{12}}{x^6} \\ &= x^{12-6} \\ &= x^6 \quad \checkmark \end{aligned}$$

$$\begin{aligned} \text{When } a=6, h=8 + \frac{18-6}{2} \\ &= 8+6 \\ &= 14 \quad \checkmark \end{aligned}$$

$$\begin{aligned} P &= x + (x+3) + (x+3) \\ P &= 3x+6 \quad \checkmark \end{aligned}$$

$$\begin{aligned} \frac{24x^2y}{15} \div \frac{8xy^3}{5a} &= \frac{24x^2y}{15} \times \frac{5a}{8xy^3} \\ &= \frac{120ax^2y}{120xy^3} \\ &= \frac{ax}{y} \quad \checkmark \end{aligned}$$

6 The gradient is  $m$  in  $y = mx + b$ , so arranging this equation into gradient-intercept form will reveal the value of the gradient.

$$\begin{aligned} 18x + 6y - 4 &= 0 \\ 6y &= 4 - 18x \\ y &= \frac{4}{6} - \frac{18}{6}x \\ y &= \frac{2}{3} - 3x \end{aligned}$$

$\therefore$  The gradient is  $-3$ .  $\checkmark$

$$\begin{aligned} 7 \quad l &= \sqrt{\frac{3V}{h}} \\ l &= \sqrt{\frac{3 \times 835}{10.5}} \\ l &= 15.445757... \text{ [Cal.]} \\ l &= 15.4 \quad \checkmark \end{aligned}$$

$$8 \quad 2(x-5) = 62 \quad \checkmark$$

$$\begin{aligned} 9 \quad 3(2x-1) - 2(x-2) &= 6x-3-2x+4 \\ &= 4x+1 \quad \checkmark \end{aligned}$$

$$\begin{aligned} 10 \quad \sqrt{9m} &= 9 \\ \text{Square both sides} \\ (\sqrt{9m})^2 &= (9)^2 \\ 9m &= 81 \\ m &= 81 \div 9 \\ m &= 9 \quad \checkmark \end{aligned}$$

11 Using the graph, the gradient of the Euro line is  $\frac{300}{400}$ , or  $\frac{3}{4}$ , so the equation of the line is: Euro =  $\frac{3}{4}$  AUD.

Likewise, the gradient of the USD line is  $\frac{600}{500}$ , or  $\frac{6}{5}$ , so the equation of the line is: USD =  $\frac{6}{5}$  AUD.

If 300 Euro = 400 AUD, substituting this into the USD equation gives: USD =  $\frac{6}{5} \times 400$   
USD = 480  $\checkmark$

$$\begin{aligned} 12 \quad \text{Gradient} &= \frac{y_2 - y_1}{x_2 - x_1} \\ m &= \frac{4 - 0}{0 - (-2)} \\ m &= \frac{4}{2} = 2 \quad \checkmark \end{aligned}$$

13  $y = 3x - 2$  is a linear equation.  $\checkmark$

$$\begin{aligned} 14 \quad \text{When the line } 3x + y = 9 \\ \text{cuts the } x\text{-axis } y = 0 \\ 3x + 0 &= 9 \\ 3x &= 9 \\ x &= 9 \div 3 \\ x &= 3 \end{aligned}$$

$\therefore$  The point is  $(3, 0)$ .  $\checkmark$

$$\begin{aligned} 15 \quad 3x - 5y = 5 \\ 3x - 5 = 5y \\ 5y &= 3x - 5 \\ \frac{5y}{5} &= \frac{3x}{5} - \frac{5}{5} \\ y &= \frac{3x}{5} - 1 \\ \therefore \text{ Gradient} &= \frac{3}{5} \quad \checkmark \end{aligned}$$

$$\begin{aligned} 16 \quad a \quad x^2 &= 4ay \text{ when } a = \frac{1}{2}, y = 8 \text{ and } x > 0 \\ x^2 &= 4 \times \frac{1}{2} \times 8 \\ x^2 &= 16 \\ x &= \sqrt{16} \\ x &= \pm 4, \text{ but } x > 0 \\ \therefore x &= 4 \quad \checkmark \end{aligned}$$

$$\begin{aligned} b \quad t &= \frac{m_1 - m_2}{1 + m_1 m_2} \\ &= \frac{\frac{1}{2} - \left(-\frac{1}{3}\right)}{1 + \frac{1}{2} \times \left(-\frac{1}{3}\right)} \end{aligned}$$

$t = 1$  (from calculator)  $\checkmark$

$$\begin{aligned} c \quad \frac{8x^2y^2}{5xyz} \times \frac{15xz^2}{48y^4} &= \frac{120x^3y^2z^2}{240xy^3z} \\ &= \frac{x^2z}{2y^3} \quad \checkmark \end{aligned}$$

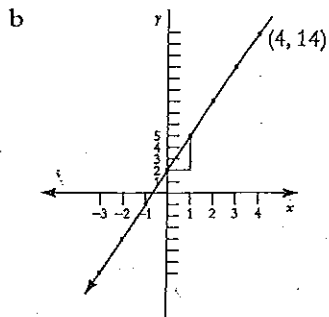
$$\begin{aligned} \frac{3x}{7} + \frac{x}{5} &= \frac{5(3x) + 7(x)}{35} \\ &= \frac{15x + 7x}{35} \\ &= \frac{22x}{35} \quad \checkmark \end{aligned}$$

$$\begin{aligned} T^5 &= 1.85 \times 10^6 \\ T &= (1.85 \times 10^6)^{\frac{1}{5}} \\ T &= 17.923975... \text{ [Cal.]} \\ T &= 17.92 \text{ [2 d.p.]} \quad \checkmark \end{aligned}$$

- 19 a The  $y$ -intercept for the Cost graph represents the fixed costs of production.  $\checkmark$
- b The gradient of the Income graph
- $$= \frac{\text{Rise}}{\text{Run}} = \frac{1000}{1000} = 1. \quad \checkmark$$
- c Each badge is sold for \$1.00.  $\checkmark$
- d The breakeven point is where the two graphs intersect, which is at 3000 units.  $\checkmark$
- e The gradient of the Cost graph represents the variable cost of producing each badge.
- $$\text{Gradient} = \frac{\text{Rise}}{\text{Run}} = \frac{1500}{3000} = \$0.50 \quad \checkmark$$

20 a  $y = 3x + 2$

$x$	-3	-2	-1	0	1	2	3
$y$	-7	-4	-1	2	5	8	11

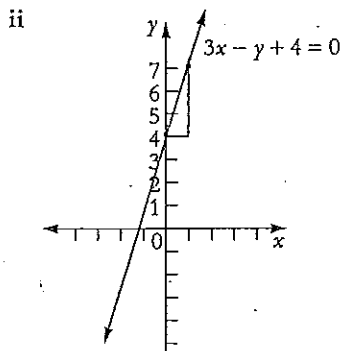


- c The line cuts the  $x$ -axis at  $y = 0$ ; that is,  $x = -\frac{2}{3}$ .  $\checkmark$

21 a  $y = 2x - 3$ .  $\checkmark$

b i  $3x - y + 4 = 0$

$$y = 3x + 4 \quad \checkmark$$



- 21 a: Cost is  $\$1.20 + \$1.80 + \$2.40 = \$5.40$ .  $\checkmark$
- b Closed circle indicates that the cost of sending a 400 g letter still falls in the 300–400 g price range, which costs \$2.40.  $\checkmark$