Further Practice: Modelling Linear and Non-linear Relationships

Remember: all questions match the numbered examples on pages 241–258.

1 Complete the table of values for $y = 4x + 7$.

x	0	1	2	3
y				

2 Complete the table of values for $P = 5n - 2$.

n	1	2	3	4
P				

3 Complete the table of values for the rule $y = x^2 + \frac{12}{x}$.

x	1	2	3	4	5	6
y						

4 Consider the rule $y = 4x + 1$.

a Complete the table of values.

x	0	1	2	3
y				

b Graph the line $y = 4x + 1$.

5 a Draw up a table of values, using 0, 2, 4 and 6 as the values of x, and complete for the rule $y = \frac{3x}{2} - 4$.

b Draw the line $y = \frac{3x}{2} - 4$.

6 Draw the straight line $y = 8 - x$.

7 Draw the line $h = 0.8t + 60$ for values of t between 0 and 100.

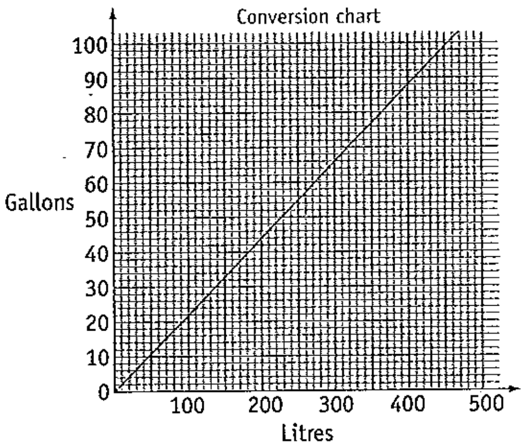
8 A graph has been drawn to convert litres to gallons and vice versa.

a What is the capacity in gallons of a container that holds 300 litres?

b What is the capacity in litres of a container that holds 90 gallons?

c Owen has three drums that each hold 40 gallons. How many litres of water will they hold in total if they are all full?

d Kate has a tank that holds 20 000 litres. She looks at the graph and finds that 200 litres is about 44 gallons so concludes that her tank will hold approximately 4400 gallons. Is she correct? Justify your answer.



9 Nelson has drawn up a table of values for a particular linear relationship.

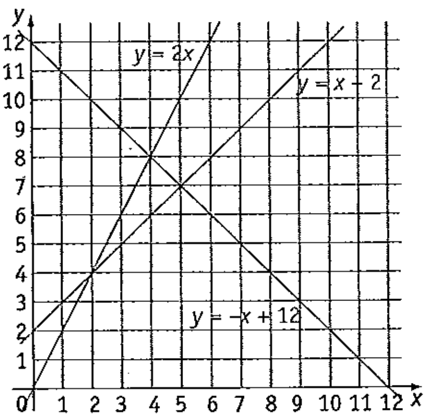
x	0	8	15
y	−5	19	40

a Draw the straight line that passes through the given points.

b What is the value of y when $x = 10$?

c What is the value of x when $y = 4$?

10 The diagram shows the lines $y = x + 2$, $y = 2x$ and $y = -x + 12$.



a Where do the lines $y = x + 2$ and $y = -x + 12$ intersect?

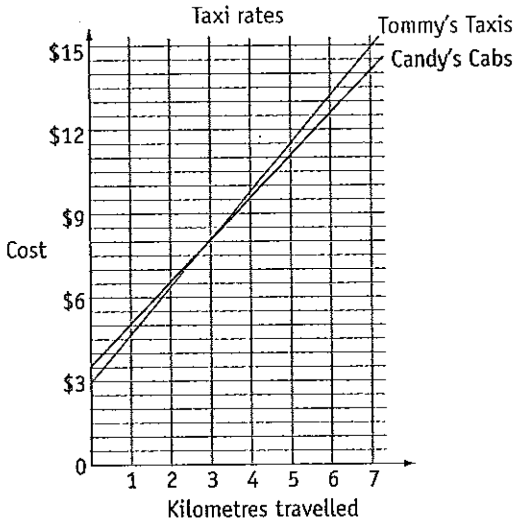
b Where does the line $y = 2x$ intersect with the line $y = -x + 12$?

c What is the point of intersection of $y = 2x$ and $y = x + 2$?

11 a On the same diagram draw the lines $y = -x + 9$ and $y = x + 5$.

b Where do the lines intersect?

12 Two different taxi companies offer slightly different rates for hiring a cab. Each charge is made up of a fixed rate and a cost depending upon the number of kilometres travelled. The graph shows the cost for hiring cabs from the two companies for different distances.



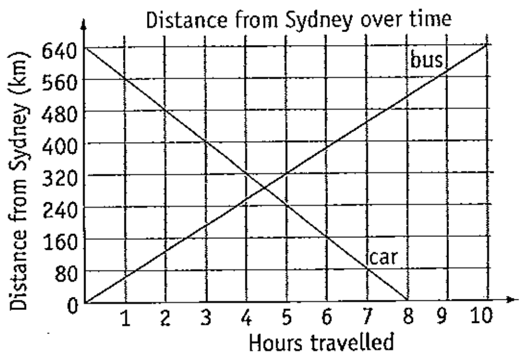
a How much does Candy's Cabs charge as a fixed rate?

b How much will it cost to hire a taxi from Tommy's taxis if you travel 7 km?

c At what distance travelled are the two rates the same?

d If you needed to travel 5 km, which would be the most economical company to choose and by how much?

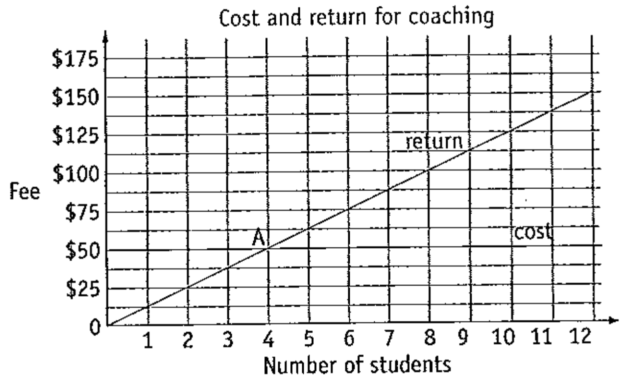
13 A bus leaves Sydney and travels to Grafton, a distance of 640 km. At the same time a car leaves Grafton and travels to Sydney. The distances from Sydney after a certain amount of time are shown on the graph.



After how many hours are the vehicles the same distance from Sydney and what is that distance?

14 Angus decides to run a coaching class and charges a fixed fee for each student who comes to the class. The costs involved with holding the class are $50.

The graph shows the cost and return to Angus for up to 12 students.



a What does the point A represent?

b What does Angus charge each student?

c How much profit does Angus make if 10 people attend?

d How much does Angus lose if only 2 people attend?

15 Complete the table of values for $y = x^2 + 4x + 3$

x	0	1	2	3	4	5	6	7	8	9	10
y											

16 a Complete the table of values for $y = x^2 - 8x + 20$.

x	0	1	2	3	4	5	6	7	8
y									

b Sketch the graph of the curve $y = x^2 - 8x + 20$.

17 Draw up a table of values for whole number values of x from 0 to 10 and complete for the quadratic function $y = 10x - x^2$. Graph the function.

18 a Complete the table of values for $y = 2x^2 - 20x + 55$.

x	0	1	2	3	4	5	6	7	8	9	10
y											

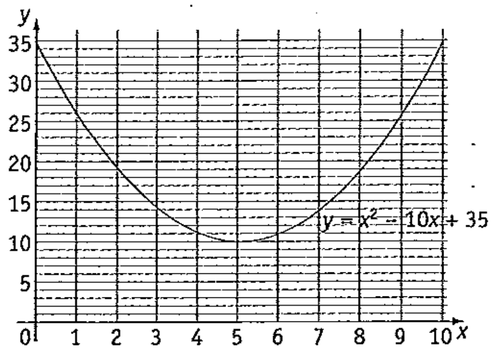
b Sketch the graph of $y = 2x^2 - 20x + 55$.

19 a Sketch $y = x^2 - 12x + 40$ for values of x between 0 and 12.

b Sketch $y = (x - 6)^2 + 4$ for values of x between 0 and 12.

c Comment on any similarities or differences between the two curves.

20 The diagram shows the graph of $y = x^2 - 10x + 35$. Use the graph to answer the following questions.



a What is the value of y when $x = 3$?

b For what values of x does $y = 19$?

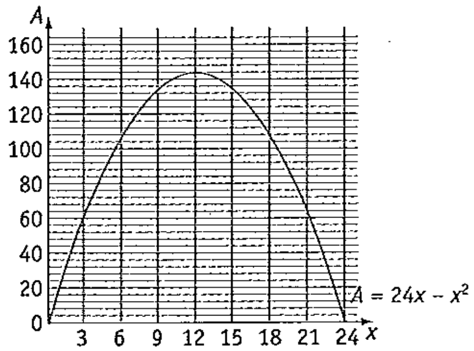
c What is the minimum value of y and for what value of x does this occur?

21 a Sketch $y = 1 + 11x - x^2$ for values of x between 0 and 11.

b For what value of x is y a maximum?

c What is the maximum value of y?

22 The area, A m², of a rectangle with perimeter 48 m is given by the expression $A = 24x - x^2$ where x is the length of one of the sides. The diagram shows the graph of $A = 24x - x^2$.



a What is the area of the rectangle if one side is 4 metres?

b What are the dimensions of the rectangle if its area is 108 m²?

c What is the maximum possible area of a rectangle with perimeter 48 m?

d What is special about the rectangle with maximum area?

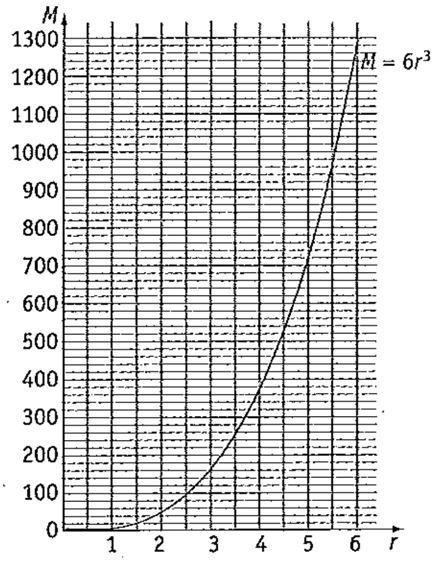
23 a Complete the table for $y = 3x^3$.

x	0	0.5	1	1.5	2	2.5	3	3.5	4
y									

b Sketch the curve $y = 3x^3$.

24 Sketch the curve $y = \frac{x^3}{10}$ for values of x between 0 and 8.

25 The mass (in grams) of a particular type of ball can be approximated by the formula $M = 6r^3$ where r is the radius of the ball in centimetres. The graph of $M = 6r^3$ is shown below.



a Using the graph, what is the approximate mass of a ball of radius 4.5 cm?

b Using the graph, what is the radius of a ball of mass 1 kg.

c Find the volume of a sphere of radius 2.4 cm using the formula $V = \frac{4}{3}\pi r^3$. Give the answer to the nearest cubic centimetre.

d Use the graph to find the mass of a ball with radius 2.4 cm and hence find the density in g/cm³ by dividing the mass by the volume.

26 Complete the table for $y = 7(4^x)$.

x	0	1	2	3	4	5	6	7	8
y									

27 a Complete the table of values for $y = 3^x$, rounding the answers to one decimal place where necessary.

x	0	0.5	1	1.5	2	2.5	3	3.5	4
y									

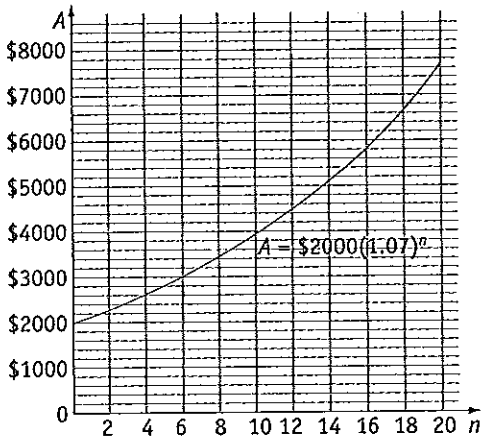
b Sketch the graph of $y = 3^x$.

28 If an amount of \$2000 is invested at 7% p.a. compound interest then the balance (A) of the account after n years is given by $A = \$2000(1.07)^n$, the graph of which is shown below. Use the graph to answer the following questions.

a How much would be in the account after twelve years?

b After how many years will the balance of the account be \$5150?

c How much interest is earned during the eighth year?



29 The population, P, of a city in millions is approximated by the formula $P = 8(1.03)^t$ where t is the time in years since 1960.

a Complete the table of values for P, rounding each answer to one decimal place where necessary.

t	0	5	10	15	20	25	30	35	40	45	50
P											

b Sketch the graph of P.

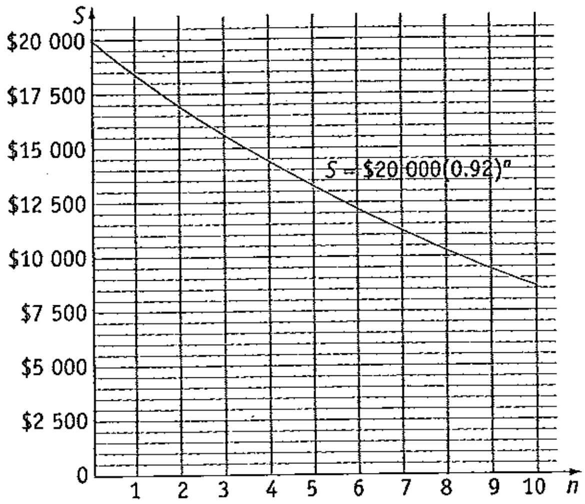
c After approximately how many years will the population reach 25 million?

30 a Complete the table of values for $y = 3000(0.5^x)$, rounding each y value to the nearest whole number where necessary.

x	0	1	2	3	4	5	6	7	8
y									

b Sketch the graph of $y = 3000(0.5^x)$.

31 If a car originally valued at \$20 000 is depreciated at the rate of 8% per year, then the salvage value S of the car after n years is given by $S = \$20\,000(0.92)^n$. The graph of $S = \$20\,000(0.92)^n$ is shown below. Use the graph to answer the following questions.



a What is the approximate value of the car after 6 years?

b When does the car fall below \$10 000 in value?

c How much does the car depreciate in the first year?

32 A radioactive substance disintegrates over time. The mass (in grams) of a particular radioactive substance present after t years is given by $M = 60(0.9)^t$.

a Complete the table of values, giving the answers to the nearest whole number.

t	0	5	10	15	20	25	30	35	40
M									

b Sketch the graph of $M = 60(0.9)^t$.

Use the graph to answer the following questions:

c How much of the substance is present after eighteen years?

d After how many years does the mass present fall below 10 g?

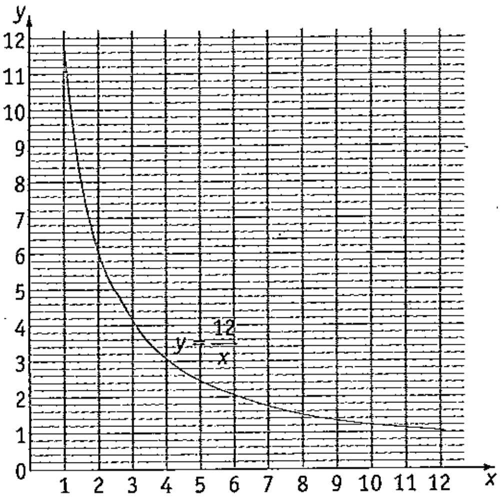
e How long does it take for half of the mass to disintegrate?

33 a Complete the table for $y = \frac{600}{x}$.

x	10	15	20	25	30	40	50	60
y								

b Sketch the graph of $y = \frac{600}{x}$ $(x > 0)$.

34 The diagram shows the graph of $y = \frac{12}{x}$ for values of x greater than 0.



Use the graph to answer the following questions:

a What is the value of y when $x = 4.5$?

b What is the value of x when $y = 1.4$?

c For what approximate value of x are the values of x and y equal?

35 Nine friends are trying to decide whether to charter a boat for a fishing trip. The cost to charter the boat is $504.

a Find the cost per person if different numbers, from 1 to 9, share the cost.

b Show the different costs on a grid.

c Samuel would like to go on the trip but can only afford to pay $70. How many other people would need to take the trip so that Samuel could go?

36 Valerie bought 8 cans of cat food for $6.80. How much would 5 cans cost?

37 $y = ax^2$ where a is a constant ($x > 0$). When $x = 3$, $y = 153$.

a Find the value of a.

b Find the value of y when $x = 8$.

c Find the value of x when $y = 2057$.

38 The distance an object falls varies directly with the square of the time it is falling. An object falls 490 metres in 10 seconds. How far would the object fall in 7 seconds?

39 Chester has a set of plates of different sizes. He knows that the area of each plate (A cm²) varies directly with the square of the distance across the plate (d cm). A plate 20 cm across has area 320 cm².

a What would be the area of a plate that is 25 cm across?

b What would be the distance across a plate that has area 980 cm²?

40 $y = kx^3$, where k is a constant. When $x = 5$, $y = 1750$.

a Find the value of k.

b Find the value of y when $x = 7$.

c Find the value of x when $y = 896$.

41 The capacity of a particular type of cylindrical tank varies directly with the cube of the radius. A tank of radius 1.5 m holds 20 250 litres.

a What would be the capacity of a tank of that type with radius 2.8 m?

b What would be the radius of such a tank that holds 48 000 litres?

42 $y = \frac{k}{x}$ where k is a constant. When $x = 8$, $y = 15$.

a Find the value of k.

b Find the value of y when $x = 16$.

c Find the value of x when $y = 45$.

43 The cost per person to go on a coach trip varies inversely with the number of people travelling. If 16 people go on the trip, the cost per person is $30.

a What is the cost per person if 48 people go on the trip?

b How many people took the trip if the final cost per person was $8?

44 The time needed to travel across a particular region is inversely proportional to the speed. It takes 2½ hours to cross the region if the speed is 8 metres per second.

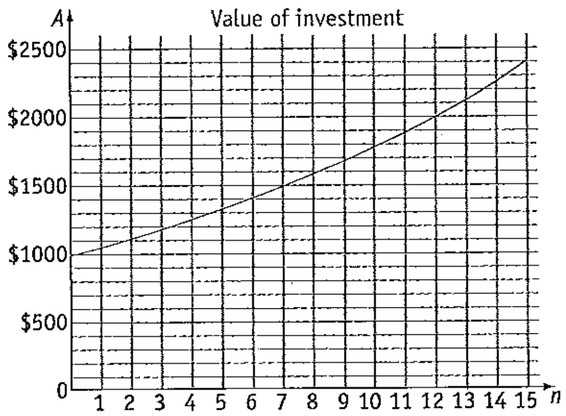
a Show that $T = \frac{72\,000}{s}$ where T is the time in seconds and s the speed (in metres per second).

b Find the time to travel across the region, in hours and minutes, if the speed is 12 metres per second.

45 Shannon decides to invest \$1000 and considers some different options.

a One option is to invest the money in an account earning \$80 simple interest every year. Write a formula for the amount of money (\$A) Shannon will have in the account after n years if he chooses this option.

b A second option is to invest the money to earn compound interest. Shannon's financial institution supplies him with a graph to show the amount he will have in the account over several years. Copy the graph and draw the graph of the amount Shannon will have if he chooses the first option on the same diagram.



c After how many years will the amounts be the same?

d After how many years will the amount double if Shannon chooses the compound interest option?

e Use the formula from part a to find the number of years it will take that option to double.

f Which option do you think Shannon should choose? Justify your answer.

46 When a ball is thrown from the roof of a building, the height (h metres) of the ball at time t seconds can be approximated by the formula $h = 96 + 14t - 5t^2$ for times up to and including six seconds.

a Find the value of h when $t = 0$ and briefly explain the significance of the answer.

b What is the value of h when $t = 6$?

c Why is the formula not valid for values of t greater than six?

47 Kelvin bought a van for \$23 000. The salvage value, \$$S$, of the van after n years is given by $S = 23\,000 - 1150n$.

a What is the salvage value of the van after 20 years?

b What limits must be placed on the formula? Justify your answer.

48 Betty is building a pen in her backyard to hold hens. She has plenty of roofing materials but only enough materials to fence 18 metres on the sides. She wants the pen to be rectangular, but is not sure how long to make each side.

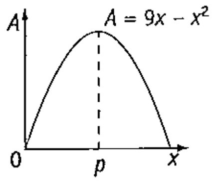
a If the length of the pen is x m and the width is y m, show that $y = 9 - x$.

b Show that the area (A m²) of the pen will be given by $A = 9x - x^2$.

c Why must x always be greater than 0?

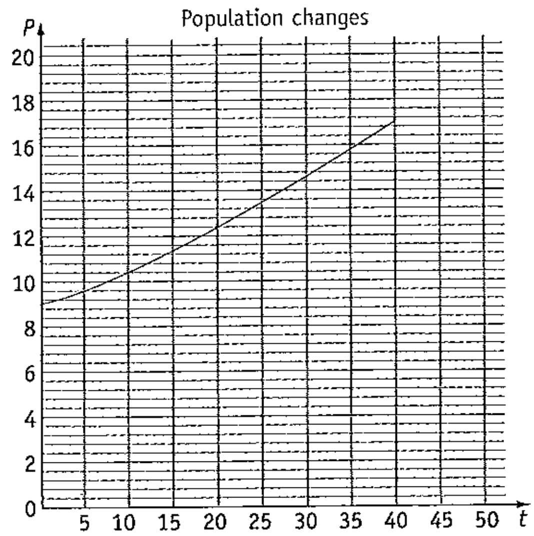
d What is the greatest possible value of x? Justify your answer.

e Betty sketched the parabola $A = 9x - x^2$. What will be the value of p? Justify your answer.



f What is the maximum possible area of the pen and what will be its dimensions?

49 The graph shows the population (in millions) of a city over forty years from 1966 until 2006.



a What was the population in 1966?

b What was the population in 2006?

c When did the population reach 12 million?

d Paris looked at the graph and predicted that the population would be 20 million in 2016. Do you agree? Justify your answer.

Go to pp 292–3 for **Quick Answers**
or to pp 356–61 for **Worked Solutions**

Challenge: Modelling Linear and Non-linear Relationships

1 Ricky has drawn up the table of values for a particular relationship. What will be the value of y when $x = 17$?

x	1	2	3	4
y	1	4	7	10

Hint 1

2
a On the same diagram sketch the graphs of $y = (x - 2)^2$ and $y = x^2 - 4x + 4$ for values of x between 0 and 4. *Hint 2*
b Comment on any similarities or differences between the curves.

3
a Complete the table of values for $y = x^3$.

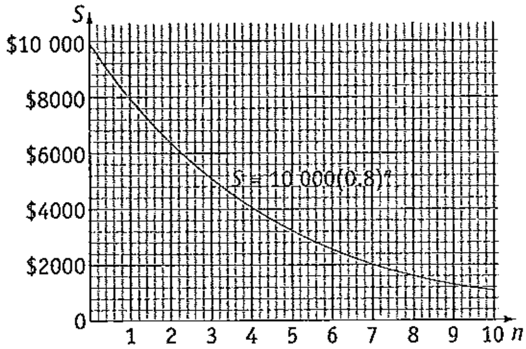
x	0	0.5	1	1.5	2	2.5	3
y							

b Sketch the graph of $y = x^3$ for values of x between 0 and 3.
c On the same diagram sketch the quadratic function $y = 12 - x^2$ for values of x between 0 and 3.
d Find a solution to the equation $x^3 = 12 - x^2$. *Hint 3*

4 The value of a machine over time is approximated by the formula $v = 30(t^2 - 28t + 192)$, where v is the value in dollars and t is the age of the machine in years.
a What was the original value of the machine? *Hint 4*
b How much did the machine lose in value in the first five years?
c Find the value after twelve years.
d What limits apply to the use of the formula? *Hint 5*

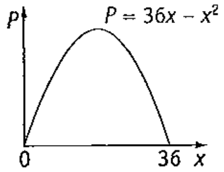
5 The cost per person for a cruise varies inversely with the number of people taking the cruise. If thirty people take the cruise, the cost per person is \$45.
a Explain why $C = \frac{1350}{n}$ where n is the number of people travelling and C the cost in dollars.
b What is the cost per person if fifty people take the cruise?
c The minimum cost per person is \$22.50. What is the maximum number of people that can take the cruise? *Hint 6*

6 The diagram shows the graph of $S = \$10\,000(0.8)^n$.



a What is the value of S when $n = 2$?
b When will S first fall below \$3000?
c After how many years will S fall below \$10? *Hint 7*

7 Let x and y be two numbers whose sum is 36.
a Show that the product of the two numbers, P, is given by $P = 36x - x^2$. *Hint 8*
b The diagram shows a sketch of the graph of $P = 36x - x^2$.



What is the maximum product of the two numbers? *Hint 9*

Go to p 294 for **Quick Answers**
or to p 361 for **Worked Solutions**

Hint 1: Find the rule linking x and y by studying the values in the table. (What is the difference between the x-values and what is the difference between the y-values?) Then substitute into the rule.
Hint 2: You will need more than four values in the table of values. Use halves.
Hint 3: Use the graphs. What is the point of intersection?
Hint 4: What is the value when t = 0?
Hint 5: Consider the answer to part c.
Hint 6: Find the number of people travelling for this cost. As the cost decreases what happens to the number of people?
Hint 7: The graph does not help. Use the equation and the estimation and refinement technique.
Hint 8: x + y = 36. Make y the subject of this equation. Then find an expression for the product of the two numbers.
*Hint 9: Use the symmetry of the diagram to find the value of x for which the product is a maximum. Substitute this value into the equation from part **a**.*

Solutions

Chapter 13: Modelling Linear and Non-linear Relationships

Further Practice p259

1 $y = 4x + 7$

x	0	1	2	3
y	7	11	15	19

2 $P = 5n - 2$

n	1	2	3	4
P	3	8	13	18

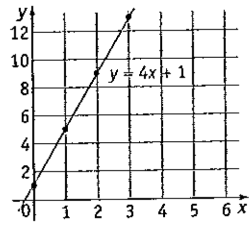
3 $y = x^2 + \frac{12}{x}$

x	1	2	3	4	5	6
y	13	10	13	19	27.4	38

4 a $y = 4x + 1$

x	0	1	2	3
y	1	5	9	13

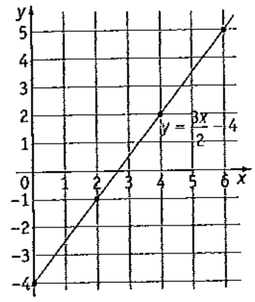
b



5 a $y = \frac{3x}{2} - 4$

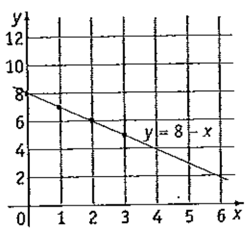
x	0	2	4	6
y	−4	−1	2	5

b



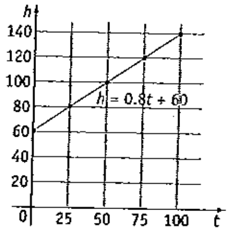
6 $y = 8 - x$

x	0	1	2	3
y	8	7	6	5



7 $h = 0.8t + 60$

t	0	25	50	75	100
h	60	80	100	120	140

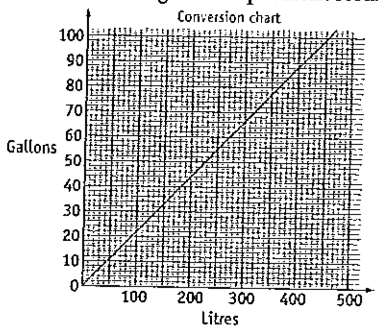


8 a 66 gallons

b 410 litres

c 40 gallons ≈ 180 litres
3×40 gallons ≈ 540 litres
Three drums would hold approximately 540 litres.

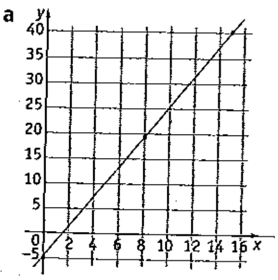
d Kate is correct. Because 0 litres is the same as 0 gallons, the rule for the number of gallons is G = kL where k is a constant. So multiplying both sides of the equation by the same number will give an equivalent result.



9

x	0	8	15
y	−5	19	40

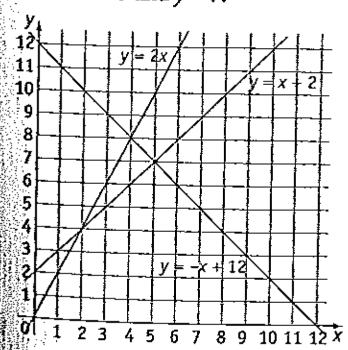
a



b When $x = 10$, $y = 25$

c When $y = 4$, $x = 3$

10 a When $x = 5$ and $y = 7$.



b When $x = 4$ and $y = 8$.

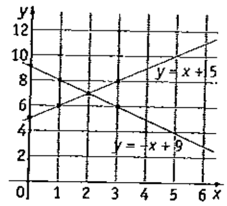
c The point of intersection is (2, 4).

11 a $y = -x + 9$

x	0	1	2	3
y	9	8	7	6

$y = x + 5$

x	0	1	2	3
y	5	6	7	8



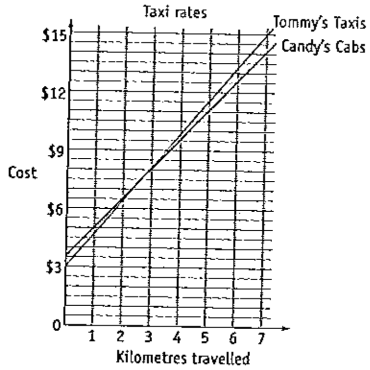
b When $x = 2$ and $y = 7$.

12 a \$3.60

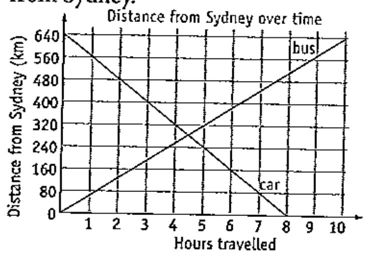
b \$15

c Approximately 2.8 km

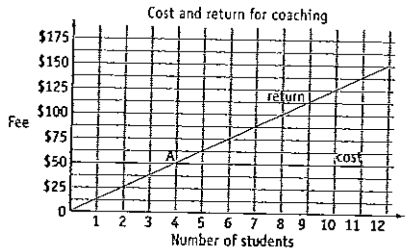
d Candy's Cabs is cheaper but only by about 45 cents.



13 After $4\frac{1}{2}$ hours; approximately 280 km from Sydney.



14 a A is the 'breakeven' point. If 4 people attend, Angus will receive \$50 and so cover his costs.



b Each person is charged \$12.50.

c If 10 people attend, Angus receives \$125.
Profit = \$125 − \$50
= \$75

d If 2 people attend, Angus receives \$25.
Loss = \$50 − \$25
= \$25

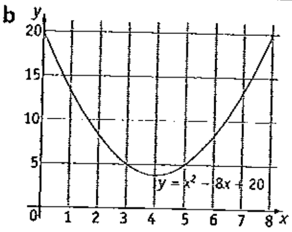
15 $y = x^2 + 4x + 3$

x	0	1	2	3	4	5	6	7	8	9	10
y	3	8	15	24	35	48	63	80	99	120	143

16 a $y = x^2 - 8x + 20$

x	0	1	2	3	4	5	6	7	8
y	20	13	8	5	4	5	8	13	20

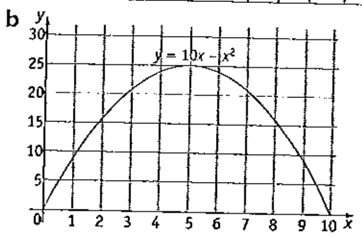
b



17 a $y = 10x - x^2$

x	0	1	2	3	4	5	6	7	8	9	10
y	0	9	16	21	24	25	24	21	16	9	0

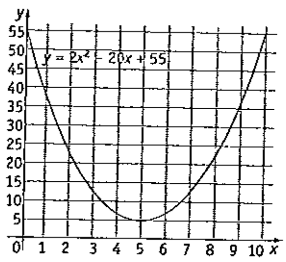
b



18 a $y = 2x^2 - 20x + 55$

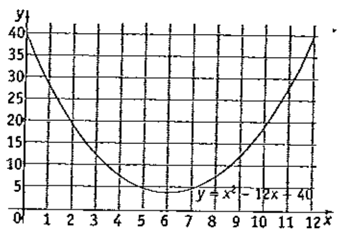
x	0	1	2	3	4	5	6	7	8	9	10
y	55	37	23	13	7	5	7	13	23	37	55

b



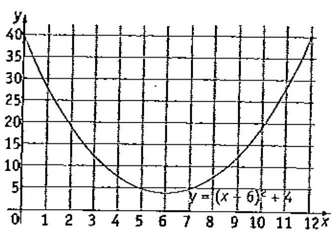
19 a $y = x^2 - 12x + 40$

x	0	1	2	3	4	5	6	7	8	9	10	11	12
y	40	29	20	13	8	5	4	5	8	13	20	29	40



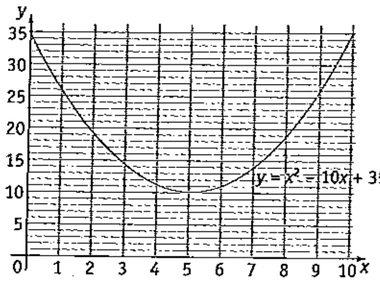
b $y = (x - 6)^2 + 4$

x	0	1	2	3	4	5	6	7	8	9	10	11	12
y	40	29	20	13	8	5	4	5	8	13	20	29	40



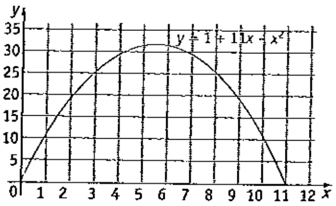
c The two curves are exactly the same.

20 a When $x = 3$, $y = 14$
b When $y = 19$, $x = 2$ or 8.
c The minimum value of y is 10 which occurs when $x = 5$.



21 a $y = 1 + 11x - x^2$

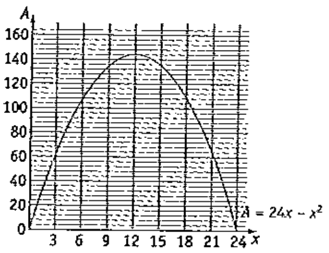
x	0	1	2	3	4	5	6	7	8	9	10	11
y	1	11	19	25	29	31	31	29	25	19	11	1



b The maximum value occurs when $x = 5.5$
[halfway between $x = 5$ and $x = 6$]

c When $x = 5.5$,
$y = 1 + 11 \times 5.5 - (5.5)^2$
$= 31.25$
The maximum value of y is $y = 31.25$.

22 a When $x = 4$, $A = 80$
The area is 80 m².



b When $A = 108$,
$x = 6$ or $x = 18$
A rectangle of area 108 m² will be 18 m by 6 m.

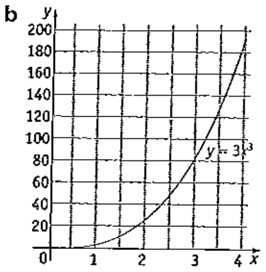
c The maximum area is 144 m².

d When $A = 144$, $x = 12$
There is only one possible value for the length of the side.
The rectangle is a square of side 12 m.

23 a $y = 3x^3$

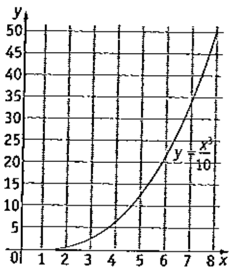
x	0	0.5	1	1.5	2	2.5	3	3.5	4
y	0	0.375	3	10.125	24	46.875	81	128.625	192

b

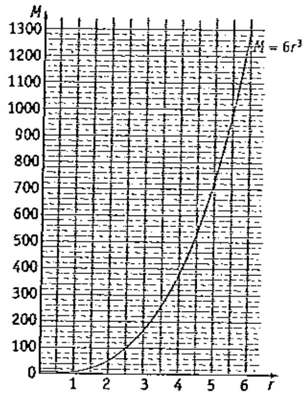


24 $y = \dfrac{x^3}{10}$

x	0	1	2	3	4	5	6	7	8
y	0	0.1	0.8	2.7	6.4	12.5	21.6	34.3	51.2



25 a When $r = 4.5$, $M \approx 550$
The mass of a ball of radius 4.5 cm is approximately 550 g.



b 1 kg = 1000 g
When $M = 1000$, $r \approx 5.5$
A ball of mass 1 kg has radius approximately 5.5 cm.

c $V = \dfrac{4}{3}\pi r^3$
When $r = 2.4$,
$V = \dfrac{4}{3} \times \pi \times (2.4)^3$
$= 57.905\,835\,79 \ldots$
$= 58$ (nearest unit)
The volume is 58 cm³ to the nearest cubic centimetre.

d From the graph, when $r = 2.4$, $M \approx 80$
Density $\approx 80 \div 58$
$= 1.379\,310\,345 \ldots$
$= 1.4$ (1 d.p.)
The density is approx. 1.4 g/cm³.

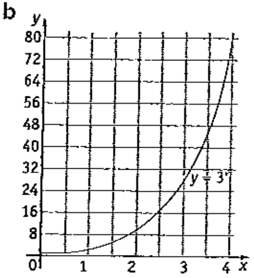
26 $y = 7(4^x)$

x	0	1	2	3	4	5	6	7	8
y	7	28	112	448	1792	7168	28 672	114 688	458 752

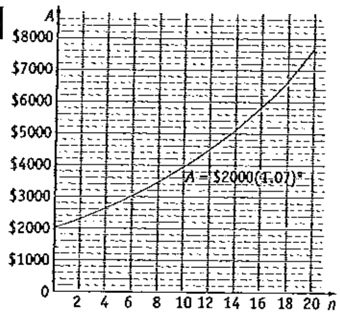
27 a $y = 3^x$

x	0	0.5	1	1.5	2	2.5	3	3.5	4
y	1	1.7	3	5.2	9	15.6	27	46.8	81

b



28



a After 12 years there is $4500 in the account.

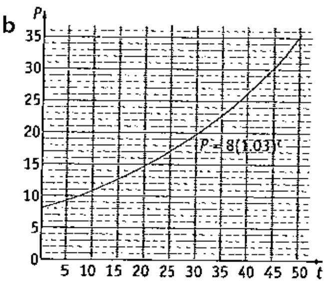
b After 14 years the balance of the account is $5150.

c When $n = 8$, $A \approx \$3400$
When $n = 7$, $A \approx \$3200$
Difference = $3400 − $3200
= $200
Approximately $200 interest is earned during the eighth year.

29 a $P = 8(1.03)^t$

t	0	5	10	15	20	25	30	35	40	45	50
P	8	9.3	10.8	12.5	14.4	16.8	19.4	22.5	26.1	30.3	35.1

b

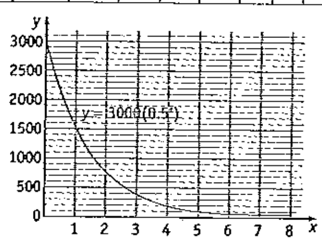


c The population would reach 25 million in approximately 38½ years.

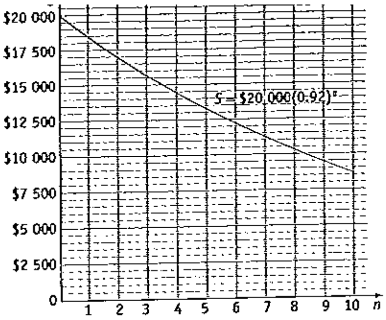
30 a $y = 3000(0.5^x)$

x	0	1	2	3	4	5	6	7	8
y	3000	1500	750	375	188	94	47	23	12

b



31



a After 6 years the value of the car is about \$12 300.

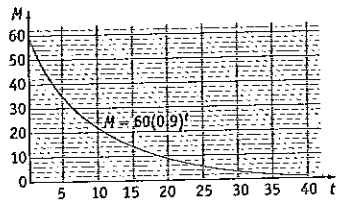
b After about $8\frac{1}{3}$ years the value of the car falls below \$10 000.

c When $n = 1$, $S = \$18\,400$
Depreciation = \$20 000 − \$18 400
= \$1600
The car depreciates by \$1600 in the first year.

32 a $M = 60(0.9)^t$

t	0	5	10	15	20	25	30	35	40
M	60	35	21	12	7	4	3	2	1

b



c After 18 years, approximately 9 grams remain.

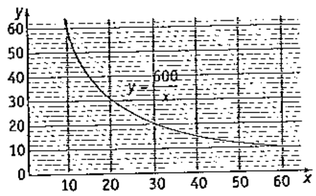
d After 17 years the mass present falls below 10 grams.

e When $M = 30$, $t = 7$
It takes approximately 7 years for half of the mass to disintegrate.

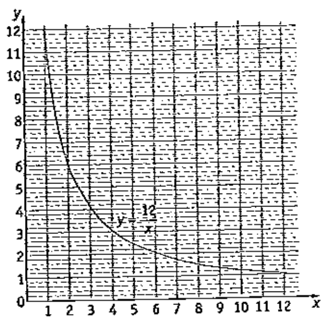
33 a $y = \dfrac{600}{x}$

x	10	15	20	25	30	40	50	60
y	60	40	30	24	20	15	12	10

b



34



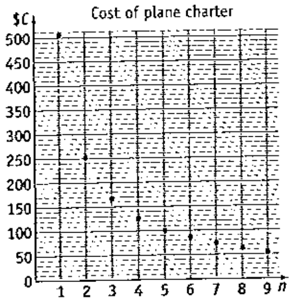
a When $x = 4.5$, $y \approx 2.7$

b When $y = 1.4$, $x \approx 8.5$

35 a Let n be the number of people and \$$C$ the cost per person.

n	1	2	3	4	5	6	7	8	9
C	504	252	168	126	100.8	84	72	63	56

b



c At least 8 people would need to travel for the cost to be less than \$70 each. Samuel would need seven other people to take the trip.

36 8 cans cost \$6.80
1 can costs \$6.80 ÷ 8 = \$0.85
5 cans cost \$0.85 × 5 = \$4.25

37 a $y = ax^2$
When $x = 3$, $y = 153$
$153 = a \times 3^2$
$153 = 9a$
$a = \dfrac{153}{9}$
$= 17$

b $y = 17x^2$
When $x = 8$,
$y = 17 \times 8^2$
$= 17 \times 64$
$= 1088$

c $y = 17x^2$
When $y = 2057$,
$2057 = 17x^2$
$x^2 = 2057 \div 17$
$= 121$
$x = \sqrt{121}\quad (x > 0)$
$= 11$

38 Let d be the distance the object falls and let t be the time taken.
$d = kt^2$
When $t = 10$, $d = 490$
$490 = k \times 10^2$
$490 = 100k$
$k = 4.9$
$\therefore d = 4.9t^2$
When $t = 7$,
$d = 4.9 \times 7^2$
$= 4.9 \times 49$
$= 240.1$
The object would fall 240.1 m in 7 seconds.

39 a $A = kd^2$
When $d = 20$, $A = 320$
$320 = k \times 20^2$
$= 400k$
$k = 320 \div 400$
$= 0.8$
$\therefore A = 0.8d^2$
When $d = 25$,
$A = 0.8 \times 25^2$
$= 500$
The area would be 500 cm^2.

b $A = 0.8d^2$
When $A = 980$,
$980 = 0.8d^2$
$d^2 = 980 \div 0.8$
$= 1225$
$d = \sqrt{1225}\quad (d > 0)$
$= 35$
The plate would be 35 cm across.

40 a $y = kx^3$
When $x = 5$, $y = 1750$
$1750 = k \times 5^3$
$1750 = 125k$
$k = \dfrac{1750}{125}$
$= 14$

b $y = 14x^3$
When $x = 7$,
$y = 14 \times 7^3$
$= 14 \times 343$
$= 4802$

c $y = 14x^3$
When $y = 896$,
$896 = 14x^3$
$x^3 = 896 \div 14$
$= 64$
$x = \sqrt[3]{64}$
$= 4$

41 a Let C be the capacity and r the radius of the tank.
$C = kr^3$
When $r = 1.5$, $C = 20\,250$
$20\,250 = k \times 1.5^3$
$20\,250 = 3.375k$
$k = 20\,250 \div 3.375$
$= 6000$
$\therefore C = 6000r^3$
When $r = 2.8$,
$C = 6000 \times 2.8^3$
$= 131\,712$
The tank would hold 131 712 litres.

b $C = 6000r^3$

When $C = 48\,000$,

$48\,000 = 6000 \times r^3$

$r^3 = 48\,000 \div 6000$

$= 8$

$r = \sqrt[3]{8}$

$= 2$

The radius of the tank would be 2 m.

42 **a** $y = \frac{k}{x}$

When $x = 8$, $y = 15$

$15 = \frac{k}{8}$

$k = 15 \times 8$

$= 120$

b $y = \frac{120}{x}$

When $x = 16$,

$y = \frac{120}{16}$

$= 7.5$

c $y = \frac{120}{x}$

When $y = 45$,

$45 = \frac{120}{x}$

$45x = 120$

$x = \frac{120}{45}$

$= 2\frac{2}{3}$

43 **a** Let $\$C$ be the cost per person and let n be the number of people travelling.

$C = \frac{k}{n}$ where k is a constant.

When $n = 16$, $C = 30$

$30 = \frac{k}{16}$

$k = 30 \times 16$

$= 480$

$\therefore C = \frac{480}{n}$

When $n = 48$,

$C = \frac{480}{48}$

$= 10$

If 48 people travel, the cost is \$10 per person.

b $C = \frac{480}{n}$

When $C = 8$,

$8 = \frac{480}{n}$

$8n = 480$

$n = 60$

Sixty people went on the trip.

44 **a** $2\frac{1}{2}$ hours $= 150$ minutes

$= 9000$ seconds

$T = \frac{a}{s}$

When $s = 8$, $T = 9000$

$9000 = \frac{a}{8}$

$a = 9000 \times 8$

$= 72\,000$

$\therefore T = \frac{72\,000}{s}$

b $T = \frac{72\,000}{s}$

When $s = 12$,

$T = \frac{72\,000}{12}$

$= 6000$

It takes 6000 seconds to cross the region.

6000 seconds $= (6000 \div 60)$ min

$= 100$ min

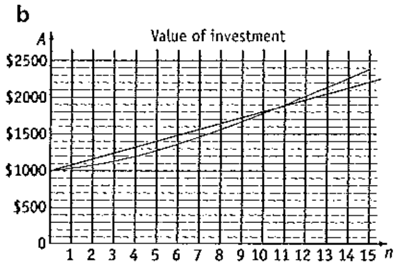
$= 1$ h and 40 min

It takes 1 hour and 40 minutes to cross the region when travelling at 12 m/s.

45 **a** $A = 1000 + 80n$

[The original amount plus \$80 per year.]

b



c 10½ years

d 12 years [When $A = 2000$]

e $A = 1000 + 80n$

When amount has doubled

$A = 2000$

$2000 = 1000 + 80n$

$1000 = 80n$

$n = 1000 \div 80$

$= 12.5$

The simple interest option doubles in $12\frac{1}{2}$ years.

f If Shannon intends to invest for less than 10 years he should choose the simple interest option. Otherwise he should use the compound interest option. Although the simple interest option produces a slightly better result for the first ten years, the compound interest produces a significantly better result after ten years.

46 **a** $h = 96 + 14t - 5t^2$

When $t = 0$,

$h = 96 + 14 \times 0 - 5 \times 0^2$

$= 96$

The height of the ball originally is 96 metres. The roof of the building is 96 m high.

b $h = 96 + 14t - 5t^2$

When $t = 6$,

$h = 96 + 14 \times 6 - 5 \times 6^2$

$= 0$

c After six seconds the height of the ball is zero. The ball has reached the ground. It cannot continue to fall (and will probably bounce). The formula gives negative values of h when $t > 6$ and this is impossible.

47 **a** $S = 23\,000 - 1150n$

When $n = 20$,

$S = 23\,000 - 1150 \times 20$

$= 0$

The salvage value is \$0 after 20 years.

b The formula must be restricted to values of n less than or equal to 20. It makes no sense for the value to be less than zero.

48 **a** Perimeter $= 18$ m

$2x + 2y = 18$

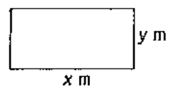
$x + y = 9$

$y = 9 - x$

b Area: $A = xy$

$= x(9 - x)$

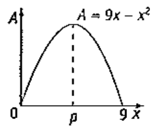
$= 9x - x^2$



c x m is the length of the yard. x must be greater than 0 or the yard will not exist.

d y m is the width of the yard so y must be greater than 0. This means that x must be less than 9.

e A is zero when $x = 0$ and $x = 9$.



Because the parabola is symmetrical p is halfway between 0 and 9.

$p = 4.5$

f The maximum area occurs when $x = 4.5$

$A = 9x - x^2$

When $x = 4.5$,

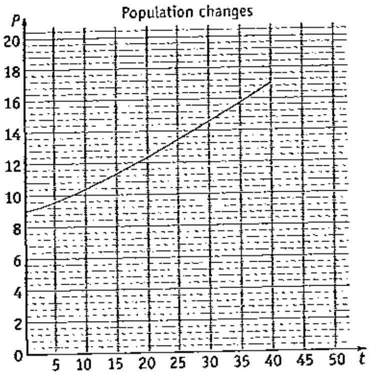
$A = 9 \times 4.5 - 4.5^2$

$= 20.25$

The maximum area is 20.25 m^2.

Each side will be 4.5 metres long.

49



a In 1966, $t = 0$
When $t = 0$, $P = 9$
The population was 9 million in 1966.

b In 2006, $t = 40$. The population was 17 million.

c $P = 12$ when $t = 18$
The population was 12 million in 1984.

d Paris might be right. It looks like P will be 20 when $t = 50$ if the graph is continued. However, it is not possible to be sure exactly where the graph will go.

Challenge p265

1

x	1	2	3	4
y	1	4	7	10

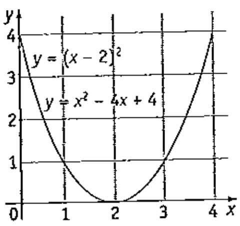
The rule is $y = 3x - 2$.
When $x = 17$,
$y = 3 \times 17 - 2$
$= 49$

2 a $y = (x-2)^2$

x	0	0.5	1	1.5	2	2.5	3	3.5	4
y	4	2.25	1	0.25	0	0.25	1	2.25	4

$y = x^2 - 4x + 4$

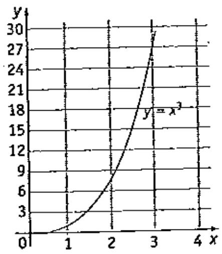
x	0	0.5	1	1.5	2	2.5	3	3.5	4
y	4	2.25	1	0.25	0	0.25	1	2.25	4



b The two graphs are exactly the same.

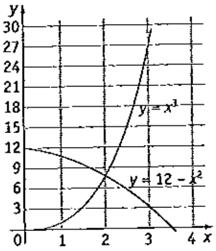
3 a $y = x^3$

x	0	0.5	1	1.5	2	2.5	3
y	0	0.125	1	3.375	8	15.625	27

b


c $y = 12 - x^2$

x	0	0.5	1	1.5	2	2.5	3
y	12	11.75	11	9.75	8	5.75	3



d $x^3 = 12 - x^2$ when $x = 2$

4 $v = 30(t^2 - 28t + 192)$

a When $t = 0$,
$v = 30(0^2 - 28 \times 0 + 192)$
$= 5760$
The original value of the machine was \$5760.

b $v = 30(t^2 - 28t + 192)$
When $t = 5$,
$v = 30(5^2 - 28 \times 5 + 192)$
$= 2310$
Loss in value $= \$5760 - \$2310$
$= \$3450$

c $v = 30(t^2 - 28t + 192)$
When $t = 12$,
$v = 30(12^2 - 28 \times 12 + 192)$
$= 0$

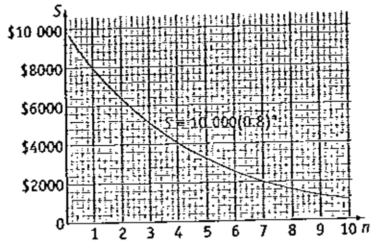
d The formula will only work for the first 12 years. The value of the machine cannot be less than zero, nor is the machine likely to increase in value.

5 a $C = \dfrac{k}{n}$
When $n = 30$, $C = 45$
$45 = \dfrac{k}{30}$
$k = 45 \times 30$
$= 1350$
$\therefore C = \dfrac{1350}{n}$

b If $n = 50$,
$C = \dfrac{1350}{50}$
$= 27$
If 50 people take the cruise, the cost per person is \$27.

c If $C = 22.50$
$22.50 = \dfrac{1350}{n}$
$22.5n = 1350$
$n = 60$
If $n > 60$, the cost per person would be less than \$22.50. The maximum number of people is 60.

6

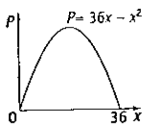


a When $n = 2$, $S = \$6400$

b $S < \$3000$ after 5.4 years.

c $S = \$10$,
$10 = 10\,000(0.8)^n$
$0.001 = 0.8^n$
$0.8^{30} = 0.001\,237\,94\ldots$
$0.8^{31} = 0.000\,990\,352\ldots$
After 31 years the value will fall below \$10.

7 a $x + y = 36$
$y = 36 - x$
$P = xy$
$= x(36 - x)$
$= 36x - x^2$

b


The maximum product occurs when $x = 18$.
When $x = 18$,
$P = 36 \times 18 - 18^2$
$= 324$
The maximum product of the two numbers is 324.