

(iii) On the graph, the point where the lines meet is the break-even point where Toby has earned the amount he spent on equipment. This occurs where  $n = 6$ , which means that after 6 parties he starts making a profit.

(iv) Amount saved in each year      \$

2001, Year 9:	900
2002, Year 10:	$900 + 30\% \times 900 = 1170$
2003, Year 11:	$1170 + 30\% \times 1170 = 1521$
2004, Year 12:	$1521 + 30\% \times 1510 = 1977.30$
$\therefore$ Total saved =	\$5568.30.

**METHOD 1**

Savings account

	Balance at start of year	Interest earned (4%)	Amount added	Balance at end of year
2005, 1st yr	5568.30	222.73	2500	8291.03
2006, 2nd yr	8291.03	331.64	2500	11 122.67
2007, 3rd yr	11 122.67	444.91	2500	14 067.58

With \$14 067.58, Toby does not quite reach his goal of \$15 000.

**METHOD 2.**

After 2004, the investment can be thought of as a combination of:

- an annuity of \$2500 invested yearly for 3 years;
- \$5568.30 invested for 3 years;

These calculations are:

- \$2500 invested yearly at 4% pa compounded annually for 3 years. Using the future value formula, with  $M = \$2500$ ,  $r = 4\% = 0.04$ ,  $n = 3$ :

$$A = \frac{M[(1+r)^n - 1]}{r}$$

$$= \frac{2500[(1+0.04)^3 - 1]}{0.04}$$

$$= \$7804;$$

- \$5568.30 invested for 3 years at 4% p.a. Using the compound interest formula, with  $P = \$5568.30$ ,  $r = 4\% = 0.04$ ,  $n = 3$ :

$$A = P(1+r)^n$$

$$= 5568.30(1+0.04)^3$$

$$= \$6263.58 \text{ (nearest cent).}$$

$$\therefore \text{Total value of investment} = \$7804 + \$6263.58 = \$14\,067.58.$$

With \$14 067.58, Toby will not reach his goal of \$15 000. (He will be short by \$932.42.)

END OF GENERAL MATHEMATICS SOLUTIONS

2001 HIGHER SCHOOL CERTIFICATE  
EXAMINATION PAPER  
GENERAL MATHEMATICS

**Section I**

Total marks (22)

Attempt Questions 1 – 22

Allow about 30 minutes for this section

1 Alex works in a shop where the normal weekday rate of pay is \$12 per hour. On Saturdays she is paid time-and-a-half.

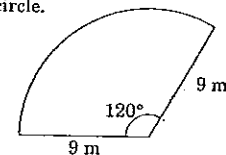
How much did Alex earn in a week in which she worked for seven hours on Thursday and three hours on Saturday?

- (A) \$84                      (B) \$120                      (C) \$138                      (D) \$180

2 If  $w = \frac{15y}{y+12}$ , and  $y = 7$ , find the value of  $w$  (correct to two decimal places).

- (A) 5.53                      (B) 8.26                      (C) 15.75                      (D) 27.00

3 This is a sketch of a sector of a circle.



Calculate the area of this sector (correct to one decimal place).

- (A) 9.4 m<sup>2</sup>                      (B) 18.8 m<sup>2</sup>                      (C) 36.8 m<sup>2</sup>                      (D) 84.8 m<sup>2</sup>

4 Frank has a credit card with an interest rate of 0.05% per day and no interest-free period.

Frank used the credit card to pay for car repairs costing \$480. He paid the credit card account 16 days later. What is the total amount (including interest) that he paid for the repairs?

- (A) \$480.24                      (B) \$483.84                      (C) \$504.00                      (D) \$864.00

5 Simplify  $3(x-2) - 2(x-1)$ .

- (A)  $x-4$                       (B)  $x-3$                       (C)  $x-1$                       (D)  $x-8$

6 The number represented by a 1 followed by one hundred zeros is called a googol.

Which of the following is equal to a googol?

- (A) 10<sup>2</sup>                      (B) 10<sup>10</sup>                      (C) 10<sup>99</sup>                      (D) 10<sup>100</sup>

- 7 Brenda surveyed the students in her year group and summarised the results in the following table.

	Play tennis	Do not play tennis	TOTALS
Right-handed	53	81	134
Left-handed	22	29	51
TOTALS	75	110	185

What percentage of the left-handed students in this group play tennis? (Round your answer to the nearest whole number.)

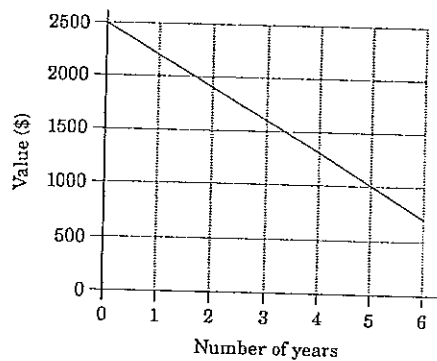
- (A) 11%      (B) 12%      (C) 29%      (D) 43%
- 8 The following frequency table shows Ravdeep's scores on a number of quizzes.

Score	Frequency
1	2
2	3
3	5
4	2
5	1

Which expression gives Ravdeep's mean score?

- (A)  $\frac{2+6+15+8+5}{13}$     (B)  $\frac{2+6+15+8+5}{5}$     (C)  $\frac{1+2+3+4+5}{13}$     (D)  $\frac{1+2+3+4+5}{5}$

- 9 A computer was purchased for \$2500 and depreciated over six years, as shown in the graph.



By how much did the computer depreciate each year?

- (A) \$200      (B) \$250      (C) \$300      (D) \$350

- 10 The table shows personal income tax rates.

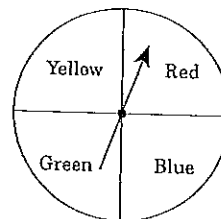
Taxable income	Tax on this income
\$0 – \$6 000	Nil
\$6 001 – \$20 000	17 cents for each \$1 over \$6 000
\$20 001 – \$50 000	\$2 380 plus 30 cents for each \$1 over \$20 000
\$50 001 – \$60 000	\$11 380 plus 42 cents for each \$1 over \$50 000
\$60 001 and over	\$15 580 plus 47 cents for each \$1 over \$60 000

Sandra has a gross income of \$60 780 and deductions that total \$2420.

What is the tax payable on Sandra's taxable income?

- (A) \$13 526.60      (B) \$14 891.20      (C) \$15 946.60      (D) \$17 084.00

11



The arrow is spun and will point to one of the four colours when it stops.

If the arrow is spun twice, what is the probability that it points to the same colour both times it stops?

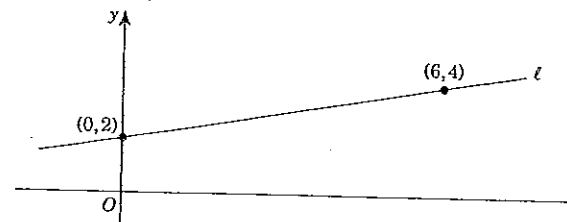
- (A)  $\frac{1}{16}$       (B)  $\frac{1}{8}$   
 (C)  $\frac{1}{4}$       (D)  $\frac{1}{2}$

- 12 Josephine invested \$1000 at the end of each year for five years. Her investment earned interest at 4.8% per annum compounded annually.

What was the total value of Josephine's investment (to the nearest dollar) at the end of the fifth year?

- (A) \$5024      (B) \$5240      (C) \$5504      (D) \$6321

13



What is the equation of the line  $l$ ?

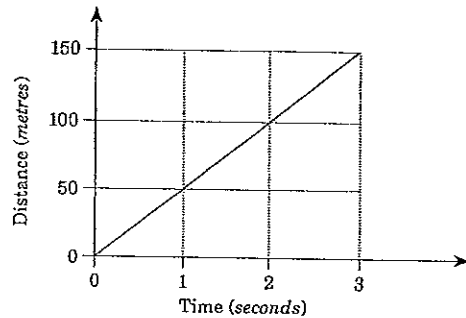
- (A)  $y = 6x+2$       (B)  $y = x+2$       (C)  $y = 3x+2$       (D)  $y = \frac{1}{3}x+2$

- 14 Joyce measures the length of a piece of wood as 250 mm, correct to the nearest mm. What is the percentage error in her measurement?  
 (A)  $\pm 0.002\%$  (B)  $\pm 0.004\%$  (C)  $\pm 0.2\%$  (D)  $\pm 0.4\%$

Use the back-to-back stem-and-leaf plot to answer Questions 15 and 16.

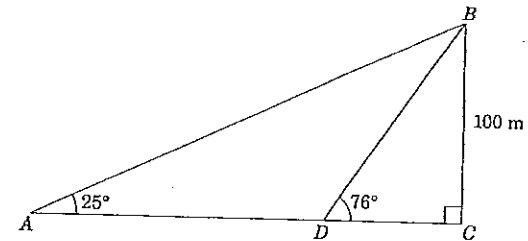
SCORES ON A CLASS TEST				
Boys			Girls	
9 8 8	0	6		
5 4 4 2 2	1	2 2 5 8		
9 3 1 1	2	1 3 3 4 5 5 6		
5 4 2	3	2 2 4		

- 15 What is the range of scores in this class test?  
 (A) 27 (B) 28 (C) 29 (D) 35
- 16 Find the median score for the boys in this class test.  
 (A) 12 (B) 15 (C) 19 (D) 21
- 17 The distance-time graph for a moving object is shown.



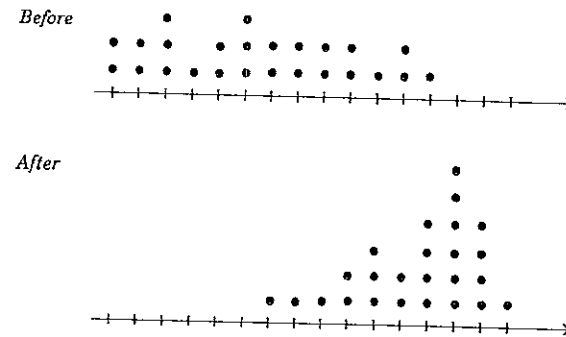
- What is the speed of the object in kilometres per hour?  
 (A) 3 km/h (B) 14 km/h (C) 50 km/h (D) 180 km/h
- 18 A sphere has a volume of  $360 \text{ cm}^3$ . What is its radius (correct to one decimal place)?  
 (A) 1.7 cm (B) 4.4 cm (C) 8.1 cm (D) 9.3 cm
- 19 A factory produces bags of flour. The weights of the bags are normally distributed, with a mean of 900 g and a standard deviation of 50 g. What is the best approximation for the percentage of bags that weigh more than 1000 g?  
 (A) 0% (B) 2.5% (C) 5% (D) 16%

- 20 Calculate the length of  $AD$  (to the nearest metre).



NOT TO SCALE

- (A) 25 m (B) 134 m (C) 190 m (D) 214 m
- 21 The dot plots below are drawn on the same scale. They show the class scores in tests taken before and after a unit of work was completed.



- Which statement about the change in scores is correct?  
 (A) The mean increased and the standard deviation decreased.  
 (B) The mean increased and the standard deviation increased.  
 (C) The mean decreased and the standard deviation decreased.  
 (D) The mean decreased and the standard deviation increased.
- 22 Sonia has written letters to four of her friends and sealed the letters in envelopes. Now she does not know which envelope contains which letter. If Sonia addresses the envelopes to her four friends at random, what is the probability that each envelope contains the correct letter?  
 (A)  $\frac{1}{256}$  (B)  $\frac{1}{24}$  (C)  $\frac{1}{16}$  (D)  $\frac{1}{4}$

**Section II**

78 marks

Attempt Questions 23 – 28

Allow about 2 hours for this section

**Question 23 (13 marks)**

(a) The 11 people in Sam's cricket team always bat in the same order. Sam recorded the batting order and the average number of runs scored by each player during the season.

Batting order	Average number of runs
1	16
2	10
3	11
4	8
5	7
6	4
7	4
8	5
9	3
10	1
11	1

Marks

(i) Display the data as a scatterplot on the graph paper provided on page 14. Make sure that you have labelled the axes.

2

(ii) Draw a line of fit on your scatterplot on the graph paper provided on page 14. (No calculations are necessary.)

1

(iii) Using your scatterplot, describe the correlation between the batting order and the average number of runs.

1

(b) Results for a reading test are given as z-scores. In this test, Kim gained a z-score equal to  $-2$ .

(i) Interpret this z-score in terms of the mean and standard deviation of the test.

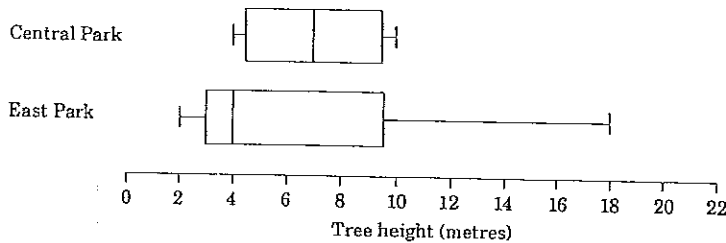
2

(ii) If the test has a mean of 75 and a standard deviation of 5, calculate the actual mark scored by Kim.

1

(c) Andy and her biology class went to two large city parks and measured the heights of the trees in metres.

In Central Park there were 25 trees. In East Park there were 27 trees. The data sets were displayed in two box-and-whisker plots.



(i) In which park is the tallest tree, and how high is it?

2

(ii) What is the median height of trees in Central Park?

1

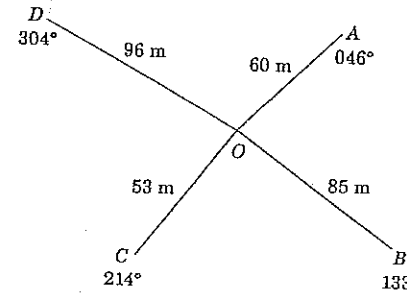
(iii) Compare and contrast the two data sets by examining the shape and skewness of the distributions, and the measures of location and spread.

3

**Question 24 (13 marks)**

Marks

(a) The following notebook entry was made during a radial survey of a field.



NOT TO SCALE

(i) What is the size of  $\angle AOB$ ?

1

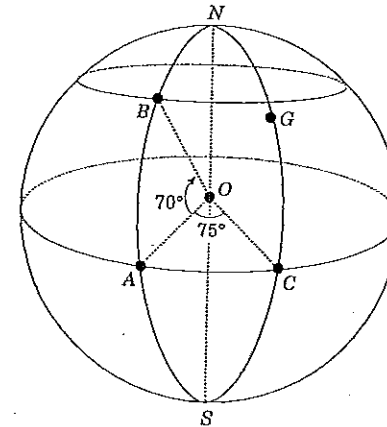
(ii) Calculate the area of triangle  $AOB$ . Round your answer to the nearest square metre.

2

(iii) Find the distance from  $A$  to  $B$ .

2

(b)



NOT TO SCALE

In this diagram of the Earth,  $O$  represents the centre and  $G$  represents Greenwich. The point  $A$  lies on the equator.

(i) What is the time difference between Greenwich and point  $A$ ? (Ignore time zones.)

1

(ii) What is the latitude of point  $B$ ?

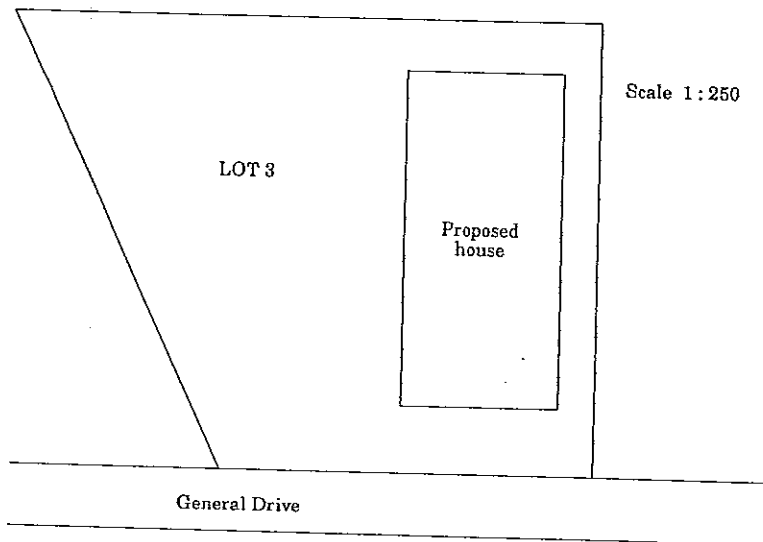
1

(iii) Calculate, to the nearest kilometre, the great circle distance from point  $A$  to point  $B$ . (You may assume that the radius of the Earth is 6400 km, and that 1 nautical mile = 1.852 km.)

2

(c) This is a site plan, drawn to scale, of Lot 3, General Drive.

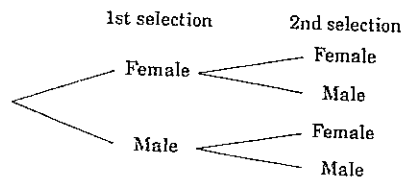
Marks



- (i) A fence is to be erected along all boundaries of Lot 3 except for the boundary on General Drive. How many metres of fencing will be required? 2
- (ii) Lot 3 is in the shape of a trapezium. By measurement and calculation, determine the actual area of Lot 3 in square metres. 2

**Question 25 (13 marks)**

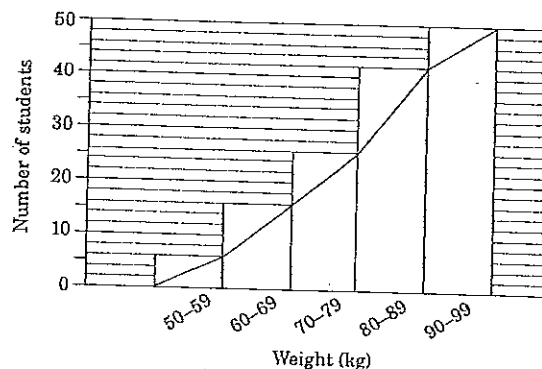
- (a) Five men and three women are living on an island, but not all will be able to stay.
  - (i) If one person is selected at random, what is the probability that this person is female? 1
  - (ii) Two people are to be randomly selected to leave the island.
    - 1 Copy the tree diagram, and complete the diagram by writing the probabilities on all the branches. 2



- 2 Calculate the probability that the selection includes exactly one female. 2
- (iii) Antoinette is one of the women on the island. Before the two people are randomly selected to leave, Antoinette calculates her chance of remaining on the island. She concludes that she has a good chance of remaining. Do you agree? Justify your answer. 2

(b) Armand recorded the weights of a random sample of male students in his Year. The cumulative frequency graph displays the results.

Marks



- (i) How many of the students surveyed were in the 80-89 kg class? 1
- (ii) Estimate the median weight of the students surveyed. 1
- (iii) Of the 300 male students in Armand's Year, how many would you expect to weigh less than 70 kg? 2
- (iv) 1 In order to select a sample, Armand's friend suggested selecting the first 50 male students in his Year to arrive at school on Monday morning. Explain why this would NOT be a random sample. 1
- 2 Describe a method that could have been used to select a random sample of the male students. 1

**Question 26 (13 marks)**

(a) Otto is the manager of a weekend market in which there are 220 stalls for rent. From past experience, Otto knows that if he charges  $d$  dollars to rent a stall, then the number of stalls,  $s$ , that will be rented is given by:

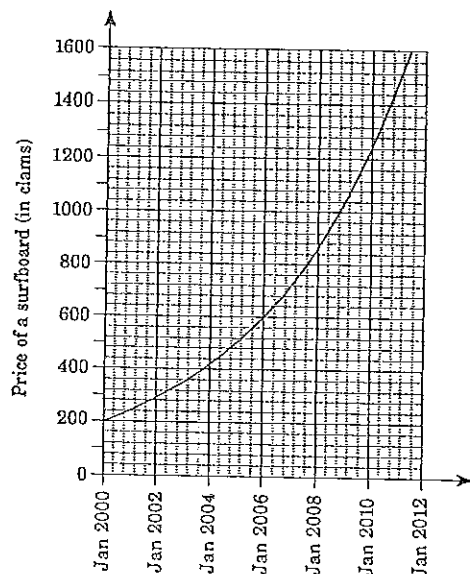
$$s = 220 - 4d.$$

- (i) How many stalls will be rented if Otto charges \$7.50 per stall? 1
- (ii) Copy and complete the following table for the function  $s = 220 - 4d$ . 1

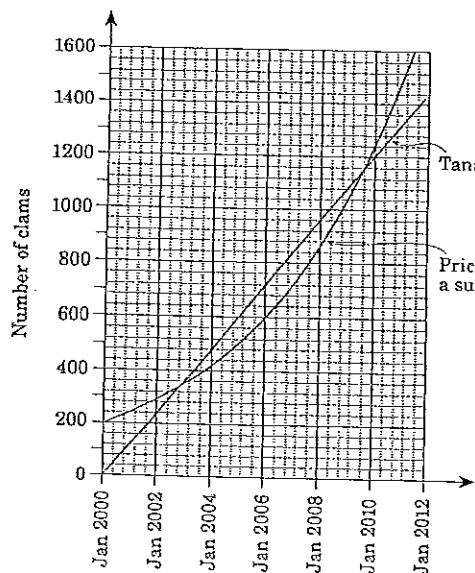
$d$	10	30	50
$s$			

- (iii) Draw a graph of the function  $s = 220 - 4d$ . Use your ruler to draw the axes. Label each axis, and mark a scale on each axis. 2
- (iv) Does it make sense to use the formula  $s = 220 - 4d$  to calculate the number of stalls rented if Otto charges \$60 per stall? Explain your answer. 1

- (b) On the island of Wupetoi the unit of currency is the clam. The rate of inflation on Wupetoi has been constant for many years. Assuming the rate of inflation remains constant, the price of a surfboard will increase as shown in the graph. Marks



- (i) The formula used to draw this graph was  $P = A \times (1.2)^t$ , where  $P$  = price of a surfboard and  $t$  = number of years after January 2000.
- 1 What is the value of  $A$ , and what does it represent? 2
  - 2 What annual rate of inflation has been assumed? 1



- (ii) In January 2000, Tana started saving a fixed number of clams each month, in order to buy a surfboard. The straight line on the graph at left represents Tana's savings.
- 1 During which year will Tana first be able to afford a surfboard? Explain your answer. 2
  - 2 If Tana uses his savings to buy a surfboard in January 2006, how many clams will he have left? 1
  - 3 Write an equation that describes the relationship between Tana's savings in clams ( $c$ ) and the number of months ( $n$ ) after January 2000. 2

Question 27 (13 marks)

Marks

- (a) George buys a television for \$574.20, including 10% GST. What is the value of the GST component? 1
- (b) A car is purchased for \$42 000. Use the declining balance method to calculate the salvage value of the car after 4 years at a depreciation rate of 15% per annum. 2
- (c) Derek and Rosetta both reached the age of 55 in 2001. They had each contributed regularly to an investment fund. Both investments earned interest at the rate of 6% per annum compounded monthly.

INVESTMENT DETAILS				
	Monthly contribution	Number of years	Total contribution	Value of investment in 2001
Derek	\$400	15 (from the age of 40)	\$72 000	\$116 327
Rosetta	\$200	30 (from the age of 25)	\$72 000	\$200 903

- (i) Explain the large difference between the values of their investments in 2001, given that Derek and Rosetta had each contributed \$72 000. 1
- (ii) If they each continue their regular monthly contributions for a further 5 years, will the difference between the values of their investments grow larger? Justify your answer with appropriate calculations. 3
- (d) Ted has borrowed \$70 000 at an interest rate of 6.24% per annum compounded monthly. The repayments have been set at \$680 per month. The loan balance sheet shows the interest charged and the balance owing for the first month.

Month	Principal (at start of month)	Monthly interest	Monthly repayment	Balance (at end of month)
1	\$70 000	$\$70\,000 \times 0.0052 = \$364$	\$680	\$69 684
2	\$69 684	A	\$680	B

- (i) Explain why 0.0052 is used to calculate the monthly interest. 1
- (ii) Find the missing amounts at A and B. 2
- (iii) Ted would like to calculate the number of months,  $n$ , it will take to repay the loan fully. He uses a 'guess-and-check' method to estimate  $n$  in the following equation:

$$\$680 \times \left\{ \frac{(1-0.0052)^n - 1}{0.0052 \times (1-0.0052)^n} \right\} = \$70\,000.$$

Here is his working.

Try  $n = 200$ :

$$\$680 \times \left\{ \frac{(1.0052)^{200} - 1}{0.0052 \times (1.0052)^{200}} \right\} \div \$84\,424$$

Hence  $n = 200$  is too big.

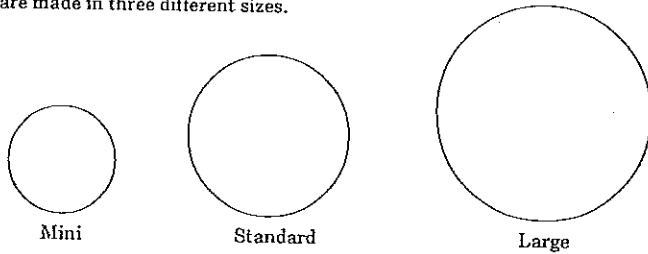
Marks

- 1 Ted's next guess is  $n = 120$ . Show Ted's working for this guess, including the calculation and the conclusion.
- 2 State a reasonable value of  $n$  for the next guess.

2  
1

**Question 28 (13 marks)**

(a) Joe's pizzas are made in three different sizes.



Joe puts olives on all his pizzas. The number of olives depends on the size of the pizza, as shown in the table.

Size	Diameter, $d$ (cm)	Number of olives, $n$
Mini	20	8
Standard	30	18
Large	40	32

The relationship between the diameter of the pizza and the number of olives can be expressed by the formula:

$$n = kd^2, \text{ where } k \text{ is a constant.}$$

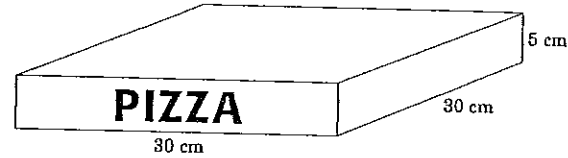
- (i) Use a pair of values from the table to show that  $k = 0.02$ .
- (ii) Joe decides to make a mega-pizza, with diameter 52 cm. Use the formula to find the number of olives needed for a mega-pizza.
- (iii) Joe is asked to make a pizza in the shape of a square with sides of length 25 cm. He decides to use the same number of olives as would be needed on a round pizza with the same area. How many olives will be needed?

1  
1  
3

Marks

- (b) Joe uses a microwave oven to heat lasagne. The time taken for heating is inversely proportional to the power setting (in watts). It takes ten minutes at a power setting of 240 watts to heat the lasagne. How long would it take at a power setting of 500 watts?
- (c) Joe's standard pizza boxes have dimensions as shown.

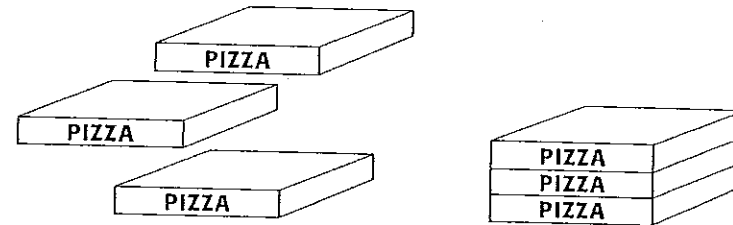
3



NOT TO SCALE

- (i) What is the surface area of one box?
- (ii) The surface area of a stack of these boxes is less than the total surface area of the individual boxes in the stack.

1  
4



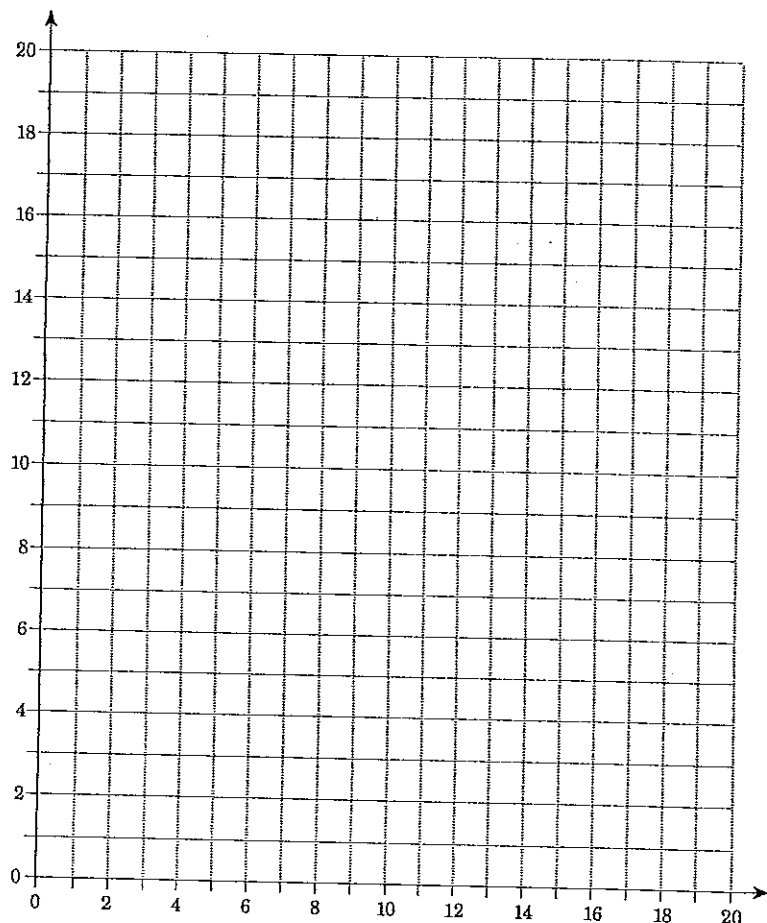
Three individual boxes

A stack of three boxes

Write down the formula for the surface area of  $N$  individual boxes AND determine a formula for the surface area of a stack of  $N$  boxes.

End of paper

Graph for Question 23(a)



## 2001 HIGHER SCHOOL CERTIFICATE SOLUTIONS GENERAL MATHEMATICS

### SECTION I

#### SUMMARY

- |      |       |       |       |
|------|-------|-------|-------|
| 1. C | 7. D  | 13. D | 19. B |
| 2. A | 8. A  | 14. C | 20. C |
| 3. D | 9. C  | 15. C | 21. A |
| 4. B | 10. B | 16. B | 22. B |
| 5. A | 11. C | 17. D |       |
| 6. D | 12. C | 18. B |       |

1. (C) Earnings =  $7 \times \$12 + 3 \times 1.5 \times \$12$   
= \$138.
2. (A)  $w = \frac{15 \times 7}{7 + 12} = \frac{105}{19} = 5.5263 \dots \div 5.53$ .
3. (D) Area =  $\frac{\theta}{360} \times \pi r^2$   
=  $\frac{120}{360} \times \pi \times 9^2$   
=  $84.823 \dots$   
 $\div 84.8 \text{ m}^2$ .
4. (B)  $I = Prn$   
=  $\$480 \times 0.0005 \times 16$   
= \$3.84.  
Total amount =  $\$480 + \$3.84 = \$483.84$ .
5. (A)  $3(x-2) - 2(x-1) = 3x - 6 - 2x + 2$   
=  $x - 4$ .
6. (D)  $10^2 = 100$  (2 zeros)  
 $10^3 = 1000$  (3 zeros)  
 $10^{100} = 10\,000 \dots$  (100 zeros).
7. (D) No. of left-handed students = 51.  
No. of left-handed students who play tennis = 22.  
Percentage of left-handed students who play tennis =  $\frac{22}{51} \times 100\% \div 43\%$ .
8. (A)  $\bar{x} = \frac{\sum fx}{\sum f}$   
=  $\frac{1 \times 2 + 2 \times 3 + 3 \times 5 + 4 \times 2 + 5 \times 1}{2 + 3 + 5 + 2 + 1}$   
=  $\frac{2 + 6 + 15 + 8 + 5}{13}$ .

9. (C) From the graph, it depreciated \$1500 (from \$2500 to \$1000) in 5 years.  
Depreciation per year =  $\frac{\$1500}{5} = \$300$ .

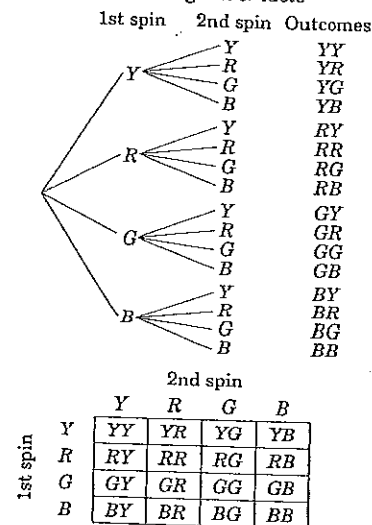
10. (B) Taxable income = income - deductions  
= \$60 780 - \$2420  
= \$58 360.

From 4th row of table, tax payable  
= \$11 380 + 0.42(\$58 360 - \$50 000)  
= \$14 891.20.

11. (C) **METHOD 1: By counting techniques**  
This is a 2-stage situation (2 spins) with 4 possibilities at each stage.  
No. of possible outcomes =  $4 \times 4 = 16$ .  
No. of outcomes with same colour = 4 (YY, RR, GG, BB).

$P(\text{same colour}) = \frac{4}{16} = \frac{1}{4}$ .

**METHOD 2: Listing outcomes by tree diagram or table**





Total no. of possible outcomes = 16.  
 No. of outcomes with same colour = 4 (YY, RR, GG, BB).  
 $P(\text{same colour}) = \frac{4}{16} = \frac{1}{4}$ .

**METHOD 3: Reasoning**  
 With this situation, we don't need to be concerned with the colour of the first spin. However, in the second spin, we are aiming to match the colour of the first spin.

On the first spin, the arrow will point to one of 4 colours: yellow, red, green or blue. On the second spin, the probability that the arrow points to the same colour (as the first spin) is  $\frac{1}{4}$ .

12. (C)  $A = M \left[ \frac{(1+r)^n - 1}{r} \right]$   
 $M = 1000, r = 0.048, n = 5$   
 $A = 1000 \left[ \frac{(1+0.048)^5 - 1}{0.048} \right]$   
 $= 5503.598 \dots$   
 $\doteq \$5504$ .

13. (D)  $y = mx + b$   
 $b = 2$ , the y intercept  
 $m = \frac{\text{vertical change}}{\text{horizontal change}}$   
 $= \frac{4-2}{6-0} = \frac{2}{6} = \frac{1}{3}$   
 Equation of line is  $y = \frac{1}{3}x + 2$ .

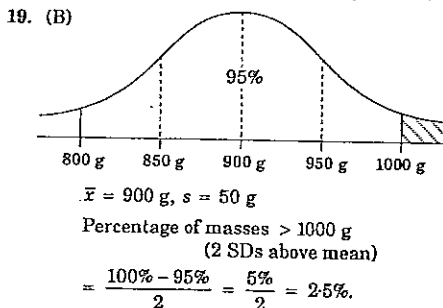
14. (C) Error =  $\pm 0.5$  mm.  
 Percentage error =  $\frac{\pm 0.5 \text{ mm}}{250 \text{ mm}} \times 100\%$   
 $= \pm 0.2\%$ .

15. (C) Range = highest score - lowest score  
 $= 35 - 6$   
 $= 29$ .

16. (B) No. of scores (boys) = 15.  
 The middle score is the 8th score.  
 Median (boys) = 15 (counting the 8th score).

17. (D) From the graph,  
 Speed =  $\frac{\text{distance}}{\text{time}} = \frac{150 \text{ m}}{3 \text{ s}} = 50 \text{ m/s}$ .  
 Converting to km/h:  
 $50 \text{ m/s} = 50 \times 60 \times 60 \text{ m/h}$   
 $= 180\,000 \text{ m/h}$   
 $= 180\,000 \div 1000 \text{ km/h}$   
 $= 180 \text{ km/h}$ .

18. (B)  $V = \frac{4}{3}\pi r^3$ .  
 Substitute  $V = 360$ :  $360 = \frac{4}{3}\pi r^3$   
 $3 \times 360 = 4\pi r^3$   
 $1080 = 4\pi r^3$   
 $\frac{1080}{4\pi} = r^3$   
 $r = \sqrt[3]{\frac{1080}{4\pi}}$   
 $= 4.413 \dots$   
 $\doteq 4.4 \text{ cm}$ .



20. (C) In  $\triangle ABC$ ,  $\tan 25^\circ = \frac{100}{AC}$   
 $AC = \frac{100}{\tan 25^\circ}$   
 $= 214.4506 \dots \text{ m}$ .  
 In  $\triangle DBC$ ,  $\tan 76^\circ = \frac{100}{DC}$   
 $DC = \frac{100}{\tan 76^\circ}$   
 $= 24.9328 \dots \text{ m}$ .  
 $AD = AC - DC$   
 $= 214.4506 \dots - 24.9328 \dots$   
 $= 189.5178 \dots$   
 $\doteq 190 \text{ m}$ .

OR In  $\triangle DBC$ ,  $\sin 76^\circ = \frac{100}{DB}$   
 $DB = \frac{100}{\sin 76^\circ}$   
 $= 103.0613 \dots \text{ m}$ .  
 In  $\triangle ABD$ ,  
 $\angle ADB = 180^\circ - 76^\circ$  ( $\angle$  sum of st. line  $ADC$ )  
 $= 104^\circ$ .  
 $\angle ABD = 180^\circ - 25^\circ - 104^\circ$  ( $\angle$  sum of  $\triangle ABD$ )  
 $= 51^\circ$ .  
 Using the sine rule,  
 $\frac{AD}{\sin 51^\circ} = \frac{DB}{\sin 25^\circ}$   
 $AD = \frac{DB \sin 51^\circ}{\sin 25^\circ}$

/continued ...

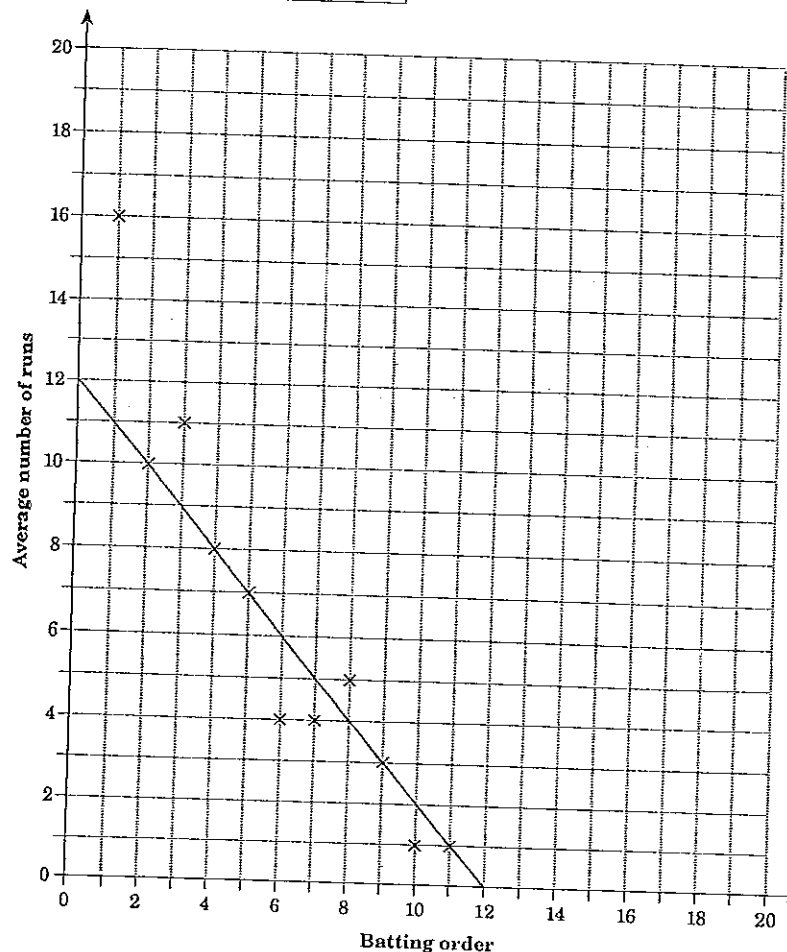
**SECTION II**

**Question 23**

- (a) (i) and (ii)  
 See graph below. Other lines of fit are possible.  
 (iii) There is a strong negative correlation. That is, the smaller the number in batting order, the higher is the average number of runs scored.  
 (b) (i) 2 standard deviations below the mean.  
 (ii) **METHOD 1**  
 $\bar{x} = 75, s = 5$ .  
 As Kim scored 2 standard deviations below the mean, Kim's mark =  $75 - 2 \times 5 = 65$ .

$= \frac{103.0613 \dots \sin 51^\circ}{\sin 25^\circ}$  (from above)  
 $= 189.5178 \dots$   
 $\doteq 190 \text{ m}$ .

21. (A) In the *After* dot plot, the dots are further to the right so the mean increased, and they are closer together (smaller spread) so the standard deviation decreased.  
 22. (B) Total no. of ways of placing 4 letters in 4 envelopes =  $4 \times 3 \times 2 \times 1 = 24$ .  
 Note: One letter per envelope.  
 No. of ways of placing correct letters in correct envelopes = 1.  
 $P(\text{each envelope contains correct letter}) = \frac{1}{24}$ .



**METHOD 2**

$$z = \frac{x - \bar{x}}{s}$$

$$-2 = \frac{x - 75}{5}$$

$$-10 = x - 75$$

$$x = 65$$

Kim's mark was 65.

- (c) (i) The tallest tree is in East Park and it is 18 m tall.
- (ii) 7 m
- (iii) In comparing the shape, Central Park trees had a symmetrical distribution, whereas East Park trees were positively skewed. The median height of trees in Central Park is higher than the median height of trees in East Park. The range and interquartile range in East Park are higher than in Central Park. 50% of trees in East Park are shorter than the shortest tree in Central Park, even though East Park contains the tallest tree.

**Question 24**

- (a) (i)  $\angle AOB = 133 - 46 = 87^\circ$ .
- (ii)  $A = \frac{1}{2} ab \sin C$
- $$= \frac{1}{2} \times 60 \times 85 \times \sin 87^\circ$$
- $$\doteq 2546.505$$
- $\therefore$  The area is 2547 m<sup>2</sup> correct to the nearest square metre.
- (iii)  $c^2 = a^2 + b^2 - 2ab \cos C$
- $$= 60^2 + 85^2 - 2 \times 60 \times 85 \times \cos 87^\circ$$
- $$\doteq 10\,291.17 \dots$$
- $$c \doteq \sqrt{10\,291.17 \dots}$$
- $$\doteq 101.45 \text{ m (correct to 2 d.p.)}$$
- (b) (i) Longitude difference between G and A is 75°.
- 15° = 1 hour
- Time difference = 75° ÷ 15° = 5 hours.
- OR 1° is equivalent to 4 minutes difference.
- $\therefore$  Time difference = 75 × 4 = 300 minutes = 5 hours.
- (ii) 70° North
- (iii) **METHOD 1**
- $$l = \frac{\theta}{360} \times 2\pi r$$
- $$= \frac{70}{360} \times 2 \times \pi \times 6400$$
- $$\doteq 7819 \text{ km (to the nearest km)}$$

**METHOD 2**

1° = 60 nautical miles

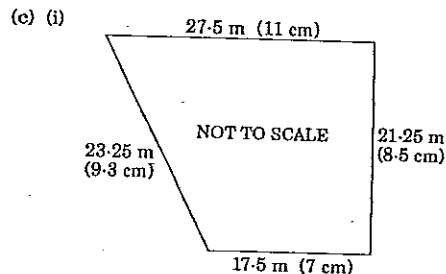
$\therefore AB = 70 \times 60$

$$= 4200 \text{ nautical miles}$$

$$= 4200 \times 1.852 \text{ km}$$

$$\doteq 7778 \text{ km, to the nearest km}$$

Note: Method 2 is more accurate as 1 M is exactly 1.852 km, but the radius of the Earth is only approximately 6400 km.



KEY: Actual length (measured length)

Perimeter = 23.25 + 27.5 + 21.25 = 72 m.

(ii)  $A = \frac{h}{2}(a + b)$

$$= \frac{21.25}{2}(27.5 + 17.5)$$

$$= 478.125 \text{ m}^2$$

**Question 25**

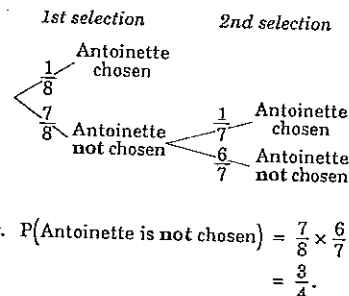
- (a) (i)  $P(\text{female}) = \frac{3}{8}$ .
- (ii) 1
- |  |                      |                      |  |
|--|----------------------|----------------------|--|
|  | <i>1st selection</i> | <i>2nd selection</i> |  |
|  | $\frac{3}{8}$ Female | $\frac{2}{7}$ Female |  |
|  |                      | $\frac{5}{7}$ Male   |  |
|  | $\frac{5}{8}$ Male   | $\frac{3}{7}$ Female |  |
|  |                      | $\frac{4}{7}$ Male   |  |
- 2  $P(\text{exactly one female})$
- $$= P(F, M) + P(M, F)$$
- $$= \left(\frac{3}{8} \times \frac{5}{7}\right) + \left(\frac{5}{8} \times \frac{3}{7}\right)$$
- $$= \frac{15}{28}$$
- (iii) **METHOD 1**
- If two people are randomly chosen to leave the island, then Antoinette has a  $\frac{2}{8}$  chance of being chosen.

/continued ...

This means she has a  $\frac{6}{8}$  chance of not being chosen and  $\frac{6}{8} = \frac{3}{4} = 75\%$  chance is a good chance of not being chosen.

Note: The number of men and women on the island has no effect on this problem.

**METHOD 2**

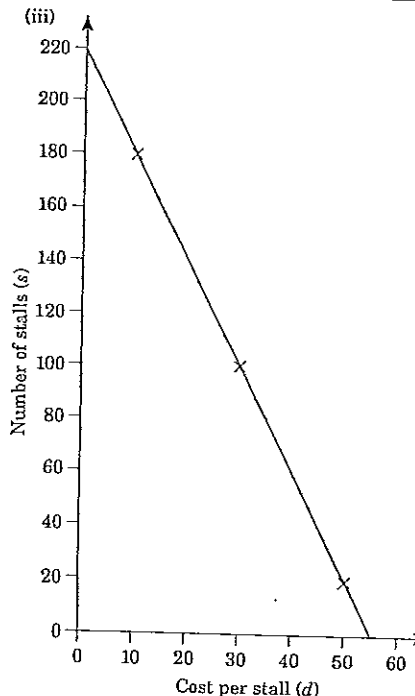


Yes, Antoinette has a 3 in 4 chance or 75% chance of remaining, which is a good chance.

- (b) (i) 42 - 26 = 16 students.
- (ii) Using the scale of the weight axis, the 25th score is approximately 77 kg.
- (iii) Number of students less than 70 kg = 16.
- Expected number in population =  $\frac{16}{60} \times 300 = 96$  students.
- (iv) 1 Selecting the first 50 male students to arrive is a systematic sample which is biased towards students who arrive early. This is not a random sample.
- 2 —Put all the male names in a hat and select 50.
- Assign each male student a number and, using a random number generator, select 50.

**Question 26**

- (a) (i)  $s = 220 - 4d$
- $$= 220 - 4 \times 7.50$$
- $$= 190 \text{ stalls}$$
- (ii)
- |          |     |     |    |
|----------|-----|-----|----|
| <i>d</i> | 10  | 30  | 50 |
| <i>s</i> | 180 | 100 | 20 |



- (iv) When  $d = 60$ ,  $s = 220 - 4 \times 60 = -20$ . No, it doesn't make sense to have a negative number of stalls rented. The formula is only valid between \$0 and \$55.

- (b) (i) 1 **METHOD 1**
- $A = 200$  (vertical intercept of graph).  $A$  represents the price of the surfboard in Jan. 2000.

**METHOD 2**

From the graph, when  $t = 6$ ,  $P = 600$ .

$$\therefore 600 = A(1.2)^6$$

$$\therefore A = 600 \div (1.2)^6$$

$$\doteq 200.94$$

$A$  represents the price of the surfboard in Jan. 2000.

- 2  $1.2 = 1 + \frac{20}{100}$
- $\therefore$  An annual rate of inflation of 20% has been assumed.
- (ii) 1 During 2002, because this is where the graphs of savings and price of the surfboard intersect for the first time.
- 2 In 2006, savings = 720.
- $\therefore 720 - 600 = 120$  clams left.

- 3 Gradient =  $\frac{1200 \text{ clams}}{10 \text{ years}}$   
 = 120 clams per year  
 = 10 clams per month.  
 y intercept is 0.  
 $\therefore$  Equation is  $c = 10n$ .

## Question 27

- (a) 110% of normal price = \$574.20.

METHOD 1 1% = \$574.20 + 110  
 = \$5.22.

10% = \$5.22  $\times$  10  
 = \$52.20.

GST = \$52.20.

METHOD 2 10% = \$574.20 + 11  
 = \$52.20.

- (b)
- $S = V_0(1-r)^n$

$V_0 = 42\ 000$ ,  $r = 0.15$ ,  $n = 4$

$$S = 42\ 000(1-0.15)^4$$

$$= 42\ 000(0.85)^4$$

$$= 21\ 924.2625$$

$$\div \$21\ 924.26.$$

Salvage value of the car is \$21 924.

- (c) (i) Rosetta's investment is higher because she has been contributing longer at 30 years, as opposed to Derek's 15 years. Even though her monthly contributions were smaller, they have had more time to accumulate compound interest.

- (ii) Current difference = \$200 903 - \$116 327  
 = \$84 576.

5 years into the future:

$$A = M \left\{ \frac{(1+r)^n - 1}{r} \right\}$$

For Derek:  $M = 400$ ,  $r = \frac{0.06}{12} = 0.005$ ,

$n = 20$  years = 240 months.

$$A = 400 \left\{ \frac{(1.005)^{240} - 1}{0.005} \right\}$$

$$= 184\ 816.3581 \dots$$

$$\div \$184\ 816.36.$$

For Rosetta:  $M = 200$ ,  $r = 0.05$ ,

$n = 35$  years = 420 months.

$$A = 200 \left\{ \frac{(1.005)^{420} - 1}{0.005} \right\}$$

$$= 284\ 942.0597 \dots$$

$$\div \$284\ 942.06.$$

Future difference  
 = \$284 942.06 - \$184 816.36  
 = \$100 125.70  
 > \$84 576 (current difference).

$\therefore$  The difference between the values of their investments will grow larger.

- (d) (i) The yearly interest rate divided by 12 gives the monthly interest rate:

$$6.24\% = 0.0624$$

$$0.0624 + 12 = 0.0052.$$

(ii) A: \$69 684  $\times$  0.0052 = \$362.3568

$$\div \$362.36.$$

B: \$69 684 + \$362.36 = \$68 000

$$= \$69\ 366.3568$$

$$\div \$69\ 366.36.$$

- (iii) 1 Try  $n = 120$ :

$$\$680 \times \left\{ \frac{(1.0052)^{120} - 1}{0.0052(1.0052)^{120}} \right\}$$

$$\div \$60\ 590.10$$

$$< \$70\ 000.$$

$\therefore n = 120$  is too small.

- 2  $\therefore$  The value of  $n$  is between 120 and 200. However, a guess of  $n = 120$  is closer to the \$70 000 target than  $n = 200$ .

A reasonable value for the next guess would be  $n = 140$  (or any value below the halfway mark  $n = 160$ ).

## Question 28

- (a) (i) Using  $d = 20$  and  $n = 8$ :

$$n = kd^2$$

$$8 = k \times 20^2 = 400k.$$

$$\therefore k = \frac{8}{400} = 0.02.$$

- (ii)  $n = kd^2 = 0.02d^2$ .

$$d = 52: n = 0.02 \times 52^2 = 54.08.$$

$\therefore$  54 olives are needed.

- (iii) Area of square pizza =  $25 \times 25 = 625 \text{ cm}^2$ .  
 Finding radius of circle with equivalent area:

$$A = \pi r^2$$

$$625 = \pi r^2$$

$$r^2 = \frac{625}{\pi} = 198.943 \dots$$

$$r = \sqrt{198.943 \dots} = 14.10 \dots \text{ cm.}$$

Finding diameter of circular pizza:

$$d = 2 \times r = 28.2 \dots \text{ cm.}$$

Number of olives:  $n = kd^2$

$$n = 0.02 \times (28.2 \dots)^2$$

$$= 15.915 \dots$$

$\therefore$  16 olives are required.

- (b) Let  $t$  = time taken for heating in minutes,  
 $p$  = power setting in watts.

$$t \propto \frac{1}{p}$$

$$\therefore t = \frac{k}{p}$$

When  $t = 10$ ,  $p = 240$ :

$$10 = \frac{k}{240}$$

$$k = 10 \times 240 = 2400.$$

$$\therefore t = \frac{2400}{p}$$

When  $p = 500$ ,  $t = \frac{2400}{500}$

$$= 4.8 \text{ minutes}$$

$$= 4 \text{ min } 48 \text{ s.}$$

- (c) (i) Surface area of one box

$$= (\text{top and bottom}) + (4 \text{ sides})$$

$$= (2 \times 30 \times 30) + (4 \times 30 \times 5)$$

$$= 1800 + 600$$

$$= 2400 \text{ cm}^2.$$

- (ii) Individual

Surface area of 1 box = 2400

2 boxes =  $2400 \times 2$

3 boxes =  $2400 \times 3$

$N$  boxes =  $2400 \times N$ .

$\therefore$  Surface area of  $N$  individual boxes is  $2400N$ .

Stacked

METHOD 1

Surface area of  $N$  boxes

$$= (\text{top and bottom}) + (N \times 4 \text{ sides})$$

$$= (2 \times 30 \times 30) + (N \times 4 \times 30 \times 5)$$

$$= 1800 + 600N.$$

$\therefore$  Surface area of  $N$  stacked boxes is  $600N + 1800$ .

METHOD 2

Surface area of  $N$  boxes

$$= (\text{surface area of } N \text{ individual boxes})$$

$$- (\text{overlapping areas})$$

$$= N \times 2400 - (N-1) \times (2 \times 30 \times 30)$$

\*from (i)

$$= 2400N - 1800(N-1)$$

$$= 2400N - 1800N + 1800$$

$$= 600N + 1800.$$

$\therefore$  The surface area of  $N$  stacked boxes is  $600N + 1800$ .

END OF GENERAL MATHEMATICS SOLUTIONS