Algebraic modelling – modelling linear relationships

Student Book - Series L

Contents

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Algebraic modelling – modelling linear relationships

Topic 1 - Tables of values

QUESTION **1** Complete each table of values.

a $y = x + 1$

x	y
0	
1	
2	
3	

b $y = 2x - 1$

x	y
-1	
0	
1	
2	

c $y = 2x + 1$

x	y
-2	
0	
2	
3	

d $y = x + 3$

x	y
-2	
-1	
0	
1	

e $y = 2x + 1$

x	y
-3	
-1	
0	
1	

f $y = 3x - 2$

x	y
-5	
-2	
0	
2	

QUESTION **2** Complete each of the following.

a $p = 2q - 3$

p	q
-1	
0	
1	
2	

b $C = d - 5$

C	d
-1	
0	
1	
2	

c $n = 2 - m$

n	m
-1	
0	
1	
2	

d $x = 5t + 6$

x	t
-1	
0	
1	
2	

e $h = 4x - 3$

h	x
-1	
0	
1	
2	

f $x + y = 4$

x	y
-1	
0	
1	
2	

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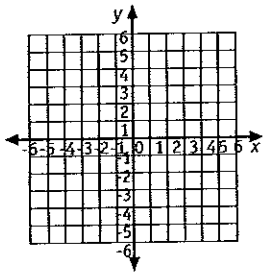
Topic 2 - Straight line graphs

QUESTION **1** Complete each table of values and then graph the equation on the number plane.

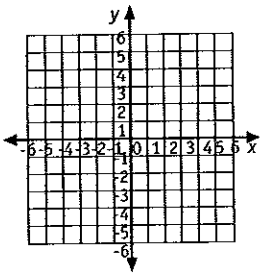
a $y = x - 1$

x	0	1	2	3
y				



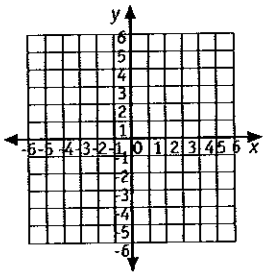
b $y = 3x$

x	0	1	2	3
y				



c $y = 2x + 2$

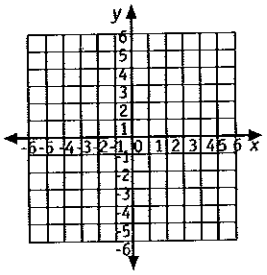
x	0	1	2	3
y				



QUESTION **2** Complete each table of values and then graph the equation on the number plane.

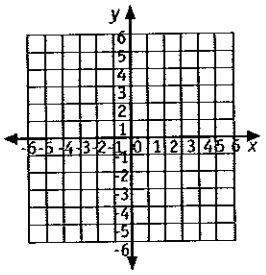
a $x = 2$

x				
y	0	1	2	3



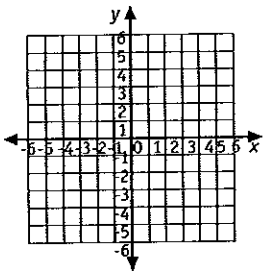
b $y = 3$

x	0	1	2	3
y				



c $y = x$

x	0	1	2	3
y				



QUESTION **3**

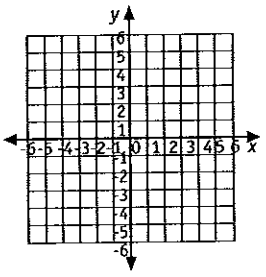
a On the same number plane, graph the following equations by first completing the tables of values.

$y = x - 3$

x				
y				

$y = -x + 3$

x				
y				



b What are the coordinates of the point of intersection of $y = x - 3$ and $y = -x + 3$?

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Topic 3 - Independent and dependent variables

QUESTION **1** Consider the equation $C = 5n + 3$.

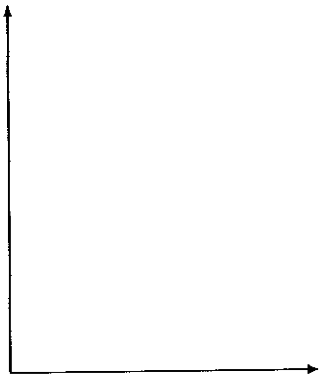
a When drawing up a table of values, for which variable (C or n) do we choose different values? _____

b Which is the independent variable? _____

c Which variable depends on the independent variable? _____

d Which is the dependent variable? _____

e Draw up a table of values for $C = 5n + 3$, $n \geq 0$



f Graph $C = 5n + 3$ on the axes provided.

QUESTION **2** Determine which is the dependent and which the independent variable for each equation.

a $y = 4x - 1$

independent: _____

dependent: _____

b $P = 6 - 2k$

independent: _____

dependent: _____

c $x = 3t + 17$

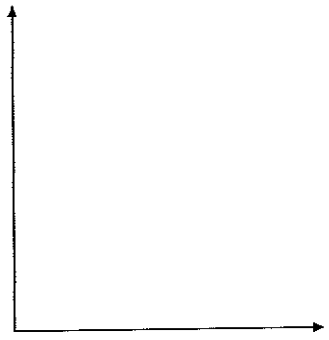
independent: _____

dependent: _____

QUESTION **3** Complete each table of values and graph, remembering to correctly label the axes.

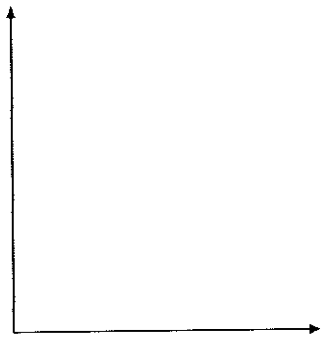
a $p = 2q + 4$

	0	1	2	3



b $C = k + 8$

	0	1	2	3



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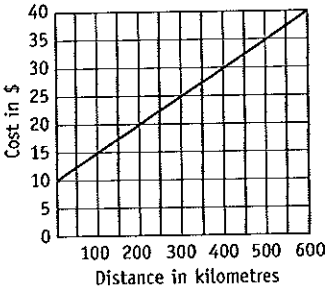
Topic 4 - Graphs of linear functions

QUESTION **1** Elizabeth drew the following graph to give the weekly cost of running her car.



a What is Elizabeth's weekly cost if she travels 400 km? _____

b One week Elizabeth calculates her weekly cost to be $37.50. How many kilometres did she travel that week? _____

c What is the cost if Elizabeth does not travel at all? _____

d Why is this cost (in part c) not $0. Briefly explain.

_____

_____

_____

QUESTION **2** A truck will deliver fuel for $1.15 per litre plus a $100 delivery charge.

a Complete the table.

Amount of fuel (litres)	500	1000	1500	2000
Total cost				

b Draw a graph to show the cost for amounts of fuel up to 2000 litres.

c Dale pays $2170 for a fuel delivery. Use the graph to find the amount of fuel he received.

_____

QUESTION **3** A car's petrol tank holds 60 litres of fuel when full. Felicity fills the tank and drives 400 km. She then fills the tank again and finds that it takes 25 litres of petrol.

a Assuming that the car uses fuel at a constant rate, draw a graph showing the amount of fuel in the petrol tank for each kilometre travelled.

b What restrictions must be put on the graph? Briefly comment.

_____

_____

_____

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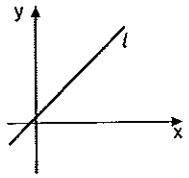
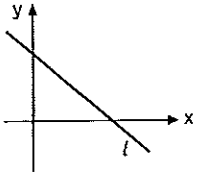
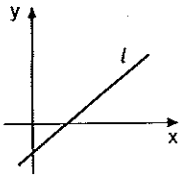
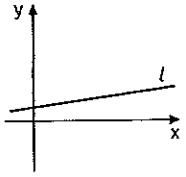
Topic 5 - Gradients

QUESTION **1** State whether the gradient of the line l will be positive or negative.

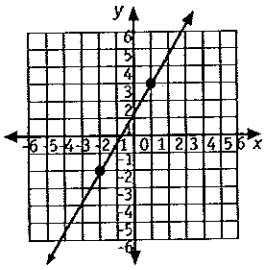
a 

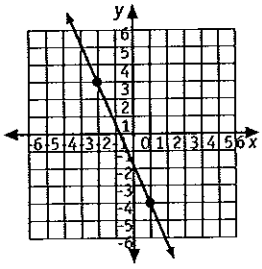
b 

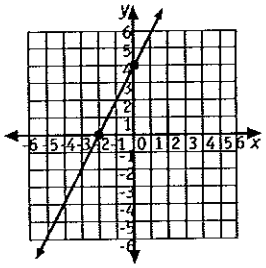
c 

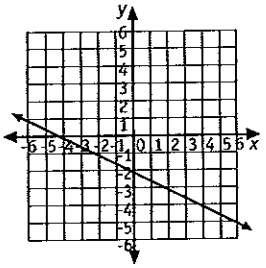
d 

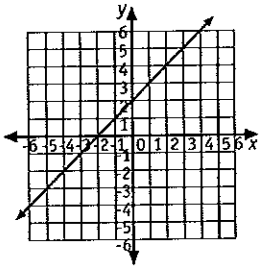
QUESTION **2** Find the gradient of each line.

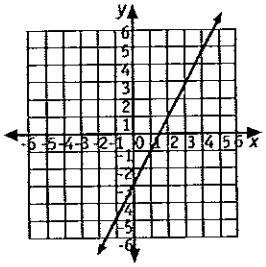
a 

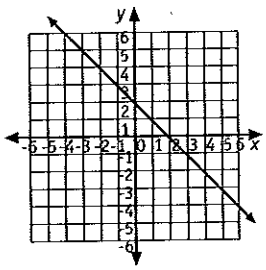
b 

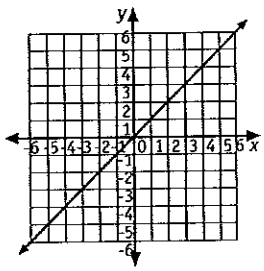
c 

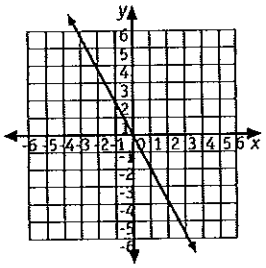
d 

e 

f 

g 

h 

i 

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Topic 6 - Meaning for gradient and vertical intercept

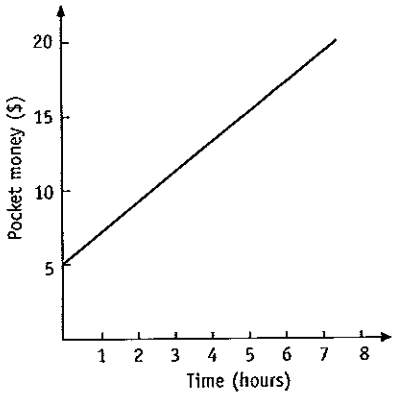
QUESTION **1** Liam receives a fixed amount of pocket money each week. In addition, if Liam chooses to help his mother she gives him an extra amount per hour for the time spent. The graph shows the amount of money Liam might receive in pocket money each week.

a What is the intercept on the vertical axis?

b What does the intercept on the vertical axis represent?

c What is the gradient of this line?

d What does the gradient represent?



QUESTION **2** Dorian intends to ride a bicycle from Aden to Barton to raise money for the local hospital. The graph shows his expected distance from Barton in kilometres over time (in hours).

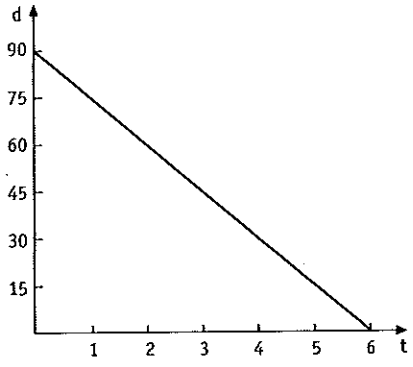
a What is the intercept on the vertical axis?

b What information does this intercept tell us?

c What is the gradient of the line?

d What information does the gradient tell us?

e What is the equation of the line?



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Topic 7 - The graph of y = mx + b

QUESTION **1** For each given equation, write down the gradient and y-intercept.

a $y = 2x + 7$

gradient: _____

y-intercept: _____

b $y = 3x + 1$

gradient: _____

y-intercept: _____

c $y = 7x$

gradient: _____

y-intercept: _____

d $y = 4x - 3$

gradient: _____

y-intercept: _____

e $y = \frac{1}{2}x + 6$

gradient: _____

y-intercept: _____

f $y = x + 4$

gradient: _____

y-intercept: _____

g $y = -3x + 8$

gradient: _____

y-intercept: _____

h $y = -x - 5$

gradient: _____

y-intercept: _____

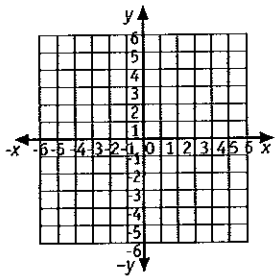
i $y = 11 - 2x$

gradient: _____

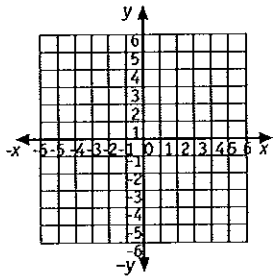
y-intercept: _____

QUESTION **2** Find the y-intercept and the gradient and hence sketch the graph of each line.

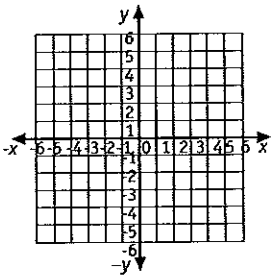
a $y = 3x + 2$



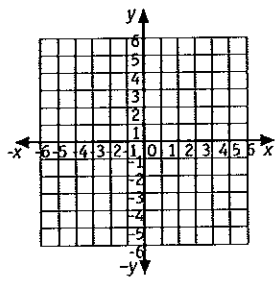
b $y = 2x - 1$



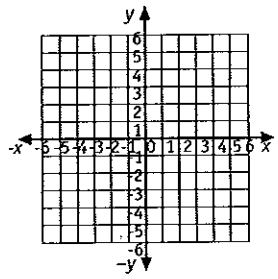
c $y = 3x - 5$



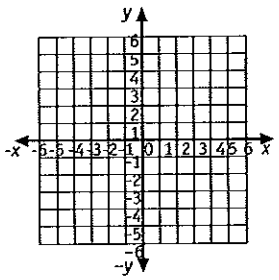
d $y = x$



e $y = -2x + 1$



f $y = \frac{1}{2}x + 4$



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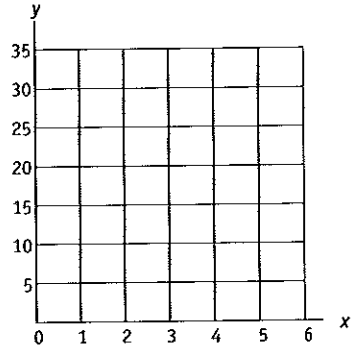
Topic 8 - Graphs involving variation

QUESTION **1** It is known that y varies directly with x. When $x = 5$, $y = 30$.

a Draw the graph of y against x.

b Find the gradient of the graph.



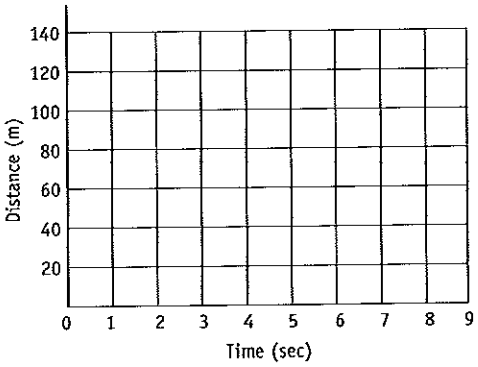
c Write the equation connecting x and y.

QUESTION **2** A car is travelling at a constant speed. It travels 80 m in 5 seconds.

a Draw the graph of distance against time.

b What distance would the car travel in 3 seconds?

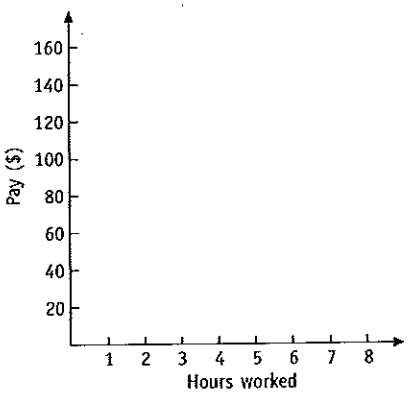
c How many seconds would it take the car to travel 120 m?



QUESTION **3** The pay Sally earns in a day is directly proportional to the number of hours she works. For an 8 hour day she receives $120.

a Draw the graph of pay against hours worked.

b Write an equation connecting pay and hours worked.



c For how many hours would Sally need to work to earn $90?

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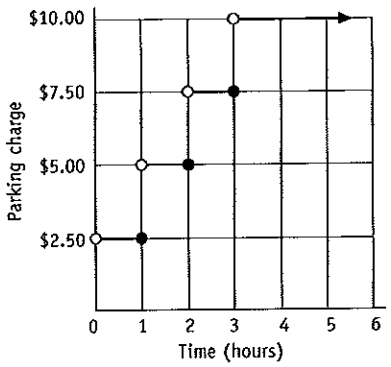
Topic 9 - Stepwise and piecewise linear functions

QUESTION **1** The step graph shows parking charges at a parking station. Use the graph to answer the following questions.



a What is the cost of parking for one hour? _____

b For how long can you park for $7.50? _____

c What is the cost for $2\frac{1}{2}$ hours of parking? _____

d What is the parking cost for 5 hours? _____

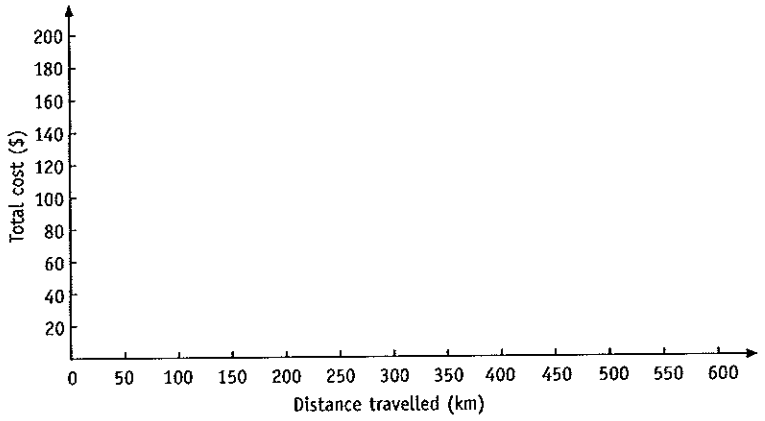
e What is the maximum cost shown on the graph? _____

QUESTION **2** The cost of hiring a small car for a day is $55 plus 30 cents per kilometre over 200 km travelled.

a Complete the table of values.

Distance travelled (km)	0	50	100	150	200	250	300	350	400
Total cost ($)									

b Draw a graph of the cost.



c Dion hired the car for one day and paid $160. How far did Dion travel that day?

_____

QUESTION **3** Calls to a certain information service are charged at 15 cents connection fee plus 45 cents per minute or part thereof. (So a call lasting 30 seconds will cost 60 cents, 15c plus 45c for the part of a minute.)

a How much will a call cost that lasts:

i 1 minute _____

ii 2 minutes _____

iii $1^1/_2$ minutes _____

b Show the charges on a graph.

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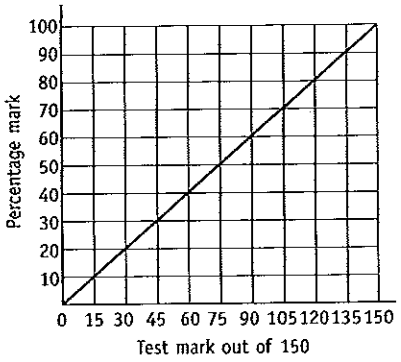
Topic 10 - Conversion graphs

QUESTION **1** The conversion graph on the right changes students' test marks out of 150 to percentages. Use the graph to answer the following questions.



a A student obtains 120 marks out of 150. What percentage is this?

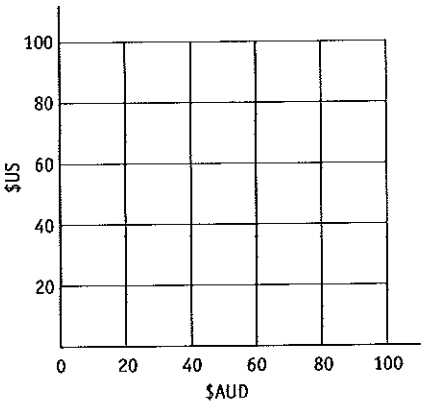
_____

b As 50% is a pass mark, how many marks out of 150 must a student obtain to pass? _____

c A distinction mark is 80% or better. How many marks are needed, out of 150, to gain a distinction? _____

d A student has to be demoted to a lower class if he gets 30 marks or less out of 150. What percentage is this? _____

QUESTION **2** When Nelly was planning her overseas trip, one hundred Australian dollars ($AUD) was worth 72 U.S. dollars ($US).



a Use this fact and the fact that the graph goes through the origin ($0 AUD = $0 US) to draw a straight line graph.

Use the graph to answer the following questions.

b What was the value in U.S. dollars of $75 AUD?

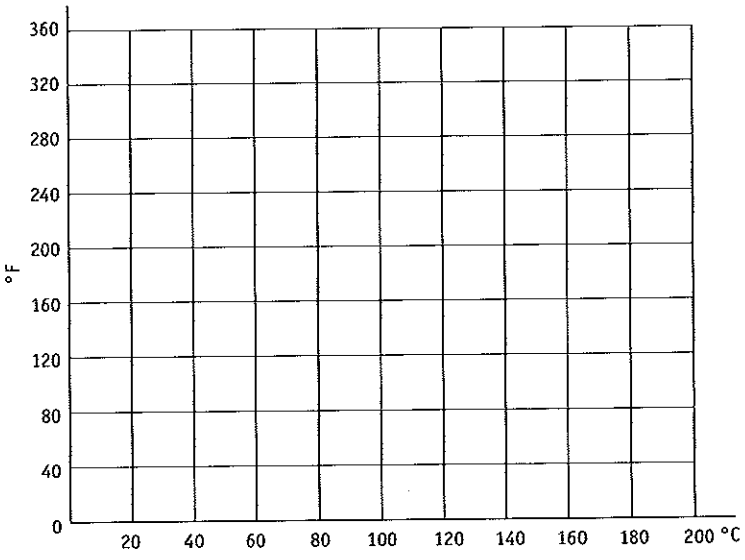
_____

c What was the value in Australian dollars of $40 US?

_____

QUESTION **3** A graph to convert degrees Celsius (°C) to degrees Fahrenheit (°F) is a straight line graph.

a Use the fact that freezing point is 0°C or 32°F and that boiling point is 100°C or 212°F to draw the conversion graph.



b Debbie finds an old recipe for a ginger cake. It needs to be cooked at 325°F. At what temperature, (°C), should Debbie set her oven to cook the ginger cake?

_____

_____

_____

c The forecast temperature is 35°C. What is that in degrees Fahrenheit? _____

Algebraic modelling – modelling linear relationships

Topic 11 - Graphical solution of simultaneous equations

QUESTION **1** The graph shows the cost charged by two different companies to cater for a party. In each case the total cost ($\$C$) depends on the number of people attending (n).

a For what number of people attending do the 2 companies charge the same amount?

_____

b What is the total cost then? _____

c If 9 people are to attend the party, what company would you recommend? Justify your answer.

_____

_____

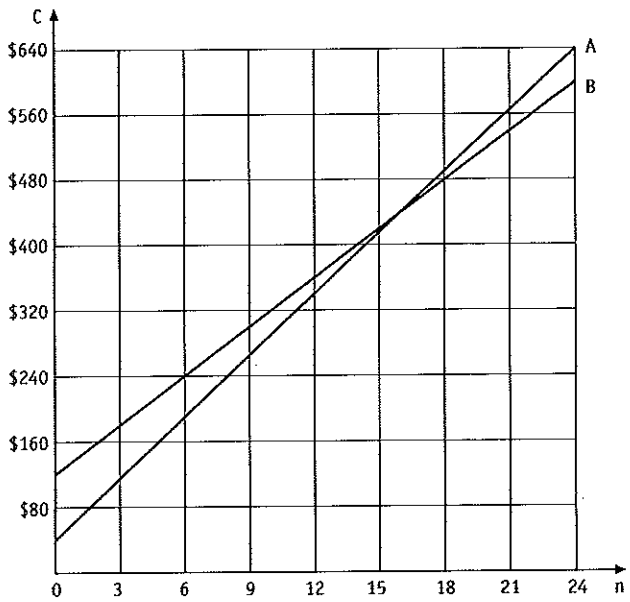
_____

d If 24 people are to attend the party, what is the difference in the cost per person between the two companies?

_____

_____

_____



QUESTION **2** For producing up to 30 items, the cost to a factory is $100 plus $30 for every item. The factory receives $35 for every item sold.

a Complete the table of values.

Number of items	0	5	10	15
Total cost ($)				
Return from sales ($)				

b Show both the total cost and the return on the graph provided at right.

c The factory 'breaks even' when the total cost and the return are equal. How many items does the factory need to produce to break even?

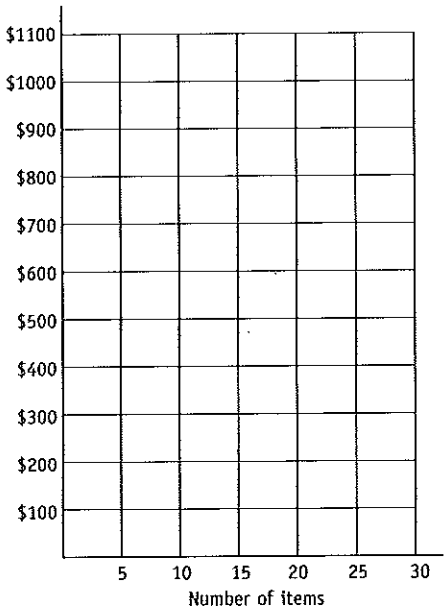
_____

d How many items do you recommend the factory produce? Justify your answer.

_____

_____

_____

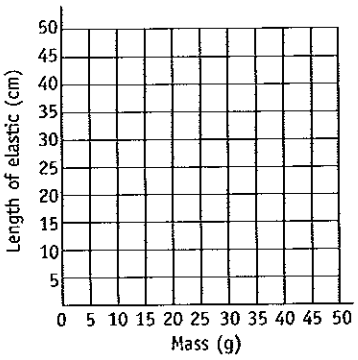


Algebraic modelling – modelling linear relationships

Topic 12 - Lines of best fit

QUESTION **1** A piece of elastic string is fixed at one end and different masses are hung on its free end. The results are shown in the table.

Mass (g)	0	5	10	15	20	25	30	35
Length of elastic (cm)	10	14	20	24	28	33	37	42



a For this information, plot the points.

b Draw a line of best fit.

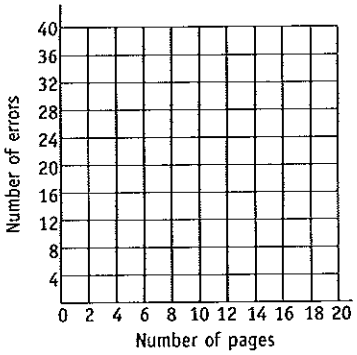
c Use the graph to estimate the length of the elastic when the mass attached weighs:

i 12 g _____ **ii** 40 g _____ **iii** 50 g _____

d Use the graph to find which mass would need to be attached to make the length of the elastic 30 cm.

QUESTION **2** The table shows the number of mistakes found in John's examination papers.

Pages (P)	3	5	6	9	10	12	17
Errors (E)	6	9	11	15	21	25	34



a Plot these points on the number plane.

b Draw a line of best fit.

c Find the equation of this line. _____

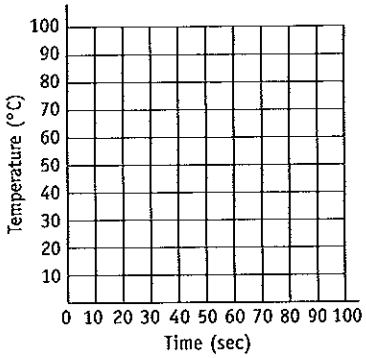
d Use this equation to estimate the number of errors in an examination paper of 20 pages. _____

e Also using the equation, find the average number of errors per page.

_____

QUESTION **3** The table shows the temperature of water in a kettle, measured at intervals of 10 seconds.

Time (s)	0	10	20	30	40	50	60
Temp. (°C)	16	27	39	50	60	71	80



a Plot the points on the number plane.

b Draw a line of best fit.

c Estimate the temperature of water after 25 seconds. _____

d At what time would you expect the water to boil (i.e. to reach 100°C)?

_____

Algebraic modelling – modelling linear relationships

Topic Test — PART A

Instructions This part consists of 10 multiple-choice questions
Each question is worth 1 mark
Calculators may be used
Fill in only ONE CIRCLE for each question

Time allowed: 30 minutes **Total marks = 30**

1 The equation of a linear graph with a y-intercept 3 and gradient -1 is:

Ⓐ $y = -x - 3$ Ⓑ $y = -x + 3$ Ⓒ $y = 3x - 1$ Ⓓ $y = -3x - 1$

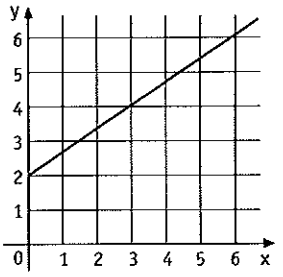
2 The line $y = 2x$ passes through the point:

Ⓐ $(0, -1)$ Ⓑ $(0, 0)$ Ⓒ $(0, 1)$ Ⓓ $(0, 2)$

3 The line $y = 2x - 2$ has:

Ⓐ gradient 2 and y-intercept 2
Ⓑ gradient -2 and y-intercept 2
Ⓒ gradient 2 and y-intercept -2
Ⓓ gradient -2 and y-intercept -2

4 The gradient of this line is:

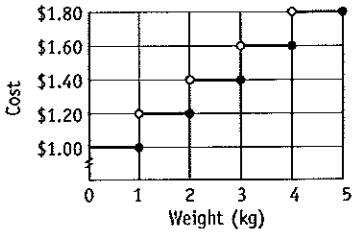


Ⓐ $\frac{2}{3}$ Ⓑ 1

Ⓒ $1\frac{1}{2}$ Ⓓ 2

5 The cost of sending parcels by post for different masses is shown in the step graph. Two parcels weighing 1 kg and 3.75 kg are sent separately to the same address.

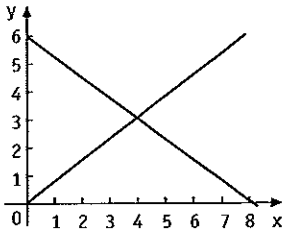
How much would have been saved by sending them together?



Ⓐ 40c Ⓑ 60c

Ⓒ 80c Ⓓ $1.00

6 The solution of these two simultaneous equations is:



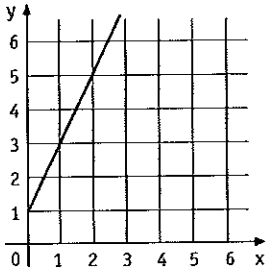
Ⓐ $x = 3$ and $y = 4$ Ⓑ $x = 6$ and $y = 8$

Ⓒ $x = 4$ and $y = 3$ Ⓓ $x = 8$ and $y = 6$

Algebraic modelling – modelling linear relationships

Topic Test — PART A

7 The equation of this line is:



Ⓐ $y = \frac{1}{2}x + 1$

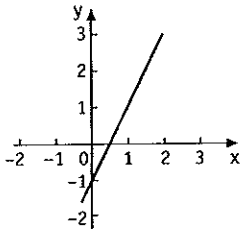
Ⓑ $y = 2x + 1$

Ⓒ $y = x + 2$

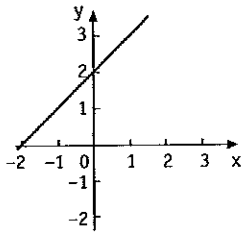
Ⓓ $y = x + \frac{1}{2}$

8 Which is the graph of $y = 2 - x$?

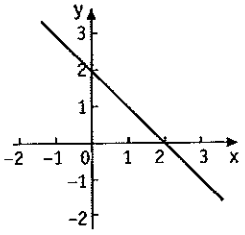
Ⓐ



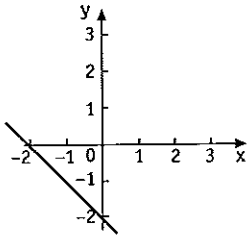
Ⓑ



Ⓒ



Ⓓ



9 This conversion graph has been drawn to convert kilometres to miles.

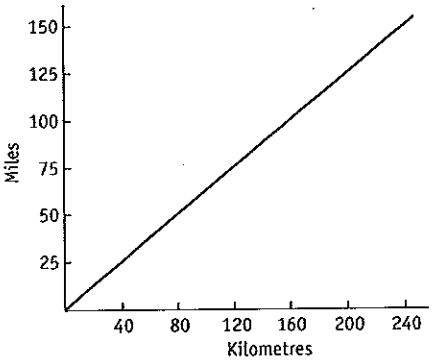
90 miles is approximately:

Ⓐ 55 km

Ⓑ 75 km

Ⓒ 145 km

Ⓓ 175 km



10 For the equation $h = 20t + 50$, consider the following statements:

I h is the independent variable

II t is the dependent variable

Which is correct?

Ⓐ I only Ⓑ II only Ⓒ both I and II Ⓓ neither I nor II

Total marks achieved for PART A ____/10

Algebraic modelling – modelling linear relationships

Topic Test — PART B

Instructions Show all necessary working

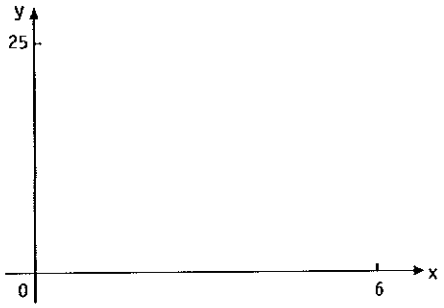
Total marks = 35

11 For the equation $y = 4x + 1$:

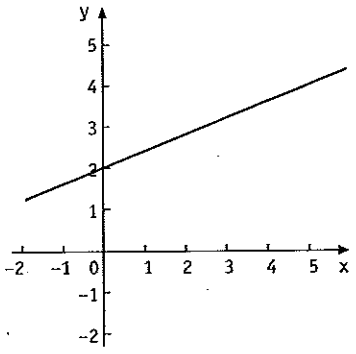
a complete the table of values **2 marks**

x	0	1	2	3
y				

b graph the line on the number plane provided. **2 marks**



12 For this line find:



a the gradient **1 mark**

b the y-intercept **1 mark**

c the equation of the line. **1 mark**

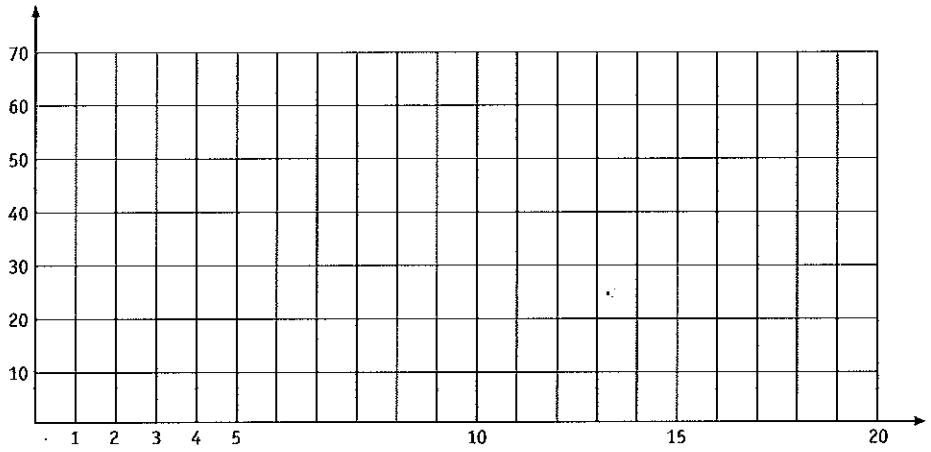
Topic Test — PART B

13 Grace makes batches of home-made lemonade which she sells to her friends by the jug. Grace has calculated that the cost of producing the jugs of lemonade is $8 plus $3 for every jug.

a Complete the table of values **2 marks**

Number of Jugs	0	1	2	3
Cost ($)				

b Draw a graph of the cost on the number plane provided. **2 marks**



c What is the intercept on the vertical axis? Briefly explain what this represents. **2 marks**

d What is the gradient of the line? Briefly explain what it represents? **2 marks**

e What is the cost of producing 12 jugs of lemonade? _____ **1 mark**

f The total cost of a batch Grace made was $56. How many jugs did this batch contain? _____ **1 mark**

g If Grace sells the lemonade for $4 per jug, draw the graph of her return from sales on the same number plane. **1 mark**

h Where do the two lines intersect? Briefly explain what this means. **2 marks**

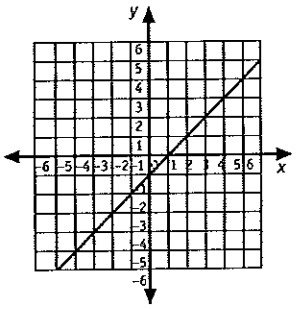
Total marks achieved for PART B ____/20

Answers – Algebraic modelling – modelling linear relationships

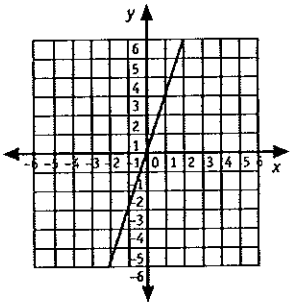
Page 1 1 a 1, 2, 3, 4 b −3, −1, 1, 3 c −3, 1, 5, 7 d 1, 2, 3, 4 e −5, −1, 1, 3 f −17, −8, −2, 4

2 a 1, $1\frac{1}{2}$, 2, $2\frac{1}{2}$ b 4, 5, 6, 7 c 3, 2, 1, 0 d $-\frac{7}{5}, -\frac{6}{5}, -1, -\frac{4}{5}$ e $\frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}$ f 5, 4, 3, 2

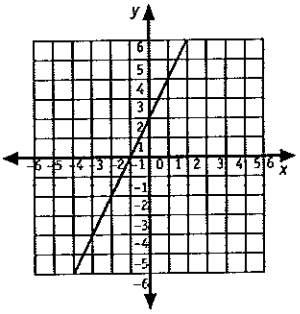
Page 2 1 a (0, −1), (1, 0), (2, 1), (3, 2)



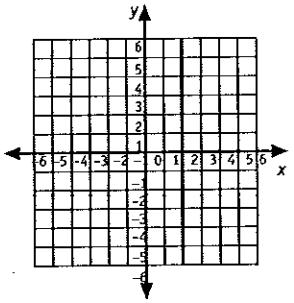
b (0, 0), (1, 3), (2, 6), (3, 9)



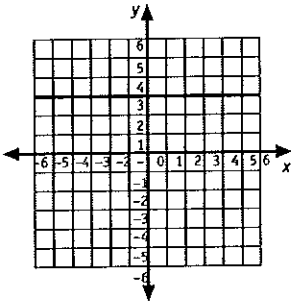
c (0, 2), (1, 4) (2, 6) (3, 8)



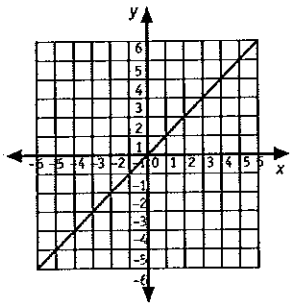
2 a (2, 0), (2, 1), (2, 2), (2, 3)



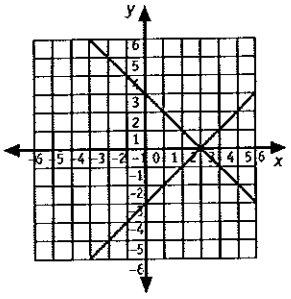
b (0, 3), (1, 3), (2, 3), (3, 3)



c (0, 0), (1, 1), (2, 2), (3, 3)



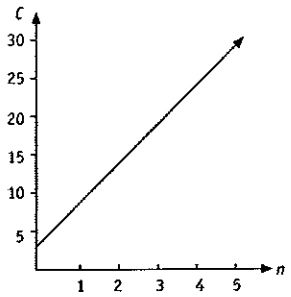
3 a (0, −3), (1, −2), (2, −1), (3, 0); (0, 3), (1, 2), (2, 1), (3, 0) b (3, 0)



Page 3 1 a n b n c Dependent variable d C

e

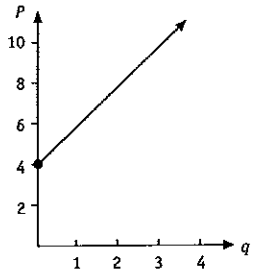
n	0	1	2	3	4	5
C	3	8	13	18	23	28

f 

2 a x, y b k, P c t, x

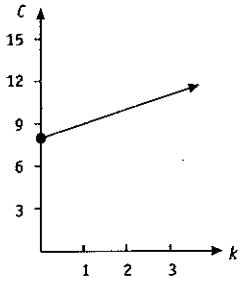
3 a

q	0	1	2	3
p	4	6	8	10



b

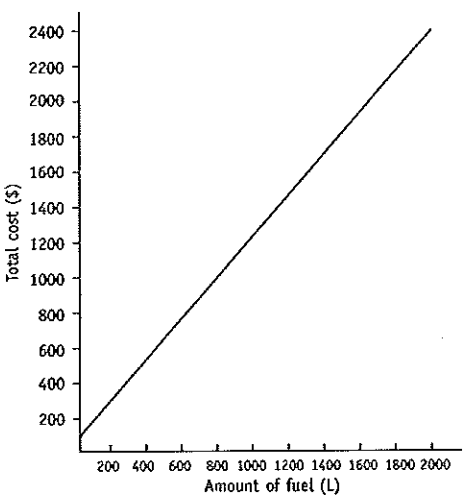
k	0	1	2	3
C	8	9	10	11



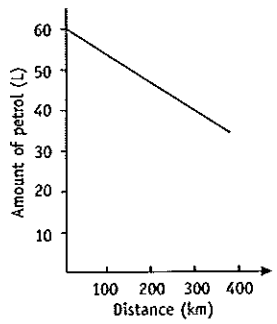
Page 4 1 a \$30 b 550 km c \$10 d Registration and insurance are fixed costs and need to be paid even if no travel was undertaken.

2 a

Amount of fuel (L)	500	1000	1500	2000
Total cost (\$)	675	1250	1825	2400

b 

c 1800 L

3 a 

b The graph could not pass 960 km if the car was not refilled, that is the limit for 60 litres of petrol

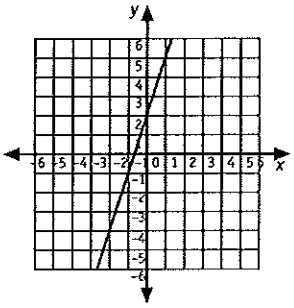
Answers – Algebraic modelling – modelling linear relationships

Page 5 1 a Positive b Negative c Positive d Positive 2 a $\frac{5}{3}$ b $-\frac{7}{3}$ c 2 d $-\frac{1}{2}$ e 1 f 2 g −1 h 1 i −2

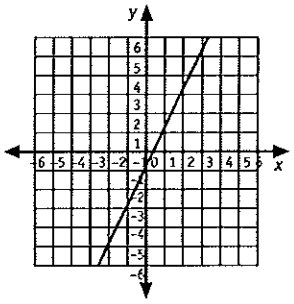
Page 6 1 a 5 b The fixed amount of the pocket money per week, \$5 c 2 d Liam's mother pays him \$2 per hour when he helps her 2 a 90 b Barton is 90 km from Aden. c −15 d Dorian rides at a constant 15 km/h e $d = -15t + 90$

Page 7 1 a 2, 7 b 3, 1 c 7, 0 d 4, −3 e $\frac{1}{2}$, 6 f 1, 4 g −3, 8 h −1, −5 i −2, 11

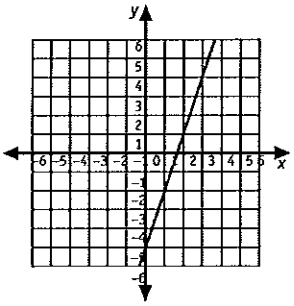
2 a 2, 3



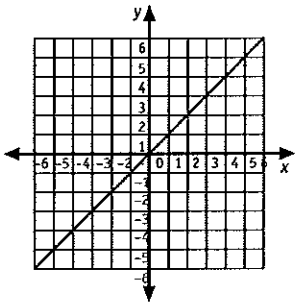
b −1, 2



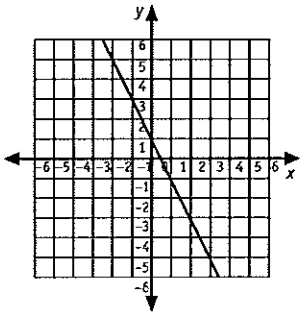
c −5, 3



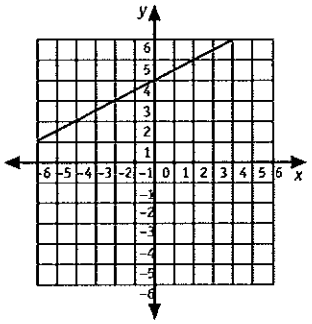
d 0, 1



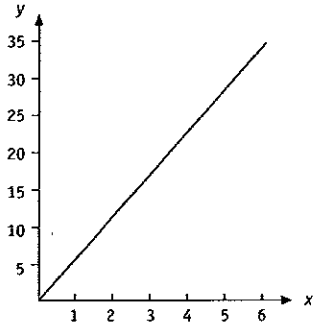
e 1, −2



f 4, $\frac{1}{2}$

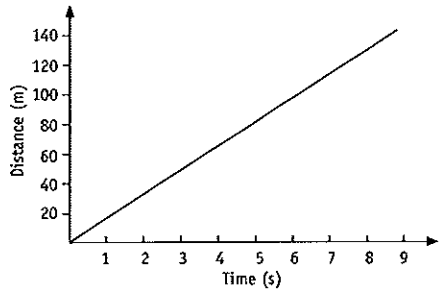


Page 8 1 a

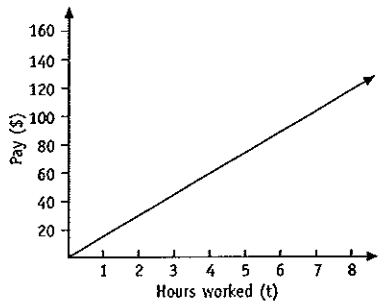


b $m = 6$ c $y = 6x$

2 a



b 48 m c 7.5 s

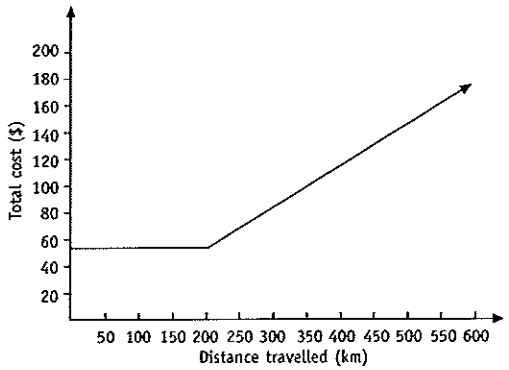
3 a 

b $P = 15t$ c 6 hours

Page 9 1 a \$2.50 b 3 hours c \$7.50 d \$10 e \$10

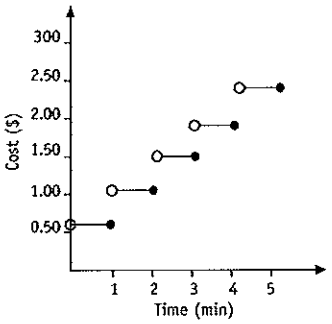
2 a

Distance (km)	0	50	100	150	200	250	300	350	400
Total cost ($)	55	55	55	55	55	70	85	100	115

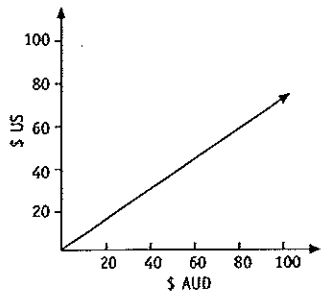
b 

c 550 km

3 a i 60c ii \$1.05 iii \$1.05

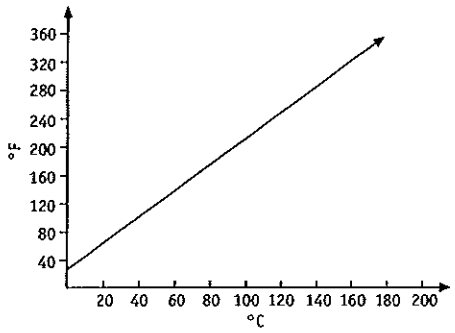
b 

Page 10 1 a 80% b 75 marks c 120 marks d 20%

2 a 

b \$54 US c \$56 AUD

3 a



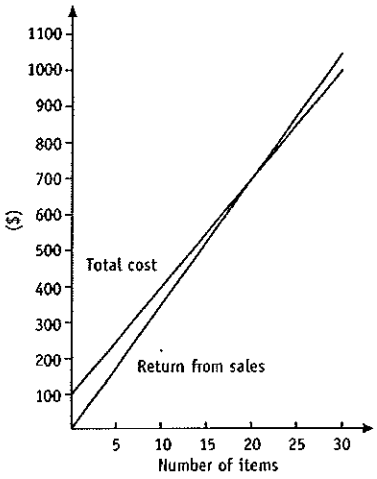
b 162°C c 95°F

Page 11 1 a 16 people b $440 c Company A, it is cheaper by $35 d $1.67 per person

2 a

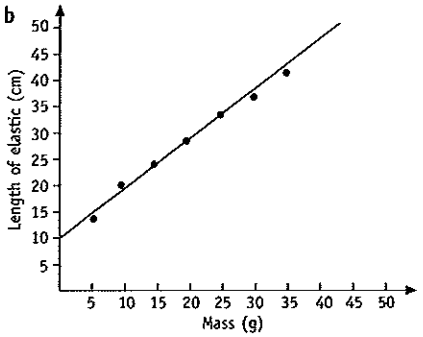
Number of items	0	5	10	15
Total cost ($)	100	250	400	550
Return from sales ($)	0	175	350	525

b



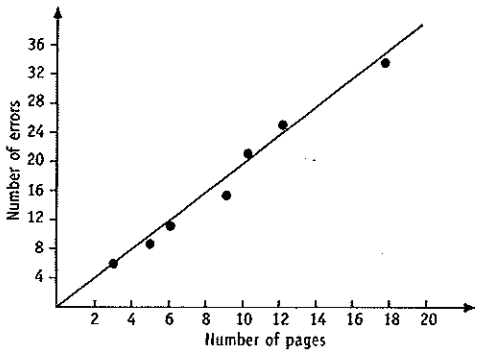
c 20 items d The maximum profit will be made when 30 items are produced, provided they are all sold

Page 12 1 a and b



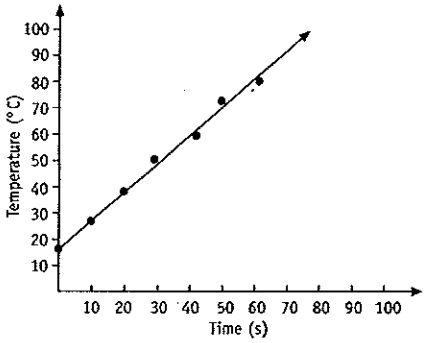
c i 21 cm ii 47 cm iii 56 cm d 22 g

2 **a** and **b**



c $E = 2p$ **d** 40 errors **e** 2 errors/page

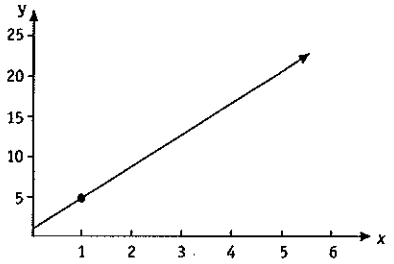
3 **a** and **b**



c 44°C **d** 78 s

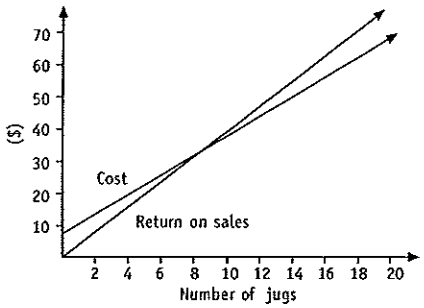
Pages 13–16 **1** B **2** B **3** C **4** A **5** C **6** C **7** B **8** C **9** C **10** D **11** **a** 1, 5, 9, 13

b



12 **a** $\frac{2}{5}$ **b** 2 **c** $y = \frac{2}{5}x + 2$ **13** **a** 8, 11, 14, 17

b and **g**



c 8, $8 is the fixed cost of making a jug of lemonade **d** 3, $3 is the additional cost per jug of lemonade **e** $44 **f** 16 jugs **h** The lines intersect at (8, 32); the break-even point is where 8 jugs of lemonade are produced and sold

