

# Further Practice: Algebraic Skills and Techniques

Remember: all questions match the numbered examples on pages 223–232.

- 1** If  $x = 2$ , evaluate  $3x - 5$ .
- 2** If  $a = 3$ ,  $b = 2$  and  $c = 6$ , find the value of  $5ab - 4c$ .
- 3** If  $m = 4$ , evaluate  $m^2 - m$ .
- 4** If  $p = 10$ ,  $3p^2 = ?$
- 5** If  $t = -3$ , find the value of  $10 - t$ .
- 6** If  $a = -2$ , find the value of  $6a^2$ .
- 7** Find the value of  $x^3 - 4x^2 + 8x - 5$  if  $x = -1$ .
- 8** Evaluate  $\sqrt{x^2 + y^2}$  if  $x = 9$  and  $y = 40$ .
- 9** Evaluate  $\frac{b^2 + c^2 - a^2}{2bc}$  when  $a = 700$ ,  
 $b = 800$  and  $c = 500$ .
- 10** Find the value, correct to two decimal places,  
of  $4\pi r^2$  when  $r = 9$ .
- 11** If  $r = \sqrt{\frac{A}{\pi}}$  find the value of  $r$ , to one decimal place,  
if  $A = 415.5$
- 12** If  $r = 0.05$ ,  $n = 6$  and  $M = 2000$ , find the value of  $N$  to  
two decimal places if  $N = M \left\{ \frac{(1+r)^n - 1}{r(1+r)^n} \right\}$ .
- 13** Find  $9k + 3k - 7k$
- 14** Simplify  $5a + 4b - 3c$ , if possible.
- 15** Simplify  $7a + 3a + 4b$
- 16** Simplify  $8p - 3q + 6p + 5q$
- 17** Simplify  $x^2 + 6x - 5 + 3x - 4x^2 - 2$
- 18** Simplify  $-5a + 4b + 3a - 7b$
- 19** Simplify  $7pq + 3p - 4q - 6qp - 3q + 1$
- 20** Simplify  $7 \times a \times b$
- 21** Find  $4p \times 5q$
- 22** Find  $2a \times 3b \times 5c \times 6d$
- 23** Simplify  $4x \times 3x$
- 24** Simplify  $3p \times 5p \times 2q$
- 25** Simplify  $x^4 \times x^6$
- 26** Find  $a^3b^2 \times ab^5$
- 27** Simplify  $m^3 \times n^4$
- 28** Find  $2a^5 \times 4a^4$
- 29** Simplify  $7x^3y^2 \times 5xy^3$
- 30** Simplify  $6a^2b^3c^2 \times 3abc \times 2a^3b^2c^2$
- 31** Find  $6x \div 2$
- 32** Find  $7m + m$
- 33** Find  $15xyz \div 5y$
- 34** Simplify  $\frac{9ab}{3b}$
- 35** Simplify  $\frac{15pq}{3r}$

- 36** Simplify  $\frac{18x^2}{12x}$
- 37** Find  $p^6 \div p^2$
- 38** Simplify  $a^5b^6 \div a^2b^3$
- 39** Find  $x^8 \div a^2$
- 40** Find  $36x^{36} \div 9x^9$
- 41** Find  $12a^7b^5c^2 \div 3a^6bc^2$
- 42** Simplify  $\frac{24a^7b}{8a^6b^2}$
- 43** Expand  $3(x + 5)$
- 44** Expand  $4(2a - 3b)$
- 45** Expand  $a(a + 6)$
- 46** Expand  $5t(4t + 3u - 1)$
- 47** Expand  $-7(2a + 5b)$
- 48** Expand  $-(p - q)$
- 49** Simplify  $7x^2 + 4x \times 2x$
- 50** Simplify  $10n^{10} \div 2n^2 - 4n \times 3n^7$
- 51** Expand and simplify  $2(8h + 7) + 3(5h - 9)$
- 52** Expand and simplify  $5(2x + 3) - 2(4x + 9)$
- 53** Expand and simplify  $x(x - 8) - 4(x - 1)$
- 54** Solve  $7k + 4 = 25$
- 55** Solve the equation  $6x + 7 = 23 - 2x$
- 56** Solve  $\frac{8a - 5}{3} = 2a + 1$
- 57** Solve  $5x^2 = 1805$  ( $x > 0$ )
- 58** Solve  $\sqrt{7x + 2} = 3$
- 59** Change the subject of the equation  $v = u + at$  to  $a$ .
- 60** Change the subject of the equation  $S = V_0 - Dn$  to  $n$ .
- 61** Make  $h$  the subject of the formula  $A = \frac{1}{2}bh$ .
- 62** Change the subject of the formula  $P = 2l + 2b$  to  $l$ .
- 63** Change the subject of the formula  $C = \frac{5}{9}(F - 32)$  to  $F$ .
- 64** Change the subject of the equation  $x = kt^2$  to  $t$ .
- 65** Change the subject of the formula  $A = \pi r^2$  to  $r$  ( $r > 0$ ).
- 66** Find the value of  $x$  if  $y = mx + b$  and, when  $y = 7$ ,  $m = 2$  and  $b = -1$ .
- 67** If  $C = \frac{k}{x^2}$  find the value of  $k$  for which  $C = 50$  and  $x = 3.4$
- 68** Evaluate  $x$  ( $x > 0$ ), when  $y = ax^2$  given  $a = 5$  and  $y = 320$ .
- 69** Find the value of  $r$ , correct to one decimal place, if  $V = \frac{1}{3}\pi r^2h$  and  $V = 5750$  when  $h = 19$  ( $r > 0$ ).
- 70** a Change the subject of the formula  $V = \frac{4}{3}\pi r^3$  to  $r$ .  
b If  $V = 1988$  find  $r$ .
- 71** a Change the subject of the formula  $A = \pi(R^2 - r^2)$  to  $R$  ( $R > 0$ ).  
b Find  $R$ , correct to one decimal place, if  $r = 13$  and  $A = 356$ .
- 72** Use the estimation and refinement technique to find the value of  $x$  for which  $6^x = 10\,077\,696$ .
- 73** Find the value of  $t$ , to the nearest whole number, for which  $2.3^t = 116\,000$ .
- 74** Find, to two decimal places, the value of  $x$  for which  $3^x = 21$ .

**75** If  $N = 12\,200$ ,  $M = 800$  and  $r = 0.04$ , use the estimation and refinement technique and your calculator to find the value of  $n$ , to the nearest whole number, for which  $N = M \left\{ \frac{(1+r)^n - 1}{r(1+r)^n} \right\}$ .

**76** Express 28 000 in scientific notation.

**77** Express in scientific notation:

- a 600 000
- b 9 417
- c 7 290 000 000

**78** Write  $7 \times 10^5$  as a normal number.

**79** Write as normal numbers:

- a  $4 \times 10^6$
- b  $3.5 \times 10^4$
- c  $1.234 \times 10^8$

**80** Express in scientific notation:

- a 0.0007
- b 0.0312
- c 0.000 45

**81** Write as decimals:

- a  $5 \times 10^{-4}$
- b  $8.7 \times 10^{-6}$
- c  $4.61 \times 10^{-5}$

**82** Find the product of  $4.6 \times 10^4$  and  $3.1 \times 10^{-6}$ .

**83** Find  $400\,000 \div 0.000\,05$ , expressing the answer in scientific notation.

**84** The population of Western Australia is  $2 \times 10^6$  and its area is  $2.5 \times 10^6$  square kilometres. Find the population density in people per square kilometre.

**85** a The planet Mercury is 57.5 million kilometres from the sun. Express this distance in metres, in scientific notation.  
b If the speed of light is  $3 \times 10^8$  m/s, find the time for light from the sun to reach Mercury.

Go to p 292 for Quick Answers  
or to pp 353–5 for Worked Solutions

# Challenge: Algebraic Skills and Techniques

**1** Simplify  $3x^2 + 5x \times 2x - 8x^3 \div 4x$ . *Hint 1*

**2** If  $a = 3$ ,  $b = 4$  and  $c = 6$ , evaluate  $\frac{7a+4b-3c}{a^2+b^2}$ . *Hint 2*

**3** Simplify  $3xy + 5x - 2y + 4yx$ . *Hint 3*

**4** Expand and simplify:  
 $5x(x^2 + 3x - 2) - 2(3x^2 + 1)$ . *Hint 4*

**5** Solve  $\frac{4x}{9} + \frac{x+1}{3} = 5$ . *Hint 5*

**6** Change the subject of the formula  $V = \frac{2\pi r^3}{3}$  to  $r$ . *Hint 6*

**7** Consider the formula  $R = \frac{a^2 + b^2 - c^2}{2ab}$ :

- Change the subject of the formula to  $c$  (given that  $c > 0$ ).
- If  $R = 0.5$ ,  $a = 8$  and  $b = 5$  find the value of  $c$ .  
*Hint 7*

**8** Use the estimation and refinement technique to find the value of  $x$ , to two decimal places, for which  $10^x = 5370$ . *Hint 8*

**9** Find the value of  $\frac{5.9 \times 10^6 + 1.5 \times 10^5}{4.4 \times 10^8 \times 2.5 \times 10^{-3}}$ . *Hint 9*

**10** Simplify  $8x - 5a + 3a$ . *Hint 10*

**11** If  $p = 3$  and  $q = 5$ , evaluate  $pq^2$ . *Hint 11*

**12** Simplify  $\frac{2ab^2}{8a^3b^2}$ . *Hint 12*

**13** Arrange in ascending order:  
 $3.1 \times 10^{-2}$ ,  $5.834 \times 10^{-4}$ ,  $9 \times 10^{-3}$ . *Hint 13*

**14** If  $y = 2x^2 + b$  and  $x$  is a positive number, find  $x$  when  $y = 19$  and  $b = 1$ . *Hint 14*

**15** Francesca tried to solve the equation  $7x + 19 = 2x - 4$ . Her solution appears below.

$$\begin{array}{ll} 7x + 19 = 2x - 4 & \\ 7x = 2x - 15 & \text{line I} \\ 5x = 15 & \text{line II} \\ x = 3 & \text{line III} \end{array}$$

- Is  $x = 3$  the correct solution to the equation?  
*Hint 15*
- Where did Francesca make a mistake? *Hint 16*

Go to p 292 for **Quick Answers**  
or to pp 355–6 for **Worked Solutions**

*Hint 1: Remember the correct order of operations.*

*Hint 2: Make sure you divide by the whole denominator.*

*Hint 3: Only add or subtract like terms.  $xy$  and  $yx$  are the same term.*

*Hint 4: All of the second bracket must be subtracted. Be careful with the signs.*

*Hint 5: Multiply every term by the common denominator. Take care when there is more than one term in part of a fraction.*

*Hint 6: First divide by  $\frac{2\pi}{3}$ .*

*Hint 7: Substitute into the rearranged formula from part a.*

*Hint 8: You should know that  $10^3 = 1000$  and  $10^4 = 10\,000$ . Between what two whole numbers must  $x$  lie?*

*Hint 9: Divide the whole numerator by the whole denominator. Use brackets or the calculator's memory.*

*Hint 10: Remember that the sign in front of the term belongs to the term. Be careful with the signs.*

*Hint 11: Only  $q$  is squared.*

*Hint 12: The answer is a fraction.*

*Hint 13: Ascending order means from lowest to highest. Write each number as a decimal for easy comparison.*

*Hint 14: Substitute, then solve an equation.*

*Hint 15: Substitute  $x = 3$  into the original equation to see if it satisfies it (makes it true).*

*Hint 16: Check each line of the solution. There may be more than one mistake.*

# Solutions

## UNIT 5: ALGEBRAIC MODELLING

### Ch 12: Algebraic Skills and Techniques

#### Further Practice . . . . . p233

**1** If  $x = 2$ ,

$$3x - 5 = 3 \times 2 - 5 = 1$$

**2** If  $a = 3$ ,  $b = 2$  and  $c = 6$ ,

$$5ab - 4c = 5 \times 3 \times 2 - 4 \times 6 = 30 - 24 = 6$$

**3** If  $m = 4$ ,

$$m^2 - m = 4^2 - 4 = 12$$

**4** If  $p = 10$ ,

$$3p^2 = 3 \times 10^2 = 300$$

**5** If  $t = -3$ ,

$$10 - t = 10 - (-3) = 13$$

**6** If  $a = -2$ ,

$$6a^2 = 6 \times (-2)^2 = 6 \times 4 = 24$$

**7** If  $x = -1$ ,

$$\begin{aligned} x^3 - 4x^2 + 8x - 5 &= (-1)^3 - 4 \times (-1)^2 + 8 \times -1 - 5 \\ &= -1 - 4 - 8 - 5 \\ &= -18 \end{aligned}$$

**8** If  $x = 9$  and  $y = 40$ ,

$$\begin{aligned} \sqrt{x^2 + y^2} &= \sqrt{9^2 + 40^2} \\ &= \sqrt{1681} \\ &= 41 \end{aligned}$$

**9** If  $a = 700$ ,  $b = 800$  and  $c = 500$ ,

$$\begin{aligned} \frac{b^2 + c^2 - a^2}{2bc} &= \frac{800^2 + 500^2 - 700^2}{2 \times 800 \times 500} \\ &= \frac{400\,000}{800\,000} \\ &= 0.5 \end{aligned}$$

**10** When  $r = 9$ ,

$$\begin{aligned} 4\pi r^2 &= 4 \times \pi \times 9^2 \\ &= 1017.876\,02 \dots \\ &= 1017.88 \quad (2 \text{ d.p.}) \end{aligned}$$

**11** If  $A = 415.5$ ,

$$\begin{aligned} r &= \sqrt{\frac{A}{\pi}} \\ &= \sqrt{\frac{415.5}{\pi}} \\ &= 11.500\,337\,29 \dots \\ &= 11.5 \quad (1 \text{ d.p.}) \end{aligned}$$

**12** If  $r = 0.05$ ,  $n = 6$  and  $M = 2000$ ,

$$\begin{aligned} N &= M \left[ \frac{(1+r)^n - 1}{r(1+r)^n} \right] \\ &= 2000 \times \left[ \frac{(1+0.05)^6 - 1}{0.05(1+0.05)^6} \right] \\ &= 10\,151.384\,13 \dots \\ &= 10\,151.38 \quad (2 \text{ d.p.}) \end{aligned}$$

**13**  $9k + 3k - 7k = 5k$

**14**  $5a + 4b - 3c = 5a + 4b - 3c$

**15**  $7a + 3a + 4b = 10a + 4b$

**16**  $8p - 3q - 6p + 5q = 2p + 2q$

**17**  $x^2 + 6x - 5 + 3x - 4x^2 - 2 = -3x^2 + 9x - 7$

**18**  $-5a + 4b + 3a - 7b = -2a - 3b$

**19**  $7pq + 3p - 4q - 6qp - 3q + 1 = pq + 3p - 7q + 1$

**20**  $7 \times a \times b = 7ab$

**21**  $4p \times 5q = 20pq$

**22**  $2a \times 3b \times 5c \times 6d = 180abcd$

**23**  $4x \times 3x = 12x^2$

**24**  $3p \times 5p \times 2q = 30p^2q$

**25**  $x^4 \times x^6 = x^{10}$

**26**  $a^3b^2 \times ab^5 = a^4b^7$

**27**  $m^3 \times n^4 = m^3n^4$

**28**  $2a^5 \times 4a^4 = 8a^9$

**29**  $7x^3y^2 \times 5xy^3 = 35x^4y^5$

**30**  $6a^2b^3c^2 \times 3abc \times 2a^3b^2c^2 = 36a^6b^6c^5$

**31**  $6x \div 2 = 3x$

**32**  $7m \div m = 7$

**33**  $15xyz \div 5y = 3xz$

**34**  $\frac{9ab}{3b} = 3a$

**35**  $\frac{15pq}{3r} = \frac{5pq}{r}$

**36**  $\frac{18x^2}{12x} = \frac{3x}{2}$

**37**  $p^6 \div p^2 = p^4$

**38**  $a^5b^6 \div a^2b^3 = a^3b^3$

**39**  $x^8 \div a^2 = x^8 \div a^2 \quad \left[ \text{or } \frac{x^8}{a^2} \right]$

$$40 \quad 36x^{36} \div 9x^9 = 4x^{27}$$

$$41 \quad 12a^7b^5c^2 \div 3a^6bc^2 = 4ab^4$$

$$42 \quad \frac{24a^7b}{8a^6b^2} = \frac{3a}{b}$$

$$43 \quad 3(x+5) = 3 \times x + 3 \times 5 \\ = 3x + 15$$

$$44 \quad 4(2a-3b) = 4 \times 2a - 4 \times 3b \\ = 8a - 12b$$

$$45 \quad a(a+6) = a \times a + a \times 6 \\ = a^2 + 6a$$

$$46 \quad 5t(4t+3u-1) = 20t^2 + 15tu - 5t$$

$$47 \quad -7(2a+5b) = -7 \times 2a - 7 \times 5b \\ = -14a - 35b$$

$$48 \quad -(p-q) = -p + q$$

$$49 \quad 7x^2 + 4x \times 2x = 7x^2 + 8x^2 \\ = 15x^2$$

$$50 \quad 10n^{10} \div 2n^2 - 4n \times 3n^7 = 5n^8 - 12n^8 \\ = -7n^8$$

$$51 \quad 2(8h+7) + 3(5h-9) \\ = 16h + 14 + 15h - 27 \\ = 31h - 13$$

$$52 \quad 5(2x+3) - 2(4x+9) \\ = 10x + 15 - 8x - 18 \\ = 2x - 3$$

$$53 \quad x(x-8) - 4(x-1) \\ = x^2 - 8x - 4x + 4 \\ = x^2 - 12x + 4$$

$$54 \quad 7k+4=25 \quad [\text{subtract } 4] \\ 7k=21 \quad [\text{divide by } 7] \\ k=3$$

$$55 \quad 6x+7=23-2x \quad [\text{add } 2x] \\ 8x+7=23 \quad [\text{subtract } 7] \\ 8x=16 \quad [\text{divide by } 8] \\ x=2$$

$$56 \quad \frac{8a-5}{3} = 2a+1 \quad [\text{multiply by } 3] \\ 8a-5=6a+3 \quad [\text{subtract } 6a] \\ 2a-5=3 \quad [\text{add } 5] \\ 2a=8 \quad [\text{divide by } 2] \\ a=4$$

$$57 \quad 5x^2 = 1805 \quad [\text{divide by } 5] \\ x^2 = 361 \quad [\text{take the square root}] \\ x = \sqrt{361} \quad (x > 0) \\ x = 19$$

$$58 \quad \sqrt{7x+2} = 3 \quad [\text{square both sides}] \\ 7x+2=9 \\ 7x=7 \\ x=1$$

$$59 \quad v = u + at \\ u + at = v \quad [\text{turn the equation around}] \\ at = v - u \quad [\text{subtract } u \text{ from both sides}] \\ a = \frac{v-u}{t} \quad [\text{divide both sides by } t]$$

$$60 \quad S = V_0 - Dn \\ S + Dn = V_0 \quad [\text{add } Dn \text{ to both sides}] \\ Dn = V_0 - S \quad [\text{subtract } S] \\ n = \frac{V_0 - S}{D} \quad [\text{divide by } D]$$

$$61 \quad A = \frac{1}{2}bh \\ 2A = bh \quad [\text{multiply by } 2] \\ \frac{2A}{b} = h \quad [\text{divide by } b] \\ h = \frac{2A}{b}$$

$$62 \quad P = 2l + 2b \\ 2l + 2b = P \\ 2l = P - 2b \quad [\text{subtracting } 2b] \\ l = \frac{P-2b}{2} \quad [\text{dividing by } 2] \\ [\text{Or: } P = 2l + 2b \\ \frac{P}{2} = l + b \quad [\text{dividing by } 2] \\ l = \frac{P}{2} - b \quad [\text{subtracting } b]]$$

$$63 \quad C = \frac{5}{9}(F-32) \\ \frac{9C}{5} = F-32 \\ F = \frac{9C}{5} + 32$$

$$64 \quad x = kt^2 \\ kt^2 = x \\ t^2 = \frac{x}{k} \\ t = \pm \sqrt{\frac{x}{k}}$$

$$65 \quad A = \pi r^2 \\ \pi r^2 = A \\ r^2 = \frac{A}{\pi} \\ r = \sqrt{\frac{A}{\pi}} \quad (r > 0)$$

$$66 \quad y = mx + b \\ m = 2, b = -1, y = 7 \\ 7 = 2x + (-1) \\ 2x - 1 = 7 \\ 2x = 8 \\ x = 4$$

$$67 \quad C = \frac{k}{x^2} \\ C = 50, x = 3.4 \\ 50 = \frac{k}{3.4^2} \\ k = 50 \times 3.4^2 \\ = 578$$

$$68 \quad y = ax^2 \\ y = 320, a = 5 \\ 320 = 5x^2 \\ x^2 = 64 \\ x = \sqrt{64} \quad (x > 0) \\ x = 8$$

$$69 \quad V = \frac{1}{3}\pi r^2 h \\ V = 5750, h = 19 \\ 5750 = \frac{1}{3} \times \pi \times r^2 \times 19 \\ 5750 = \frac{19\pi}{3} r^2 \\ r^2 = 5750 \div \left(\frac{19\pi}{3}\right) \\ = 288.9918704 \dots \\ r = \sqrt{288.9918704 \dots} \quad (r > 0) \\ = 16.999 \ 760 \ 89 \dots \\ = 17.0 \quad (1 \text{ d.p.})$$

$$70 \text{ a } V = \frac{4}{3}\pi r^3 \\ 3V = 4\pi r^3 \\ r^3 = \frac{3V}{4\pi} \\ r = \sqrt[3]{\frac{3V}{4\pi}}$$

$$\text{b } V = 1988, \\ r = \sqrt[3]{\frac{3V}{4\pi}} \\ = \sqrt[3]{\frac{3 \times 1988}{4\pi}} \\ = 7.800 \ 263 \ 197 \dots \\ = 7.8 \quad (1 \text{ d.p.})$$

$$71 \text{ a } A = \pi(R^2 - r^2) \\ \frac{A}{\pi} = R^2 - r^2 \\ R^2 = \frac{A}{\pi} + r^2 \\ R = \sqrt{\frac{A}{\pi} + r^2} \quad (R > 0)$$

$$\text{b } A = 356, r = 13 \\ R = \sqrt{\frac{A}{\pi} + r^2} \quad (R > 0) \\ = \sqrt{\frac{356}{\pi} + 13^2} \\ = 16.802 \ 330 \ 78 \dots \\ = 16.8 \quad (1 \text{ d.p.})$$

- 72**  $6^x = 10\,077\,696$   
 Guess  $x = 7$ ,  
 [Any value of  $x$  could be used.]  
 $6^7 = 279\,936$  [Too low]  
 Guess  $x = 10$ ,  
 $6^{10} = 60\,466\,176$  [Too high]  
 Guess  $x = 9$ ,  
 $6^9 = 10\,077\,696$  [Correct answer]  
 The solution is  $x = 9$ .

- 73**  $2.3^t = 116\,000$   
 Try  $t = 12$ ,  
 $2.3^{12} = 21\,914.624\,43 \dots$   
 Try  $t = 20$ ,  
 $2.3^{20} = 17\,161\,558.31 \dots$   
 Try  $t = 15$ ,  
 $2.3^{15} = 266\,635.2355 \dots$   
 Try  $t = 14$ ,  
 $2.3^{14} = 115\,928.3632 \dots$   
 $\therefore$  the nearest whole number  $t$   
 for which  $2.3^t \approx 116\,000$  is  $t = 14$ .

- 74**  $3^x = 21$   
 $[3^2 = 9 \text{ and } 3^3 = 27 \text{ so } x \text{ is between } 2 \text{ and } 3]$   
 Guess  $x = 2.5$ ,  
 $3^{2.5} = 15.588\,457\,27 \dots$   
 Guess  $x = 2.7$ ,  
 $3^{2.7} = 19.419\,023\,52 \dots$   
 Guess  $x = 2.8$ ,  
 $3^{2.8} = 21.674\,022\,17 \dots$   
 Guess  $x = 2.77$ ,  
 $3^{2.77} = 20.97132523 \dots$   
 Guess  $x = 2.78$ ,  
 $3^{2.78} = 21.202989 \dots$   
 If  $3^x = 21$ ,  $x = 2.77$  to two decimal places.

**75** 
$$N = M \left[ \frac{(1+r)^n - 1}{r(1+r)^n} \right]$$
  

$$12\,200 = 800 \left[ \frac{(1.04)^n - 1}{0.04(1.04)^n} \right]$$
  
 $n = 24$  (by calculator to the nearest whole number)

**76**  $28\,000 = 2.8 \times 10^4$  [ $2.8 \times 10\,000$ ]

- 77** a  $600\,000 = 6 \times 10^5$   
 b  $9417 = 9.417 \times 10^3$   
 c  $7\,290\,000\,000 = 7.29 \times 10^9$

**78**  $7 \times 10^5 = 7 \times 100\,000$   
 $= 700\,000$

- 79** a  $4 \times 10^6 = 4 \times 1\,000\,000$   
 $= 4\,000\,000$   
 b  $3.5 \times 10^4 = 35\,000$   
 c  $1.234 \times 10^8 = 123\,400\,000$

- 80** a  $0.0007 = 7 \times 10^{-4}$   
 b  $0.0312 = 3.12 \times 10^{-2}$   
 c  $0.000\,45 = 4.5 \times 10^{-4}$

- 81** a  $5 \times 10^{-4} = 0.0005$   
 b  $8.7 \times 10^{-6} = 0.000\,0087$   
 c  $4.61 \times 10^{-5} = 0.000\,0461$

**82**  $4.6 \times 10^4 \times 3.1 \times 10^{-6} = 0.1426$

**83**  $400\,000 \div 0.000\,05 = 8\,000\,000\,000$   
 $= 8 \times 10^9$

**84** Population density  
 $= (2 \times 10^6) \div (2.5 \times 10^6)$   
 $= 0.8$

There are 0.8 people/km<sup>2</sup>.

**85** a 57.5 million km  
 $= 57\,500\,000 \text{ km}$   
 $= 5.75 \times 10^7 \text{ km}$   
 $= (5.75 \times 10^7) \times 1000 \text{ m}$   
 $= 5.75 \times 10^{10} \text{ m}$

- b Time = distance  $\div$  speed  
 $= (5.75 \times 10^{10}) \div (3 \times 10^8)$   
 $= 191.666\,66 \dots$   
 $= 192$  (nearest whole number)  
 192 seconds = 3 minutes and  
 12 seconds  
 It takes approximately 3 minutes and  
 12 seconds for light from the sun to  
 reach Mercury.

### Challenge . . . . . p236

**1**  $3x^2 + 5x + 2x - 8x^3 + 4x$   
 $= 3x^2 + 10x^2 - 2x^2$   
 $= 11x^2$

**2**  $a = 3, b = 4, c = 6$   

$$\frac{7a + 4b - 3c}{a^2 + b^2} = \frac{7 \times 3 + 4 \times 4 - 3 \times 6}{3^2 + 4^2}$$
  

$$= \frac{19}{25}$$
  
 $= 0.76$

**3**  $3xy + 5x - 2y + 4yx = 7xy + 5x - 2y$

**4**  $5x(x^2 + 3x - 2) - 2(3x^2 + 1)$   
 $= 5x^3 + 15x^2 - 10x - 6x^2 - 2$   
 $= 5x^3 + 9x^2 - 10x - 2$

**5** 
$$\frac{4x}{9} + \frac{x+1}{3} = 5$$
  

$$9 \times \frac{4x}{9} + 9 \times \left( \frac{x+1}{3} \right) = 9 \times 5$$
  

$$4x + 3(x+1) = 45$$
  

$$4x + 3x + 3 = 45$$
  

$$7x + 3 = 45$$
  

$$7x = 42$$
  

$$x = 6$$

**6** 
$$V = \frac{2\pi r^3}{3}$$
  

$$3V = 2\pi r^3$$
  

$$\frac{3V}{2\pi} = r^3$$
  

$$r = \sqrt[3]{\frac{3V}{2\pi}}$$

**7** a 
$$R = \frac{a^2 + b^2 - c^2}{2ab}$$
  

$$2abR = a^2 + b^2 - c^2$$
  

$$c^2 = a^2 + b^2 - 2abR$$
  

$$c = \sqrt{a^2 + b^2 - 2abR} \quad (c > 0)$$

b  $R = 0.5, a = 8, b = 5$   

$$c = \sqrt{a^2 + b^2 - 2abR}$$
  

$$= \sqrt{8^2 + 5^2 - 2 \times 8 \times 5 \times 0.5}$$
  

$$= \sqrt{49}$$
  
 $= 7$

**8**  $10^x = 5370$   
 Try  $x = 3.5$ ,  
 $10^{3.5} = 3162.277\,66 \dots$   
 Try  $x = 3.7$ ,  
 $10^{3.7} = 5011.872\,336 \dots$   
 Try  $x = 3.75$ ,  
 $10^{3.75} = 5623.413\,252 \dots$   
 Try  $x = 3.73$ ,  
 $10^{3.73} = 5370.3179\,64 \dots$   
 $\therefore x = 3.73$  to two decimal places.

**9** 
$$\frac{5.9 \times 10^6 + 1.5 \times 10^5}{4.4 \times 10^8 \times 2.5 \times 10^{-3}} = \frac{6.05 \times 10^6}{1.1 \times 10^6}$$
  
 $= 5.5$

**10**  $8x - 5a + 3a = 8x - 2a$

**11**  $p = 3, q = 5$   
 $pq^2 = 3 \times 5^2$   
 $= 3 \times 25$   
 $= 75$

**12** 
$$\frac{2ab^2}{8a^3b^2} = \frac{1}{4a^2}$$

**13**  $5.834 \times 10^{-4}, 9 \times 10^{-3}, 3.1 \times 10^{-2}$   
 $[0.000\,5834, 0.009\,0000, 0.031\,0000]$

**14**  $y = 2x^2 + b$   
 When  $y = 19$  and  $b = 1$   
 $19 = 2x^2 + 1$   
 $18 = 2x^2$   
 $9 = x^2$   
 $x = 3 \quad (x > 0)$

**15**  $7x + 19 = 2x - 4$   
 $7x = 2x - 15$     Line I  
 $5x = 15$     Line II  
 $x = 3$     Line III

a When  $x = 3$ ,

$$7x + 19 = 7 \times 3 + 19$$
$$= 40$$

$$2x - 4 = 2 \times 3 - 4$$
$$= 2$$

$x = 3$  is not the correct solution.

b There is a mistake in line I.

Subtracting 19 from both sides leaves

$$7x = 2x - 23.$$

There is a mistake in line II.

Subtracting  $2x$  from both sides of line I leaves  $5x = -23$ .

The correct solution is:

$$7x + 19 = 2x - 4$$

$$7x = 2x - 23$$

$$5x = -23$$

$$x = -4.6$$