METHOD 2

$C = 0{\cdot}05b^3$.

Substitute $b = 10$: $C = 0{\cdot}05 \times 10^3 = 50$.

Substitute $b = 20$ (since the base is doubled):

$C = 0{\cdot}05 \times (20)^3 = 400$.

When the base is doubled the cost is 8 times more, so Felicity is incorrect.

(c) (i) Median height of middle section = 170.
Median weight of middle section = 55.
∴ Coordinates of C are (170, 55).

(ii) Jill needs to draw a line parallel to AB which is a third of the distance towards C.

(iii) (1) Weight in kg $= \frac{2}{3}(\text{height in cm}) - 50$.

Substitute weight = 75.

$$75 = \frac{2}{3}(\text{height in cm}) - 50$$

$$125 = \frac{2}{3}\text{ height in cm}$$

$$\text{height in cm} = \frac{125}{2} \times 3 = 187{\cdot}5.$$

∴ The height is predicted to be 187·5 cm.

(2) Suggested possible answers are:
- the sample used is small (only 9 people)

OR
- tall people are not necessarily heavier

OR
- the model cannot be used outside the range of the sample

OR
- the accuracy with which the graph is drawn.

END OF GENERAL MATHEMATICS SOLUTIONS

2003 Higher School Certificate Examination Paper
GENERAL MATHEMATICS

Section I

22 marks
Attempt Questions 1 – 22
Allow about 30 minutes for this section

1 A number of men and women were surveyed at a railway station. They were asked whether or not they were travelling to work. The table shows the results.

	Going to work	*Not going to work*
Men	64	24
Women	60	42

How many men were surveyed?

(A) 64 (B) 88 (C) 124 (D) 190

2 Simplify $3y^3 \div 12y^2$.

(A) $4y$ (B) $\frac{4}{y}$ (C) $\frac{1}{4y}$ (D) $\frac{y}{4}$

3 Dora works for \$9·60 per hour for eight hours each day on Thursday and Friday. On Saturday she works for six hours at time-and-a-half.

How much does Dora earn in total for Thursday, Friday and Saturday?

(A) \$192·00 (B) \$211·20 (C) \$240·00 (D) \$316·80

4 If $d = \sqrt{\frac{h}{5}}$, what is the value of d, correct to one decimal place, when $h = 28$?

(A) 1·1 (B) 2·4 (C) 2·8 (D) 5·6

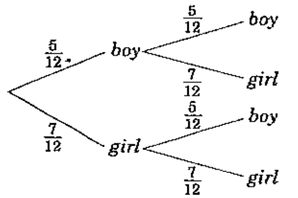
5 Jim bought a new car at the beginning of 2001 for \$40 000. At the end of 2001 the value of the car had depreciated by 30%. In 2002 the value of the car depreciated by 25% of the value it had at the end of 2001.

What was the value of the car at the end of 2002?

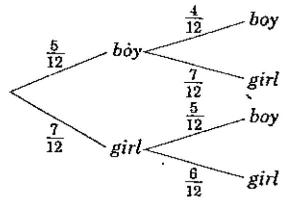
(A) \$18 000 (B) \$19 600 (C) \$21 000 (D) \$22 000

6 From 5 boys and 7 girls, two children will be chosen at random to work together on a project. Which of the following probability trees could be used to determine the probability of choosing a boy and a girl?

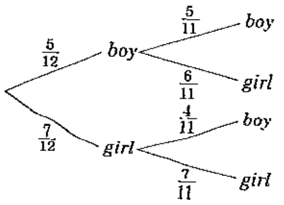
(A)



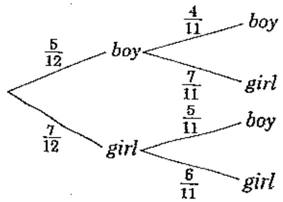
(B)



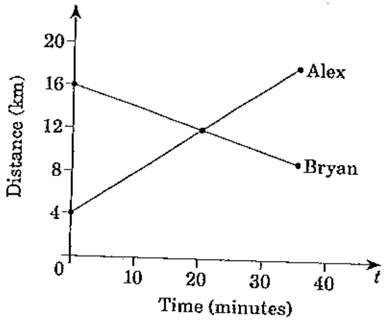
(C)



(D)



7 At the same time, Alex and Bryan start riding towards each other along a road. The graph shows their distances (in kilometres) from town after t minutes.

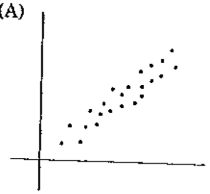


How many kilometres has Alex travelled when they meet?

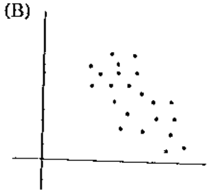
(A) 4 (B) 8 (C) 12 (D) 20

8 Which scatterplot shows a low (weak) positive correlation?

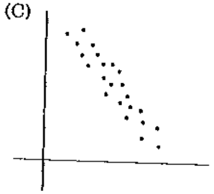
(A)



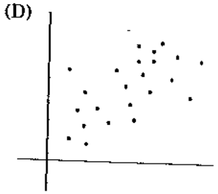
(B)



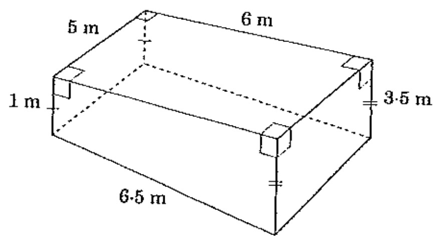
(C)



(D)



9 A swimming pool has a length of 6 m and a width of 5 m. The depth of the pool is 1 m at one end and 3·5 m at the other end, as shown in the diagram.



NOT TO SCALE

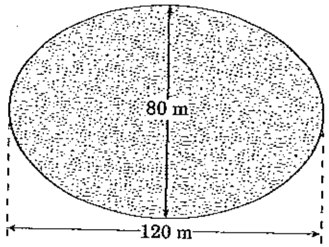
What is the volume of this pool in cubic metres?

(A) 67·5 (B) 105 (C) 109·375 (D) 113·75

10 Kathmandu is 30° west of Perth. Using the longitude difference, what is the time in Kathmandu when it is noon in Perth?

(A) 10:00 am (B) 11:30 am (C) 12:30 pm (D) 2:00 pm

11 The council wants to put new grass on a park which is in the shape of an ellipse.



NOT TO SCALE

If grass costs $7·50 per square metre, what is the total cost to the nearest dollar?

(A) $7540 (B) $30 159 (C) $56 549 (D) $226 195

Use the following information to answer Questions 12 and 13.

Joy asked the students in her class how many brothers they had. The answers were recorded in a frequency table as follows:

Number of brothers	*Frequency*
0	5
1	10
2	3
3	1
4	1

12 What is the mean number of brothers?

(A) 1·15 (B) 2 (C) 2·3 (D) 4

13 One of the students is chosen at random. What is the probability that this student has at least two brothers?

(A) 0·10 (B) 0·15 (C) 0·25 (D) 0·75

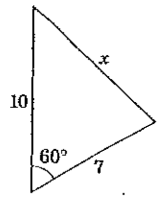
14 Which equation should be used to obtain the value of x in this triangle?

(A) $\dfrac{x}{\sin 60^\circ} = \dfrac{7}{\sin 10^\circ}$

(B) $x^2 = 10^2 + 7^2 - 2 \times 10 \times 7 \cos 60^\circ$

(C) $\cos 60^\circ = \dfrac{x^2 + 10^2 - 7^2}{2 \times 10 \times 7}$

(D) $x^2 = 10^2 - 7^2$



NOT TO SCALE

15 Kylie and Danny work in a music store. The weekly wage $\$W$ of an employee at the store is given by

$$W = 0{\cdot}75n + 50,$$

where n is the number of CDs the employee sells.

If Kylie sells two more CDs than Danny in one week, how much more will she earn?

(A) \$0·75 (B) \$1·50 (C) \$50·75 (D) \$51·50

16 Pauline calculates the present value (N) of an annuity. The interest rate is 4% per annum, compounded monthly. In five years the future value will be \$100 000.

Which calculation will result in the correct answer?

(A) $N = \dfrac{100\,000}{(1+0{\cdot}04)^{5}}$ (B) $N = \dfrac{100\,000}{(1+0{\cdot}04 \div 12)^{5}}$

(C) $N = \dfrac{100\,000}{(1+0{\cdot}04)^{60}}$ (D) $N = \dfrac{100\,000}{(1+0{\cdot}04 \div 12)^{60}}$

17 If an electrical current varies inversely with resistance, what is the effect on the current when the resistance is doubled?

(A) The current is doubled (B) The current is exactly the same.
(C) The current is halved. (D) The current is squared.

18 George measures the breadth and length of a rectangle to the nearest centimetre. His answers are 10 cm and 15 cm. Between what lower and upper values must the actual area of the rectangle lie?

(A) 10×15 cm² (lower) and 11×16 cm² (upper)
(B) 10×15 cm² (lower) and $10{\cdot}5 \times 15{\cdot}5$ cm² (upper)
(C) $9{\cdot}5 \times 14{\cdot}5$ cm² (lower) and 10×15 cm² (upper)
(D) $9{\cdot}5 \times 14{\cdot}5$ cm² (lower) and $10{\cdot}5 \times 15{\cdot}5$ cm² (upper)

19 The roof of the Sydney Opera House is covered with 1·056 million tiles. If each tile covers 175 cm², what area is covered by the tiles?

(A) 184·8 m² (B) 18 480 m² (C) 184 800 m² (D) 1 848 000 m²

20 Iliana buys several items at the supermarket. The docket for her purchases is shown.

XYZ Supermarket
27/04/03 12.53

MILK 1 L	\$1.19
* PEPSI 1.25 L	\$1.29
* DISINFECTANT	\$7.23
* TEA TREE OIL	\$4.13
SPINACH	\$1.64
SOUP	\$1.57
TOTAL	\$17.05

10% GST INCLUDED IN COST OF TAXABLE ITEMS
* = TAXABLE ITEMS

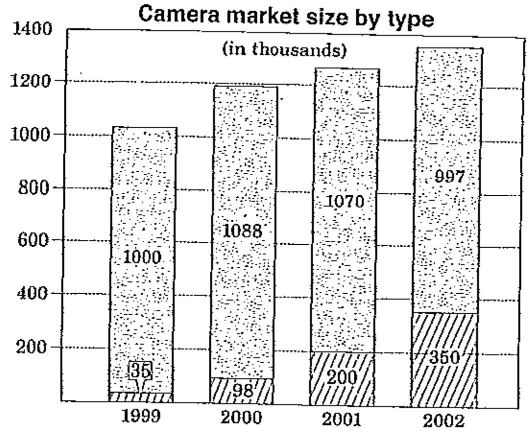
What is the amount of GST included in the total?

(A) \$1·15
(B) \$1·27
(C) \$1·55
(D) \$1·71

21 The graph shows the numbers of the two major types of cameras, analog and digital, sold in Australia in the years 1999–2002.

In 2001, what percentage of the cameras sold were digital cameras? (To the nearest per cent.)

(A) 16%
(B) 19%
(C) 23%
(D) 84%



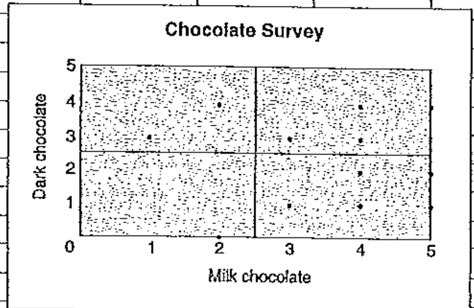
22 Charlie surveyed 12 school friends to find out their preferences for chocolate. They were asked to indicate their liking for milk chocolate on the following scale.

Dislike			Like		
strongly	moderately	a little	a little	moderately	strongly
0	1	2	3	4	5

They were also asked to do this for dark chocolate. Charlie displayed the results in a spreadsheet and graph as shown below.

Chocolate Survey Results

Milk chocolate	Dark chocolate
4	2
3	3
4	3
2	4
5	1
4	1
2	0
1	3
5	4
4	4
3	1
5	2



Charlie assumes that these 12 students are representative of the 600 students at the school.

What is Charlie's estimate of the number of students in the school who **like** milk chocolate but **dislike** dark chocolate?

(A) 50 (B) 200 (C) 250 (D) 450

Section II

78 marks

Attempt Questions 23 – 28
Allow about 2 hours for this section

Question 23 (13 marks)

Marks

(a) Keryn is designing a new watering system for the shrubs in her garden. She knows that each shrub needs 1·2 litres of water per day. To minimise evaporation, Keryn designs a system to drip water into a tube that takes the water to the roots.

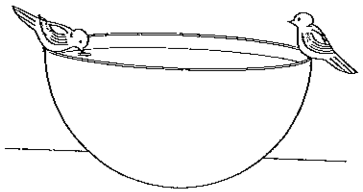
(i) What is the number of litres of water required daily for 13 shrubs? **1**

(ii) Keryn pays 94·22 cents per kilolitre for water. Calculate the total cost of watering 13 shrubs for one week. **1**

(iii) Keryn knows that 1 mL = 15 drops. Find the number of drops that one shrub needs daily. **1**

(iv) How many drops per minute are required for one shrub if the system is in use for 10 hours per day? **1**

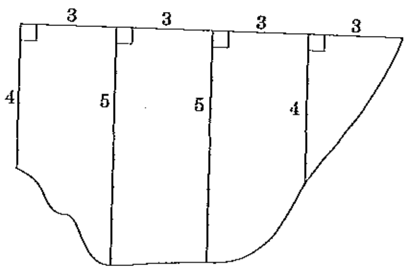
(b) In her garden, Keryn has a birdbath in the shape of a hemisphere (half a sphere). The internal diameter is 45 cm. **2**



NOT TO SCALE

What is the internal surface area of this birdbath? (Give your answer to the nearest square centimetre.)

(c) A river has a cross-section as shown below, with measurements in metres. **2**

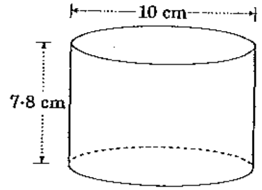


NOT TO SCALE

Calculate the area of this cross-section using Simpson's rule.

Marks

(d) Peta is designing an eight-cylinder racing engine. Each cylinder has a bore (diameter) of 10·0 cm and a stroke (height) of 7·8 cm, as shown below.



NOT TO SCALE

(i) Calculate the volume of each cylinder, correct to the nearest cubic centimetre. **2**

(ii) The capacity of the engine is the sum of the capacities of the eight cylinders. Does Peta's engine meet the racing requirement that the capacity should be under 5 litres? Justify your answer with a mathematical calculation. **3**

Question 24 (13 marks)

(a) Minh invests $24 000 at an interest rate of 4·75% per annum, compounded monthly. What is the value of the investment after 3 years? **2**

(b) Vicki earns a taxable income of $58 624 from her job with an insurance company. She pays $14 410·80 tax on this income.

(i) Vicki has a second job which pays $900 gross income per month. What is Vicki's total annual taxable income from both jobs, assuming that she has no allowable tax deductions? **1**

(ii) Use the tax table below to calculate the total tax payable on her income from both jobs. **2**

Taxable income	*Tax payable*
$0 – $6000	NIL
$6001 – $22 000	18 cents for each $1 over $6000
$22 001 – $55 000	$2880 plus 30 cents for each $1 over $22 000
$55 001 – $66 000	$12 780 plus 45 cents for each $1 over $55 000
$66 001 and over	$17 730 plus 48 cents for each $1 over $66 000

(iii) Show that Vicki's monthly net income from her second job is $486·44. **3**

(iv) Vicki plans to take a holiday in two years time which she estimates will cost $12 000. At the end of each month, Vicki invests the net income from her second job in an account which pays 4% per annum, compounded monthly. **2**

Will she have enough in this account, immediately after the twenty-fourth payment, to pay for her holiday? Justify your answer with calculations.

Marks

(c) Zoë plans to borrow money to buy a car and considers the following repayment guide: 3

Fortnightly car loan repayment guide

Amount borrowed ($)	*Length of loan* 1 year ($)	2 years ($)	3 years ($)
10 000	410	217	153
10 500	430	228	161
11 000	451	239	168
11 500	471	249	176
12 000	492	260	183
12 500	512	271	191
13 000	532	282	199
13 500	553	293	206
14 000	573	303	214
14 500	594	314	221
15 000	614	325	229
15 500	635	336	237
16 000	655	347	244

Zoë wishes to borrow $15 500 and pay back the loan in fortnightly instalments over two years. What is the flat rate of interest per annum on this loan?

Question 25 (13 marks)

(a) A census was conducted of the 33 171 households in Sunnytown. Each household was asked to indicate the number of cars registered to that household. The results are summarised in the following table.

Number of cars	*Frequency*
0	2 735
1	12 305
2	13 918
3	3 980
4	233
Total	33 171

(i) (1) Determine the mode number of cars in a household. 1

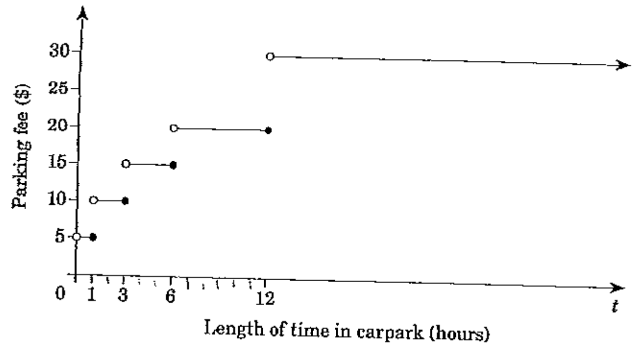
(2) Explain what is meant by *the mode number of cars in a household.* 1

(ii) Sunnytown Council issued a 'free parking' sticker for each car registered to a household in Sunnytown. How many parking stickers were issued? 2

(iii) The council represented the results of the census in a sector graph. 1
What is the angle in the sector representing the households with no cars? Give your answer to the nearest degree.

(iv) Visitors to Sunnytown Airport have to pay for parking. The following step graph shows the cost of parking for t hours. 1

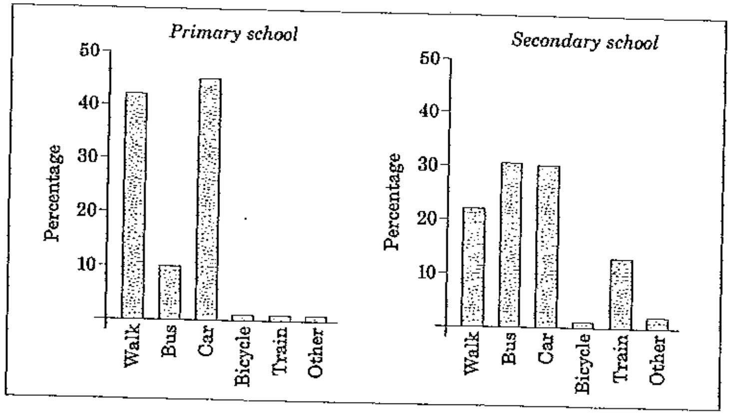
Marks



What is the cost for a car that is parked one evening from 6 pm to 8:30 pm?

(b) Equal large numbers of primary and secondary school students in a city were surveyed about their method of travel to school.
The results are summarised in the relative frequency column graphs below.

Method of travel to school



(i) Describe TWO differences in the method of travel between these primary school and secondary school students. 2

(ii) Suggest a possible reason for ONE of these differences. 1

(iii) There were 25 000 primary school students surveyed. How many of these students travelled to school by bus? 1

(c) Results for an aptitude test are given as z-scores. In this test, Hardev gains a z-score of 1.

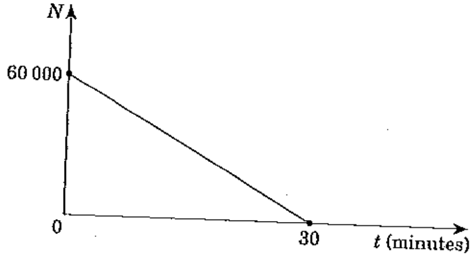
(i) Interpret Hardev's score with reference to the mean and standard deviation of the test. 2

(ii) The scores for the test are normally distributed. What proportion of people sitting the test obtain a higher score than Hardev? 1

Question 26 (13 marks) **Marks**

(a) At a World Cup rugby match, the stadium was filled to capacity for the entire game. At the end of the game, people left the stadium at a constant rate.

The graph shows the number of people (N) in the stadium t minutes after the end of the game.



The equation of the line is of the form $N = a - bt$, where a and b are constants.

(i) Write down the value of a, and give an explanation of its meaning. **2**

(ii) (1) Calculate the value of b. **1**

(2) What does the value of b represent in this situation? **1**

(iii) Rearrange the formula $N = a - bt$ to make t the subject. **2**

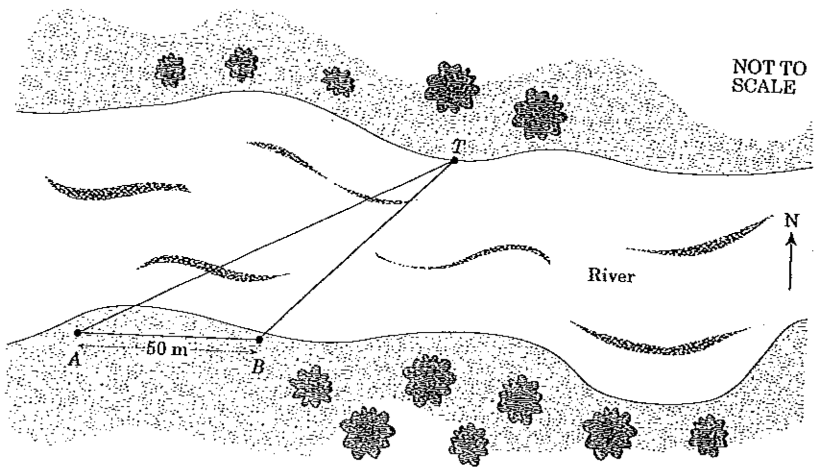
(iv) How long did it take 10 000 people to leave the stadium? **1**

(v) Copy or trace the graph of N against t shown above. **2**

Suppose that 15 minutes after the end of the game, several of the exits had been closed, reducing the rate at which people left.

On the same axes, carefully draw another graph of N against t that could represent this new situation. You new graph should show N from $t = 15$ until all the people had left the stadium.

(b)



In the diagram above, the following measurements are given: $\angle TAB = 30°$.
B is 50 m due east of A.
The bearing of T from B is 020°.

Copy or trace ΔABT. **Marks**

(i) Explain why $\angle ABT$ is 110°. **1**

(ii) Calculate the distance BT (to the nearest metre). **3**

Question 27 (13 marks)

(a) A celebrity mathematician, Karl, arrives in Sydney for one of his frequent visits. Karl is known to stay at one of three Sydney hotels.

Hotel X is his favourite, and he stays there on 50% of his visits to Sydney. When he does not stay at Hotel X, he is equally likely to stay at Hotels Y or Z.

(i) What is the probability that he will stay at Hotel Z? **1**

(ii) On his first morning in Sydney, Karl always flips a coin to decide if he will have a cold breakfast or a hot breakfast. If the coin comes up heads he has a cold breakfast. If the coin comes up tails he has a hot breakfast.

(1) List all the possible combinations of hotel and breakfast choices. **2**

(2) Give a brief reason why these combinations are not all equally likely. **1**

(3) Calculate the probability that Karl stays at Hotel Z and has a cold breakfast. **1**

(b) Two unbiased dice are thrown. The dice each have six faces. The faces are numbered 1, 2, 3, 4, 5 and 6.

(i) What is the probability that neither shows a 6? **1**

(ii) Dale plays a game with these dice. There is no entry fee. **3**

When the dice are thrown:

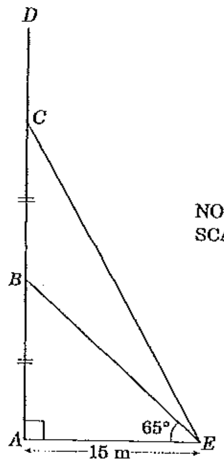
- Dale wins \$20 if both dice show a 6.
- He wins \$2 if there is only one 6.
- He loses \$2 if neither shows a 6.

What is his financial expectation from this game?

(c) The diagram shows a radio mast AD with two of its supporting wires, BE and CE. The point B is half-way between A and C.

(i) Calculate the height AB in metres, correct to one decimal place. **2**

(ii) Calculate the distance CE in metres, correct to one decimal place. **2**



Question 28 (13 marks) **Marks**

(a) Sandra is on holiday in Mexico and she plans to buy some silver jewellery. Various silver pendants are on sale. The cost varies directly with the square of the length of the pendant. A pendant of length 30 mm costs 130 Mexican pesos. **2**

How much does a pendant of length 40 mm cost? (Answer correct to the nearest Mexican peso.)

(b) In 2002, the population of Mexico was approximately 103 400 000.

(i) The growth rate of Mexico's population is estimated to be 1·57% per annum. If y represents the estimated number of people in Mexico at a time x years after 2002, write a formula relating x and y in the form $y = b(a^x)$. Use appropriate values for a and b in your formula. **2**

(ii) Using your formula, or otherwise, find an estimate for the size of Mexico's population two years after 2002. Express your answer to the nearest thousand. **2**

(c) (i) While Sandra is on holiday she visits countries where the Fahrenheit temperature scale is used. She knows that the correct way to convert from Celsius to Fahrenheit is: **1**

'Multiply the Celsius temperature by 1·8, then add 32.'

Find the value of A in the following table.

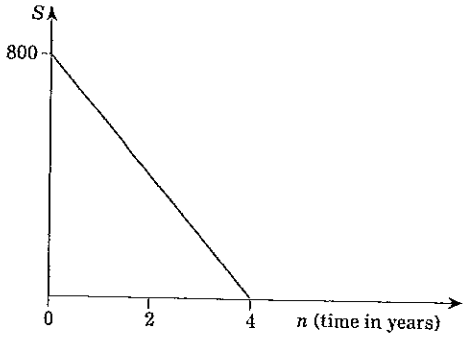
Celsius	*Fahrenheit*
5	41
15	59
A	77

(ii) Peter uses the following method to approximate the conversion from Celsius to Fahrenheit: **2**

'Add 12 to the Celsius temperature, then double your result.'

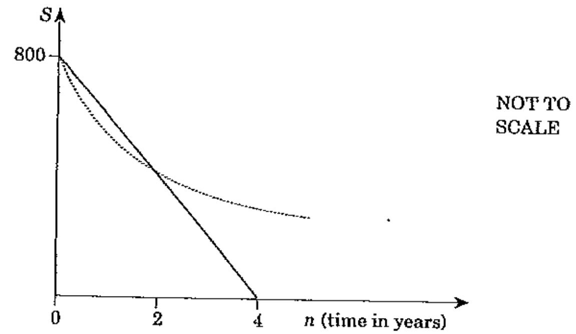
Express Peter's rule as an algebraic equation. Use C for the Celsius temperature and F for the approximate Fahrenheit temperature.

(d) While Sandra is on holidays she buys a digital camera for $800. Using the straight-line method of depreciation, the salvage value, S, of the camera will be zero in four years time, as shown on the graph below. **4**



Marks

Using the declining-balance method of depreciation, at a rate of R% per annum, the salvage value is represented by the dotted curve on the graph below.



Both methods give the same salvage value after two years. Find the value of R.

End of paper

2003 HIGHER SCHOOL CERTIFICATE SOLUTIONS

GENERAL MATHEMATICS

SECTION I

SUMMARY

1. B	7. B	13. C	18. D
2. D	8. D	14. B	19. B
3. C	9. A	15. B	20. A
4. B	10. A	16. D	21. A
5. C	11. C	17. C	22. C
6. D	12. A		

1. (B) $64 + 24 = 88$.

2. (D) $\dfrac{3y^3}{12y^2} = \dfrac{y}{4}$.

3. (C) Pay for Thursday and Friday
$= (8 \times \$9{\cdot}60) \times 2$
$= \$153{\cdot}60$.
Pay for Saturday $= 6 \times 1{\cdot}5 \times \$9{\cdot}60 = \$86{\cdot}40$.
Total pay $= \$153{\cdot}60 + \$86{\cdot}40 = \$240{\cdot}00$.

4. (B) $d = \sqrt{\dfrac{h}{5}}$
$= \sqrt{\dfrac{28}{5}}$
$= 2{\cdot}366\ 431\ 913 \ldots$
$\doteqdot 2{\cdot}4$ (1 decimal place).

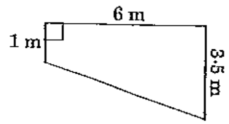
5. (C) *METHOD 1*
Value of car $= \$40\ 000 \times 0{\cdot}7 \times 0{\cdot}75 = \$21\ 000$.
METHOD 2
Value of the car at the end of 2001
$= \$40\ 000 - 30\% \times \$40\ 000$
$= \$40\ 000 - \$12\ 000$
$= \$28\ 000$.
Value of the car at the end of 2002
$= \$28\ 000 - 25\% \times \$28\ 000$
$= \$28\ 000 - \$7000$
$= \$21\ 000$.

6. (D) *Note*: After the first child is chosen, there are only 11 children left. If the first child is a boy, there are only 4 boys left.

7. (B) From the graph, Alex and Bryan met when they were 12 km from town (at time 20 minutes).
Alex had then travelled
12 km − 4 km = 8 km.

8. (D) The graphs show:
(A) high positive correlation
(B) low negative correlation
(C) high negative correlation
(D) low positive correlation

9. (A) Pool has the shape of a trapezoidal prism. That is, the uniform cross-section is a trapezium.
Area of trapezium
$= \dfrac{h}{2}(a+b)$
$= \dfrac{6}{2}(1 + 3{\cdot}5)$
$= 13{\cdot}5\ \text{m}^2$.



Note:
Alternatively, the area of the trapezium could be found by adding the area of the rectangle $\left(1 \times 6 = 6\ \text{m}^2\right)$ to the area of the triangle $\left(\frac{1}{2} \times 6 \times 2{\cdot}5 = 7{\cdot}5\ \text{m}^2\right)$.
$V = Ah$
$= 13{\cdot}5 \times 5$
$= 67{\cdot}5\ \text{m}^2$.

10. (A) *METHOD 1*
Time difference $= 30° \div 15 = 2$ hours.
Kathmandu is west of Perth, so Kathmandu is 2 hours *behind* Perth time.
When Perth is 12 noon, time in Kathmandu is 10:00 am.
METHOD 2
Time difference $= 30° \times 4$ min $= 120$ min $= 2$ hours.
Kathmandu is west of Perth, so Kathmandu is 2 hours *behind* Perth time.
When Perth is 12 noon, time in Kathmandu is 10:00 am.

11. (C)
$a = \frac{1}{2} \times 120 = 60$
$b = \frac{1}{2} \times 80 = 40$
$A = \pi ab$
$= \pi \times 60 \times 40$.
Area $= 7539{\cdot}822\ 369 \ldots$
Cost $= 7539{\cdot}822\ 369 \ldots \times \$7{\cdot}50$
$= \$56\ 548{\cdot}667 \ldots$
$\doteqdot \$56\ 549$.

12. (A) *METHOD 1*

x	f	fx
0	5	0
1	10	10
2	3	6
3	1	3
4	1	4
Totals	20	23

Mean $= \dfrac{\Sigma fx}{\Sigma f} = \dfrac{23}{20} = 1{\cdot}15$.

METHOD 2
Using the statistics mode on a calculator,
$\bar{x} = 1{\cdot}15$.

13. (C) *METHOD 1*
Number of students surveyed
$= 5 + 10 + 3 + 1 + 1$
$= 20$.
Number with at least 2 brothers
$= 3 + 1 + 1$
$= 5$.
P(student has at least 2 brothers) $= \dfrac{5}{20} = 0{\cdot}25$.
METHOD 2
P(student has at least 2 brothers)
$=$ P(2 brothers) or P(3 brothers) or P(4 brothers)
$= \dfrac{3}{20} + \dfrac{1}{20} + \dfrac{1}{20}$
$= \dfrac{5}{20}$
$= \dfrac{1}{4}$
$= 0{\cdot}25$.

14. (B) Use the cosine rule when given 2 sides and the angle included by them.
$x^2 = 10^2 + 7^2 - 2 \times 10 \times 7 \times \cos 60°$.

15. (B) *METHOD 1*
From the formula $W = 0{\cdot}75n + 50$, an employee earns \$0·75 per CD sold. If Kylie sells 2 more CDs than Danny, she will earn $2 \times \$0{\cdot}75 = \$1{\cdot}50$ more.
METHOD 2

Let Kylie sell 2 CDs	Let Danny sell 0 CDs
Kylie's weekly pay	Danny's weekly pay
$= 0{\cdot}75 \times 2 + 50$	$= 0{\cdot}75 \times 0 + 50$
$= \$51{\cdot}50$.	$= \$50$.

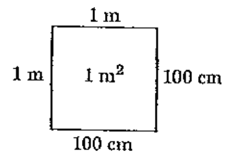
∴ Kylie's extra pay $= \$51{\cdot}50 - \$50 = \$1{\cdot}50$.

16. (D)
$A = 100\ 000$
$r = 4\% \div 12$ (compounded monthly)
$\therefore\ r = 0{\cdot}04 \div 12$
$n = 5 \times 12$
$= 60$ months.
Using the present value formula:
$N = \dfrac{A}{(1+r)^n}$
$= \dfrac{100\ 000}{(1 + 0{\cdot}04 \div 12)^{60}}$.

17. (C) Since the current varies *inversely* with resistance, if the resistance is *doubled* (multiplied by 2), then the current must be *halved* (divided by 2).

18. (D) Measurements are correct to the nearest centimetre, so the error $= \pm 0{\cdot}5$ cm.
True breadth is between 9·5 cm and 10·5 cm. True length is between 14·5 cm and 15·5 cm.
∴ Lowest possible area is $9{\cdot}5 \times 14{\cdot}5\ \text{cm}^2$.
∴ Upper possible area is $10{\cdot}5 \times 15{\cdot}5\ \text{cm}^2$.

19. (B) *Note*: $1\ \text{m}^2 = 100 \times 100\ \text{cm}^2$,
so $1\ \text{m}^2 = 10\ 000\ \text{cm}^2$.



METHOD 1
1 tile $= 175\ \text{cm}^2$
$= 175 \div 10\ 000\ \text{m}^2$
$= 0{\cdot}0175\ \text{m}^2$.
Area of 1·056 million tiles
$= 1{\cdot}056 \times 1\ 000\ 000 \times 0{\cdot}0175\ \text{m}^2$
$= 18\ 480\ \text{m}^2$.
METHOD 2
Area of 1·056 million tiles
$= 1{\cdot}056 \times 1\ 000\ 000 \times 175\ \text{cm}^2$
$= 184\ 800\ 000\ \text{cm}^2$
$= 184\ 800\ 000 \div 10\ 000\ \text{m}^2$
$= 18\ 480\ \text{m}^2$.

20. (A) Taxable items $= \$1{\cdot}29 + \$7{\cdot}23 + \$4{\cdot}13$
$= \$12{\cdot}65.$

This total includes 10% GST on the original price, so 110% of the original price $= \$12{\cdot}65.$

$\therefore$ 10% of original price $= \$12{\cdot}65 \div 11$
$= \$1{\cdot}15$

$\therefore$ GST $= \$1{\cdot}15.$

21. (A) Total number of cameras sold in 2001
$= 1070 + 200$
$= 1270.$

Percentage of cameras that were digital
$= \dfrac{200}{1270} \times 100\%$
$= 15{\cdot}748\,0315\ldots\,\%$
$\doteqdot 16\%$ (correct to nearest per cent).

22. (C) By counting the dots on the scatter graph, there were 5 students who liked milk chocolate but disliked dark chocolate.

Estimate $= \dfrac{5}{12} \times 600$
$= 250.$

SECTION II

Question 23

(a) (i) Water required $= 13 \times 1{\cdot}2$ L
$= 15{\cdot}6$ L.

(ii) Water required per week $= 15{\cdot}6 \text{ L} \times 7$
$= 109{\cdot}2$ L
$= 0{\cdot}1092$ kL.

Cost of water $= 0{\cdot}1092 \times 94{\cdot}22$ cents
$= 10{\cdot}288\,824\ldots$ cents
$\doteqdot 10$ cents.

(iii) One shrub requires 1·2 L = 1200 mL.
Number of drops $= 1200 \times 15$
$= 18\,000.$

(iv) *METHOD 1*
Drip rate $= 18\,000$ drops / 10 hours
$= 1800$ drops / h
$= \dfrac{1800}{60}$ drops / min
$= 30$ drops / min.

METHOD 2
Number of minutes in 10 hours $= 10 \times 60$
$= 600.$

Drip rate $= \dfrac{18\,000}{600}$ drops / min
$= 30$ drops / min.

(b) Surface area of a sphere $= 4\pi r^2.$
Radius $r = \frac{1}{2} \times 45$ cm $= 22{\cdot}5$ cm.
Internal surface area $= \frac{1}{2} \times 4\pi r^2$
$= \frac{1}{2} \times 4 \times \pi \times 22{\cdot}5^2$
$= 3180{\cdot}862\,562\ldots$ cm²
$\doteqdot 3181$ cm².

(c) Two applications of Simpson's rule
$A \doteqdot \frac{h}{3}\left[d_f + 4d_m + d_l\right].$

Left 'half': $A_1 \doteqdot \frac{3}{3}\left[4 + 4(5) + 5\right]$
$= 29$ m².

Right 'half': $A_2 \doteqdot \frac{3}{3}\left[5 + 4(4) + 0\right]$
$= 21$ m².

$\therefore$ Total area $\doteqdot 29 \text{ m}^2 + 21 \text{ m}^2$
$= 50$ m².

(d) (i) Volume of a cylinder $V = \pi r^2 h.$
$r = \frac{1}{2} \times 10 = 5$ cm.
$V = \pi \times 5^2 \times 7{\cdot}8$
$= 612{\cdot}610\,5675\ldots$
$\doteqdot 613$ cm³.

(ii) Engine capacity
$= 8 \times 612{\cdot}610\,5675\ldots$ cm³
$= 4900{\cdot}884\,54\ldots$ cm³
$= 4900{\cdot}884\,54\ldots$ mL $\left(1 \text{ mL} = 1 \text{ cm}^3\right)$
$= 4{\cdot}900\,884\,54\ldots$ L
< 5 L.
$\therefore$ Peta's engine meets the racing requirement of being under 5 litres.

Question 24

(a) $P = 24\,000,\ r = \dfrac{0{\cdot}0475}{12}$ (monthly),
$n = 3 \times 12 = 36$ months.
$A = P(1+r)^n$
$= 24\,000\left(1 + \dfrac{0{\cdot}0475}{12}\right)^{36}$
$= 27\,667{\cdot}890\,17\ldots$
$\doteqdot \$27\,667{\cdot}89.$

(b) (i) Taxable income $= \$58\,624 + 12 \times \$900$
$= \$69\,424.$

(ii) Tax payable
$= \$17\,730 + 0{\cdot}48 \times (\$69\,424 - \$66\,000)$
$= \$19\,373{\cdot}52.$

(iii) Annual tax on second job
$= \$19\,373{\cdot}52 - \$14\,410{\cdot}80$ (from question)
$= \$4962{\cdot}72.$

METHOD 1
Monthly tax $= \$4962{\cdot}72 \div 12$
$= \$413{\cdot}56.$
Monthly net income $= \$900 - \$413{\cdot}56$
$= \$486{\cdot}44.$

METHOD 2
Annual net income $= 12 \times \$900 - \$4962{\cdot}72$
$= \$5837{\cdot}28.$
Monthly net income $= \$5837{\cdot}28 \div 12$
$= \$486{\cdot}44.$

(iv) Using future value of an annuity formula
$$A = M\left\{\frac{(1+r)^n - 1}{r}\right\},$$
$M = 486{\cdot}44,\ r = \dfrac{0{\cdot}04}{12}$ (monthly),
$n = 24$ months.
$$A = 486{\cdot}44\left\{\frac{\left(1 + \frac{0{\cdot}04}{12}\right)^{24} - 1}{\frac{0{\cdot}04}{12}}\right\}$$
$= 12\,133{\cdot}2183\ldots$
$\doteqdot \$12\,133{\cdot}21$
$> \$12\,000.$
$\therefore$ Vicki will have enough in her account to pay for her holiday.

(c) Instalment $= \$336$, from the table.
Total instalments $= \$336 \times 26 \times 2$
$= \$17\,472.$
$\therefore$ Interest $= \$17\,472 - \$15\,500$
$= \$1972.$

METHOD 1
Interest rate p.a.
$= \dfrac{\$1972}{\$17\,472} \times 100\% \div 2$ years
$= 6{\cdot}361\ldots\,\%$
$\doteqdot 6{\cdot}36\%$ (correct to 2 decimal places).

METHOD 2
Using the simple interest formula $I = Prn$,
$P = 15\,500,\ r = ?,\ n = 2,\ I = 1972.$
$1972 = 15\,500 \times r \times 2$
$= 31\,000r$
$r = \dfrac{1972}{31\,000}$
$= 0{\cdot}063\,612\,903$
$= 6{\cdot}361\,290\,323\ldots$
$\doteqdot 6{\cdot}36\%$ (correct to 2 dec. pl.).

Question 25

(a) (i) (1) Mode = 2.

(2) The most common number of cars per household, or the number of cars per household that has the highest frequency.

(ii) The number of stickers/car
$= 0 \times 2735 + 1 \times 12\,305 + 2 \times 13\,918 + 3 \times 3980 + 4 \times 233$
$= 53\,013.$

(iii) Sector angle $= \dfrac{2735}{33\,171} \times 360°$
$= 29{\cdot}682\,554\,04\ldots°$
$\doteqdot 30°.$

(iv) Number of hours from 6 pm to 8:30 pm $= 2{\cdot}5$. The parking fee $= \$10$ (from the step graph).

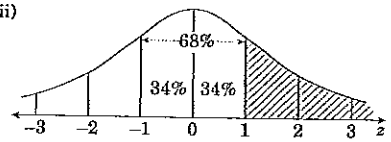
(b) (i) Possible answers:
- More secondary students travel by train/bus.
- More / most primary students travel by car/walking.
- Secondary students spread more evenly across the methods of travel.

(ii) Possible answers:
- Secondary students are more mature and can take public transport such as the bus or train.
- Secondary students often need to travel further to get to school and rely on public transport.

(iii) Primary students travelling by bus
$= 10\% \times 25\,000$
$= 2500.$

(c) (i) Hardev's score is one standard deviation above the mean.

(ii)



Shaded region is the required percentage.

METHOD 1
Percentage $= \dfrac{100\% - 68\%}{2} = 16\%.$

METHOD 2
Percentage $= 100\% - 34\% - 50\%$
$= 16\%.$
$\therefore$ 16% of people scored higher than Hardev.

Question 26

(a) (i) *METHOD 1*
$a = 60\,000$ (a is the vertical intercept of the graph of $N = a - bt$).

METHOD 2
When $t = 0$, $N = 60\,000$ (from graph).
$N = a - bt$
$60\,000 = a - b(0)$
$60\,000 = a$
$a = 60\,000.$

Explanation:
a was the number of people in the stadium immediately at the end of the game.

(ii) (1) *METHOD 1*

When $t = 30$, $N = 0$.

$$N = a - bt$$
$$0 = 60\,000 - b(30)$$
$$= 60\,000 - 30b$$
$$30b = 60\,000$$
$$b = \frac{60\,000}{30}$$
$$b = 2000.$$

METHOD 2

$-b$ is the gradient of the line.

$$-b = -\frac{60\,000}{30} \quad \left(\frac{\text{rise}}{\text{run}}\right)$$
$$-b = -2000$$
$$b = 2000.$$

(2) b is the rate at which people are leaving the stadium, in people / minute.

(iii) $N = a - bt$.
$$bt + N = a$$
$$bt = a - N$$
$$t = \frac{a - N}{b}.$$

(iv) *METHOD 1*

When 10 000 people have left, there are 60 000 − 10 000 = 50 000 in the stadium.

∴ $N = 50\,000$.

Using the formula from (iii):

$$t = \frac{60\,000 - 50\,000}{2000}$$
$$= \frac{10\,000}{2000}$$
$$= 5 \text{ minutes.}$$

METHOD 2

Using $N = a - bt$:

When $N = 50\,000$ (see Method 1),

$$50\,000 = 60\,000 - 2000t$$
$$2000t + 50\,000 = 60\,000$$
$$2000t = 10\,000$$
$$t = \frac{10\,000}{2000}$$
$$= 5 \text{ minutes.}$$

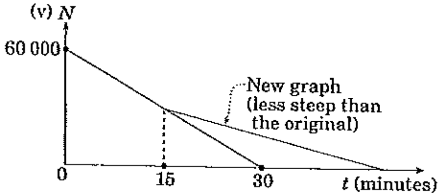
METHOD 3

2000 people / minute is the rate at which people are leaving the stadium, from (ii).

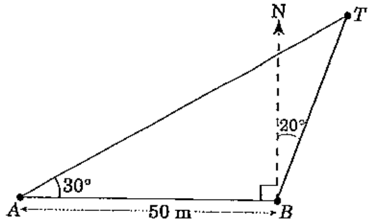
∴ The time it takes 10 000 people to leave

$$= \frac{10\,000}{2000} = 5 \text{ minutes.}$$

(v)



(b)



(i) N is north.

$\angle NBT = 20°$ (T is 020° from B)

$\angle NBA = 90°$ (B is due east of A)

∴ $\angle ABT = 20° + 90°$

$= 110°$.

(ii) $\angle T = 180° - 30° - 110°$ (∠ sum of ΔTBA)

$= 40°$.

By the sine rule,

$$\frac{BT}{\sin 30°} = \frac{50}{\sin 40°}$$
$$BT = \frac{50 \sin 30°}{\sin 40°}$$
$$= 38{\cdot}893\,095\,67\ldots$$
$$\doteqdot 39 \text{ m.}$$

Question 27

(a) (i) P(staying at hotel X) = 50%.

∴ P(staying at hotel Y or Z)

= 100% − 50%

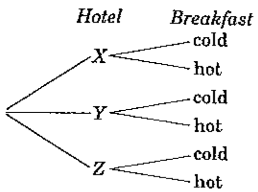
= 50%.

∴ P(staying at hotel Z)

$= \frac{1}{2} \times 50\%$ (equally likely)

$= 25\% \left(\text{or } \frac{1}{4}\right)$.

(ii) (1)



6 possible combinations:

Hotel X, cold
X, hot
Y, cold
Y, hot
Z, cold
Z, hot.

(2) These combinations are not all equally likely because the choice of hotels is not equally likely. (Karl is more likely to choose X over Y or Z.)

(3) P(Z, hot) $= 25\% \times \frac{1}{2} = 12{\cdot}5\% \left(\text{or } \frac{1}{8}\right)$.

(b)

1st die \ 2nd die	1	2	3	4	5	6
1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

There are 36 possible outcomes.

(i) Number of outcomes that don't show 6 = 25 (as shaded on the above table).

∴ P(neither shows 6) $= \frac{25}{36}$.

(ii) Number of outcomes that have both showing 6 = 1, (6, 6).

P(both show 6) $= \frac{1}{6}$.

Number of outcomes that show only one 6 = 10 (row 6 and column 6, but not (6, 6) in the table above).

∴ P(only one 6) $= \frac{10}{36}$.

P(neither shows 6) $= \frac{25}{36}$, from (i).

Financial expectation

$$= \$20\left(\frac{1}{36}\right) + \$2\left(\frac{10}{36}\right) + (-\$2)\left(\frac{25}{36}\right)$$
$$= -\$0{\cdot}277\,777\,777\ldots$$
$$\doteqdot -\$0{\cdot}28 \text{ (correct to the nearest cent).}$$

(c) (i)
$$\tan 65° = \frac{AB}{15}$$
$$AB = 15 \tan 65°$$
$$= 32{\cdot}167\,603\,81\ldots$$
$$\doteqdot 32{\cdot}2 \text{ m.}$$

(ii)
$$CA = 2 \times 32{\cdot}167\,603\,81\ldots$$
$$= 64{\cdot}335\,207\,62\ldots$$

By Pythagoras' theorem on ΔCEA:

$$CE^2 = CA^2 + AE^2$$
$$= (64{\cdot}335\,207\,62\ldots)^2 + 15^2$$
$$= 4364{\cdot}018\,939\ldots$$
$$CE = \sqrt{4364{\cdot}018\,939\ldots}$$
$$= 66{\cdot}060\,7216\ldots$$
$$\doteqdot 66{\cdot}1 \text{ m.}$$

Note: Do not round off partial answers such as 64·3352 ... and 4364·0189 ... until the end of the calculation.

Question 28

(a) Let C = cost of pendant, L = length of pendant,

$C \propto L^2$, ∴ $C = kL^2$.

When $L = 30$, $C = 130$: $130 = k(30^2) = 900k$

$$k = \frac{130}{900} = \frac{13}{90}$$

∴ $$C = \frac{13}{90}L^2.$$

When $L = 40$, $C = \frac{13}{90}(40^2)$

$= 231{\cdot}111\,1111\ldots$

$\doteqdot 231$ pesos.

(b) (i) In $y = b(a^x)$, a is the growth constant and b is the initial population.

∴ $a = 1 + 1{\cdot}57\%$

$= 1 + 0{\cdot}0157$

$= 1{\cdot}0157$

$b = 103\,400\,000$

∴ $y = 103\,400\,000(1{\cdot}0157)^x$.

(ii) When $x = 2$, $y = 103\,400\,000(1{\cdot}0157)^2$

$= 106\,672\,247{\cdot}1\ldots$

$\doteqdot 106\,672\,000$.

(c) (i) (Celsius temp.) × 1·8 + 32

= (Fahrenheit temp.).

$$A \times 1{\cdot}8 + 32 = 77$$
$$1{\cdot}8A + 32 = 77$$
$$1{\cdot}8A = 45$$
∴ $$A = \frac{45}{1{\cdot}8} = 25.$$

(ii) [(Celsius temp.) + 12] × 2

= Fahrenheit temp.).

$$(C + 12) \times 2 = F$$
$$F = 2(C + 12) \text{ or } 2C + 24.$$

(d) **Straight-line depreciation:**

METHOD 1

The camera depreciates \$800 in 4 years.

∴ It depreciates \$400 in 2 years (half the time).

∴ When $n = 2$, $S = \$800 - \$400 = \$400$.

METHOD 2

Using the formula $S = V_0 - Dn$, $V_0 = 800$.

When $n = 4$, $S = 0$:
$$0 = 800 - D(4)$$
$$= 800 - 4D$$
$$4D = 800$$
$$D = \frac{800}{4} = 200.$$

∴ $S = 800 - 200n$.

When $n = 2$, $S = 800 - 200(2) = 400$.

Declining-balance depreciation:

Using the formula $S = V_0(1 - r)^n$, $V_0 = 800$.

When $n = 2$,

$S = 400$ (same as straight-line depreciation)

$$400 = 800(1 - r)^2$$
$$\frac{400}{800} = (1 - r)^2$$
$$0{\cdot}5 = (1 - r)^2$$
$$\sqrt{0{\cdot}5} = 1 - r$$
$$r + \sqrt{0{\cdot}5} = 1$$
$$r = 1 - \sqrt{0{\cdot}5}$$
$$= 0{\cdot}292\,893\,218\ldots$$
$$R = 29{\cdot}289\,3218\ldots\%$$
$$\doteqdot 29{\cdot}3\%.$$

END OF GENERAL MATHEMATICS SOLUTIONS