

2002 HIGHER SCHOOL CERTIFICATE  
EXAMINATION PAPER  
GENERAL MATHEMATICS

**Section I**

22 marks

Attempt Questions 1 - 22

Allow about 30 minutes for this section

- 1 The results of a geography test are displayed in a stem-and-leaf plot.

2	3	3	4	5
3	5	6	6	6 7 7
4	1	2		
5	0	0	4	

What is the range of the data?

- (A) 15                      (B) 27                      (C) 29                      (D) 31
- 2 Which of the following is the correct simplification of  $8x^3 - 5x^3$ ?
- (A)  $3x^6$                       (B)  $3x^3$                       (C)  $3x$                       (D) 3
- 3 The Great Pyramid of Egypt has a square base of side 230 m. Its perpendicular height is 135 m. What is the volume of the pyramid?
- (A) 10 350 m<sup>3</sup>                      (B) 1 397 250 m<sup>3</sup>                      (C) 2 380 500 m<sup>3</sup>                      (D) 7 141 500 m<sup>3</sup>

**Boomerang Council**

**RATES AND CHARGES NOTICE**  
1st July 2001 to 30th June 2002

Customer Ref No 1111111  
Due Date 31/08/2001  
Issue Date 17/07/2001  
Assessment No 11111-00000-1

Z Smith  
14 The Crescent  
Boomerang

PROPERTY RATING CATEGORY	VALUER GENERAL'S LAND VALUE	VALUATION BASE DATE
Residential	\$377 000	01/07/2000

RATES & CHARGES	RATEABLE VALUE OR QTY	CENTS IN \$ OR CHARGE	AMOUNT
Residential rate	\$377 000	0.272950	[ ]
Waste Mgt Chg 80 litres	1	\$195.00	\$195.00
<b>TOTAL PAYABLE</b>			[ ]

Payments made after 29/06/2001 will not be shown on this notice.

What is the total payable?

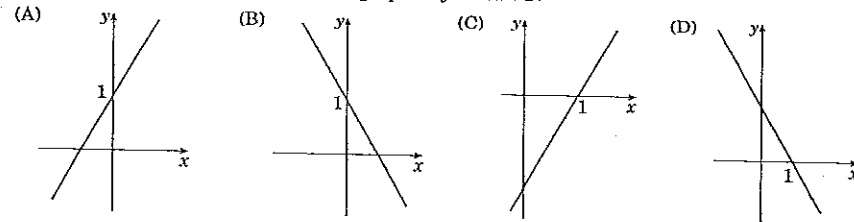
- (A) \$1029.02                      (B) \$1224.02                      (C) \$102 902.15                      (D) \$103 097.15

2002 HIGHER SCHOOL CERTIFICATE

- 5 Sarah has two packets of jelly beans. Each packet contains one black and five yellow jelly beans. Sarah takes one jelly bean from each packet without looking. What is the probability that both of the jelly beans are black?

- (A)  $\frac{1}{36}$                       (B)  $\frac{1}{12}$                       (C)  $\frac{1}{6}$                       (D)  $\frac{1}{3}$

- 6 Which one of the following could be the graph of  $y = 3x + 1$ ?



- 7 Richard has 2000 shares with a current market value of \$4.80 each. During the past twelve months, Richard received a total dividend of \$240.

What is the current dividend yield on these shares?

- (A) 0.025%                      (B) 2%                      (C) 2.5%                      (D) 40%

- 8 Results for an aptitude test are given as z-scores. In this test Di gained a z-score of 3. The test has a mean of 55 and a standard deviation of 6.

What was Di's actual mark in this test?

- (A) 57                      (B) 58                      (C) 64                      (D) 73

- 9 The table shows monthly repayments for loans over 30 years.

		Loan amount			
		\$100 000	\$150 000	\$200 000	\$250 000
Interest rate per annum	5.0%	\$537	\$806	\$1074	\$1343
	5.5%	\$568	\$852	\$1136	\$1420
	6.0%	\$600	\$900	\$1200	\$1499
	6.5%	\$633	\$949	\$1265	\$1581
	7.0%	\$666	\$998	\$1331	\$1664
	7.5%	\$700	\$1049	\$1399	\$1749

James borrows \$200 000 over a period of 30 years at 6.5% per annum. Repayments are to be made monthly according to the table.

How much would James repay over 30 years if the interest rate were to remain the same?

- (A) \$1265                      (B) \$37 950                      (C) \$390 000                      (D) \$455 400

- 10 The game of Beach Quidditch is played with a large hollow spherical ball made from gold vinyl. The diameter of the ball is 1.2 metres.

If the vinyl costs \$32 per square metre, which of the following is closest to the cost of the vinyl for one ball?

- (A) \$29                      (B) \$145                      (C) \$232                      (D) \$579

- 11 At the end of 2000, Zara purchased a new computer for \$4999. Use the declining balance method to determine the value of the computer at the end of 2002, assuming a depreciation rate of 40% per annum. (Answer to the nearest dollar.)

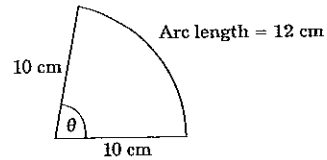
- (A) \$800                      (B) \$1000                      (C) \$1800                      (D) \$2000

- 12 In the town of Burrow the ages of the residents are normally distributed. The mean age is 40 years and the standard deviation is 12 years.

Approximately what percentage of the residents are younger than 52?

- (A) 16% (B) 32% (C) 68% (D) 84%

- 13 This is a sketch of a sector of a circle.



NOT TO SCALE

Find the value of  $\theta$  to the nearest degree.

- (A)  $47^\circ$  (B)  $48^\circ$  (C)  $68^\circ$  (D)  $69^\circ$

- 14 Arrange the numbers  $5.6 \times 10^{-2}$ ,  $4.8 \times 10^{-1}$ ,  $7.2 \times 10^{-2}$  from smallest to largest.

- (A)  $5.6 \times 10^{-2}$ ,  $7.2 \times 10^{-2}$ ,  $4.8 \times 10^{-1}$  (B)  $4.8 \times 10^{-1}$ ,  $5.6 \times 10^{-2}$ ,  $7.2 \times 10^{-2}$   
 (C)  $7.2 \times 10^{-2}$ ,  $5.6 \times 10^{-2}$ ,  $4.8 \times 10^{-1}$  (D)  $4.8 \times 10^{-1}$ ,  $7.2 \times 10^{-2}$ ,  $5.6 \times 10^{-2}$

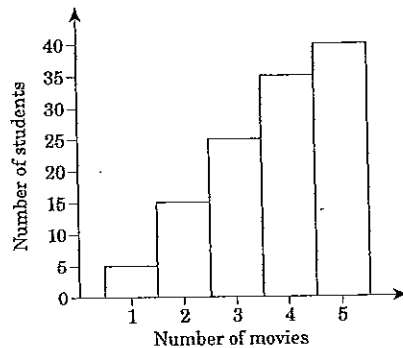
- 15 Calculate the present value of an annuity in which \$1200 is invested at the end of every year for ten years and interest is paid annually at a rate of 5% per annum. (Answer to the nearest dollar.)

- (A) \$1922 (B) \$9266 (C) \$15 093 (D) \$30 654

- 16 If  $w = 2y^3 - 1$ , what is the value of  $y$  when  $w = 13$ ?

- (A)  $\frac{\sqrt[3]{14}}{2}$  (B)  $\sqrt[3]{6}$  (C)  $\sqrt[3]{7}$  (D)  $\sqrt[3]{14}$

- 17 Students were surveyed about the number of movies they had watched in the last week. The results are shown in this cumulative frequency histogram.



How many students said they watched four movies last week?

- (A) 5 (B) 10 (C) 25 (D) 35

- 18 Amy buys a \$1 ticket in a raffle. There are 200 tickets in the raffle and two prizes. First prize is \$100 and second prize is \$50.

Find Amy's financial expectation.

- (A)  $-\$1.00$  (B)  $-\$0.75$  (C)  $-\$0.25$  (D)  $+\$0.25$

- 19 In one year, the population of a city increased by 20%. The next year, it decreased by 10%. What was the percentage increase in the population over the two years?

- (A) 8% (B) 10% (C) 15% (D) 30%

- 20 Rob, Alex and Tan plan a swimming race against each other. Rob and Alex are each twice as likely as Tan to win the race.

What is the probability that Tan will win the race?

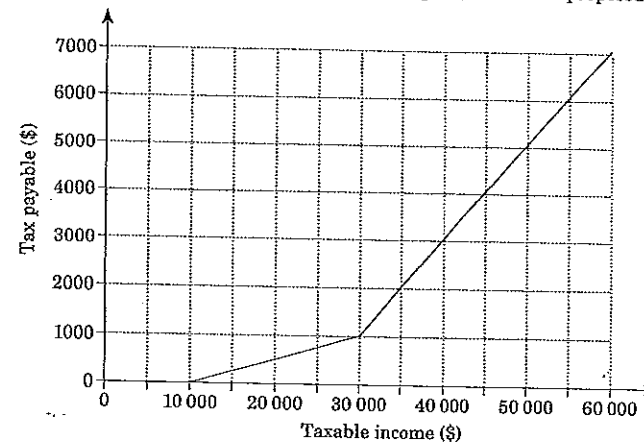
- (A)  $\frac{1}{6}$  (B)  $\frac{1}{5}$  (C)  $\frac{1}{4}$  (D)  $\frac{1}{3}$

- 21 The sheets of paper Jenny uses in her photocopier are 21 cm by 30 cm. The paper is 80 gsm, which means that one square metre of this paper has a mass of 80 grams. Jenny has a pile of this paper weighing 25.2 kg.

How many sheets of paper are in the pile?

- (A) 500 (B) 2000 (C) 2500 (D) 5000

- 22 The graph shows the tax payable for taxable incomes up to \$60 000 in a proposed tax system.



How much of each dollar earned over \$30 000 is payable in tax?

- (A) 10 cents (B) 12 cents (C) 20 cents (D) 23 cents

**Section II**

78 marks  
 Attempt Questions 23 – 28  
 Allow about 2 hours for this section

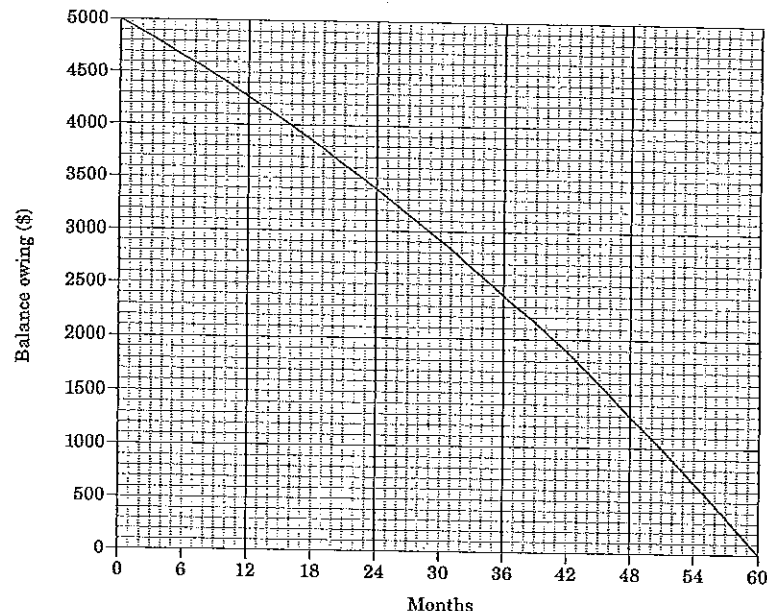
**Question 23 (13 marks)**

**Marks**

- (a) Jordan's gross pay is \$1500 per fortnight.
- (i) Fortnightly deductions from Jordan's gross pay are: 1
- \$269.17 for tax;
  - \$7.88 for union fees;
  - \$16.25 for private health insurance.
- Calculate his fortnightly net pay.
- (ii) Jordan is paid an annual leave loading of  $17\frac{1}{2}\%$  of 4 weeks' gross pay. 1  
 Calculate his annual leave loading.
- (iii) Jordan visits Italy on his holidays. He pays €180 (180 euros) for a pair of boots. 1  
 This price includes a value added tax of 20%. 1
- (1) What was the price of the boots before the tax was added? 1
- (2) How much is €180 in Australian dollars if \$A1 is worth €0.58? 1
- (b) (i) Katherine invests \$50 000 with Standard Credit Union for a term of 5 years. 2  
 Her investment earns interest at 3.1% per annum compounded annually.  
 How much will Katherine's investment be worth at the end of 5 years?  
 Give your answer to the nearest dollar.
- (ii) Katherine's sister Liz also has \$50 000 to invest for a term of 5 years. She invests with 2  
 General Bank. Her investment earns interest at 3% per annum, compounded monthly.  
 Which sister makes the better investment?  
 Justify your answer with appropriate calculations.
- (c) Minh wants to buy a hi-fi system from Advanced Sound Systems for \$5000.
- (i) Minh considers buying the system on hire purchase. 3  
 What would Minh repay monthly if he were to buy the hi-fi system on the following  
 hire purchase terms?

**ADVANCED SOUND SYSTEMS**  
 Hire Purchase Terms:  
**10% deposit**  
 15% pa simple interest on the balance  
 Equal monthly repayments over 3 years

- (ii) Minh decides instead to borrow the required \$5000 from his local bank. The loan is 1  
 a reducing balance loan over 5 years, with monthly repayments.  
 The graph (on the next page) shows the balance owing on the loan over time. 1
- (1) Use the graph to determine the balance owing after 2 years. 1
- (2) Use the graph to determine when the loan is half-paid. 1



**Question 24 (13 marks)**

**Marks**

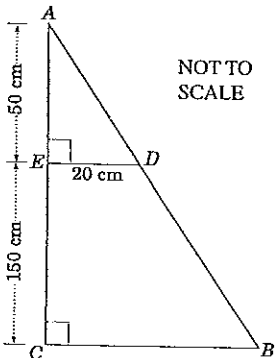
- Jane and Sam are in a Geography class of 12 students. The class is going on a three-day excursion by bus.
- (a) The students are asked to each pack one bag for the trip. The bags are weighed, and the weights (in kg) are listed in order as follows: 1
- 8, 9, 10, 10, 15, 18, 22, 25, 29, 35, 38, 41
- (i) A bag is selected at random. What is the probability that the chosen bag weighs 1  
 more than 30 kg?
- (ii) While Sam waits for the bus to be ready, he works out the five number summary 2  
 for the weights of the bags:
- 8, 10, 20, 32, 41
- Using this five number summary, construct an accurate box-and-whisker plot 1  
 to display the distribution of the weights of the bags.
- (iii) Calculate the interquartile range of the weights. 1
- (b) While waiting in the carpark, Jane notices that some of the cars entering the carpark have headlights on. For each car, Jane notes whether or not the lights are on, and whether the driver is male or female. Her results are presented in the two-way table below. There are two missing numbers at A and B.

	Headlights on	Headlights off	Total
Male drivers	10	A	53
Female drivers	8	62	70
Total	B	105	

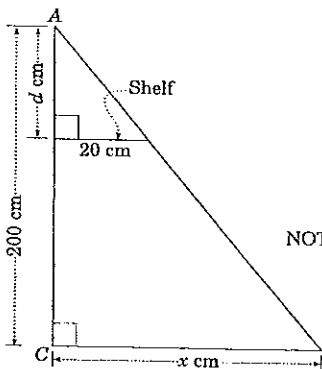
- |  |            |
|--|------------|
| (i) Determine the values of $A$ and $B$ .                          | Marks<br>2 |
| (ii) How many cars are included in this data set?                  | 1          |
| (iii) What fraction of the cars had female drivers?                | 1          |
| (iv) Of the cars driven by women, what fraction had headlights on? | 1          |
- (c) There is one seat at the back of the excursion bus that is very popular among the students. Before the excursion, a draw is conducted to determine who will sit in the popular seat. The names of the 12 students are placed in a hat and 3 names are drawn without replacement. The first name drawn determines who will sit in the seat on the first day. The second name drawn determines who will sit in the seat on the second day. The third name drawn determines who will sit in the seat on the third day.
- |   |   |
|---|---|
| (i) What is the probability that Jane's name is the first drawn?                                      | 1 |
| (ii) What is the probability that Jane's name is the second drawn?                                    | 1 |
| (iii) What is the probability that Jane's name will NOT be one of the three names drawn from the hat? | 2 |

**Question 25 (13 marks)**

- (a) A shelf 20 cm wide is attached to a wall, under a light.
- (i) The diagram shows the end view,  $ED$ , of the shelf attached to a wall  $AC$ .

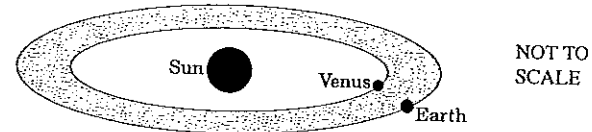


- When the wall light at  $A$  is turned on, the shelf casts a shadow  $CB$  on the floor.
- |   |   |
|---|---|
| (1) Name a pair of similar figures in the diagram.                      | 1 |
| (2) Calculate the enlargement factor between these two similar figures. | 1 |
| (3) What is the length of the shadow $CB$ ?                             | 1 |



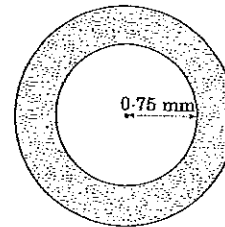
- (ii) The shelf is moved to a new position  $d$  cm below the light. The length of the shadow is now  $x$  cm. Write down an equation relating  $d$  and  $x$ .

- |   |       |
|---|-------|
| (b) The table shows the approximate coordinates for two cities. | Marks |
|---|-------|
- | City         | Latitude | Longitude |
|--------------|----------|-----------|
| Buenos Aires | 35°S     | 60°W      |
| Adelaide     | 35°S     | 140°E     |
- |  |   |
|--|---|
| (i) What is the time difference between Adelaide and Buenos Aires? (Ignore time zones.)  | 1 |
| (ii) Roy lives in Adelaide and his cousin Juan lives in Buenos Aires. Roy wants to telephone Juan at 7 pm on a Friday night, Buenos Aires time. At what time, and on what day, should Roy make the call? | 2 |
- (c) (i) The orbits of Earth and Venus around the Sun are almost circular, and in the same plane.



Earth is  $1.496 \times 10^8$  km from the Sun. Venus is  $1.082 \times 10^8$  km from the Sun. Treating the space between the orbits as an annulus, calculate its area. Write your answer in scientific notation correct to two significant figures.

- |  |   |
|--|---|
| (ii) Rearrange the formula for the area of an annulus, $A = \pi(R^2 - r^2)$ , to make $R$ the subject. | 2 |
| (iii) A small metal washer is to be made in the shape of an annulus with inner radius 0.75 mm.         | 2 |



The area of the face of the washer (shaded on the diagram) is to be  $6.79 \text{ mm}^2$ . Calculate the outer radius correct to two decimal places.

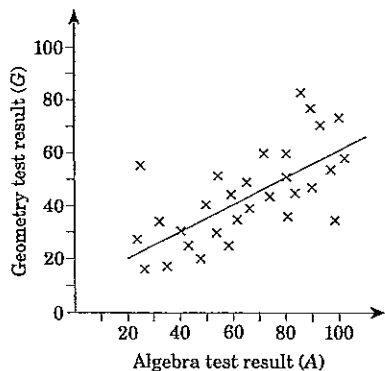
**Question 26 (13 marks)**

- |  |   |
|--|---|
| (a) After three small quizzes, Vicki has an average mark of 5. She wants to increase her average to 6. What mark must she score in the next quiz for her average mark to be exactly 6?   | 2 |
| (b) Roxy selected 30 students at random from Year 12 at her high school, and asked each of them how many text messages they had sent from a mobile phone within the last day. The results are summarised in the following table. |   |

Number of text messages sent	Frequency
0	3
1	3
2	4
3	4
4	9
5	7

Marks

- (i) Calculate the mean number of text messages sent. (Give your answer correct to two decimal places.) 1
  - (ii) Calculate the sample standard deviation. (Give your answer correct to two decimal places.) 1
  - (iii) Determine the median number of text messages sent. 1
  - (iv) Describe the skewness of the data. 1
  - (v) There are 150 students in Year 12 at Roxy's school. Use the sample data in the table to estimate how many of these Year 12 students would have sent more than three text messages within the last day. 1
- (c) A class of 30 students sat for an algebra test and a geometry test. The results were displayed in a scatterplot, and a line of fit was drawn, as shown.

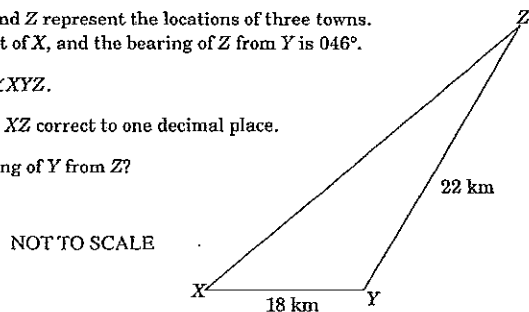


- (i) How many students scored less than 30 on the algebra test? 1
- (ii) Calculate the gradient of the line of fit drawn. 1
- (iii) What is the equation of the line of fit drawn? 2
- (iv) Describe the correlation between geometry test results and algebra test results. 1
- (v) Mitchell looked at the scatterplot and said: 'In this class, all students who are near the top in algebra are also near the top in geometry'. Explain why his statement is incorrect. 1

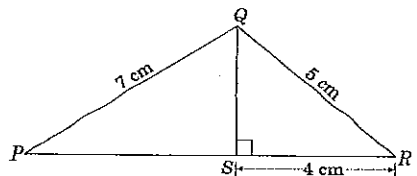
**Question 27 (13 marks)**

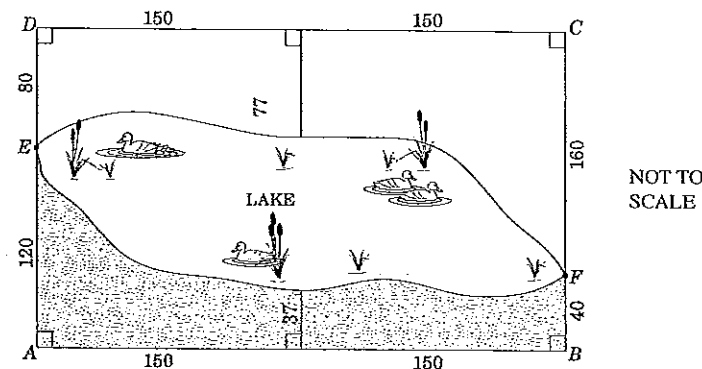
- (a) In the diagram  $X$ ,  $Y$  and  $Z$  represent the locations of three towns. The town  $Y$  is due east of  $X$ , and the bearing of  $Z$  from  $Y$  is  $046^\circ$ .

- (i) Find the size of  $\angle XYZ$ . 1
- (ii) Find the distance  $XZ$  correct to one decimal place. 2
- (iii) What is the bearing of  $Y$  from  $Z$ ? 1



Marks

- (b)  NOT TO SCALE
- (i) Find the perimeter of  $\triangle PQR$ . (Give your answer to one decimal place.) 3
  - (ii) Find the size of  $\angle QPS$  to the nearest degree. 1
- (c) In order to find the area of a lake, Bob took some measurements (in metres) and drew the following diagram.

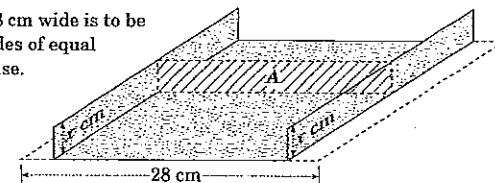


- (i) Use Simpson's rule to find the shaded area  $ABFE$ . 2
- (ii) Calculate the area of the lake. 3

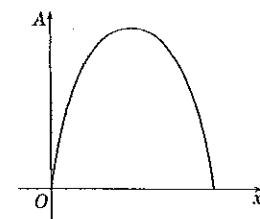
**Question 28 (13 marks)**

- (a) A long rectangular sheet of metal 28 cm wide is to be made into a gutter by turning up sides of equal height  $x$  cm, perpendicular to the base.

- (i) Show that a formula for the cross-sectional area,  $A$ , of the gutter is  $A = 28x - 2x^2$ . 1
- (ii) Explain why the formula in part (i) is only valid for values of  $x$  between 0 and 14. 1



- (iii) The graph of  $A$  against  $x$ , for values of  $x$  between 0 and 14, is a parabola, as shown. What is the maximum value of  $A$ ? 2



- (b) In 2001, when Toby was in Year 9, he started earning money by juggling at children's parties. He charged \$50 per party.
- (i) Write a formula for the amount,  $Q$  (in dollars), that Toby had earned in 2001 after he had juggled at  $n$  parties. 1
- (ii) By the end of 2001 Toby had juggled at 30 parties. 3  
 Draw the straight line graph of  $Q$  against  $n$ , with  $n$  on the horizontal axis and  $Q$  on the vertical axis. Use your ruler to draw the axes, and mark a scale on each axis.
- (iii) Before Toby started juggling at parties he spent \$300 on juggling equipment. 1  
 On your graph in part (b) (ii) draw a horizontal line through the point on the vertical axis where  $Q = 300$ . Give an interpretation of the point at which this horizontal line crosses the straight line graph of  $Q$  against  $n$ .
- (iv) Toby has a long-term plan. When he has finished Year 12, he wants to go to university for 3 years and then take a back-packing trip around the world. 4  
 By the end of 2001 Toby had saved \$900 from his earnings, and sketched out the following plan for saving money:

- In Year 9 (2001), I saved \$900 (interest included).
- For each year while still at school, save 30% more than I saved the previous year (interest included).
- At the end of Year 12 (2004), deposit total savings in an account paying 4% per annum interest, compounded annually.
- Add \$2500 to the account at the end of each of the three years while at university.

Toby's goal is to have \$15 000 in his account at the end of 2007. Will he achieve this goal if he follows the above plan? Show all your calculations to justify your answer.

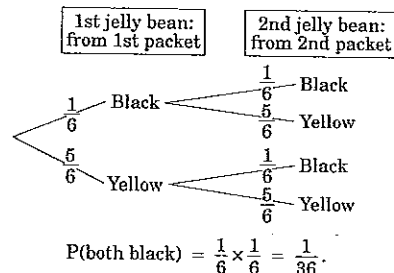
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## 2002 HIGHER SCHOOL CERTIFICATE SOLUTIONS GENERAL MATHEMATICS

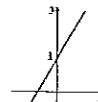
### SECTION I SUMMARY

- |      |       |       |       |
|------|-------|-------|-------|
| 1. D | 7. C  | 13. D | 18. C |
| 2. B | 8. D  | 14. A | 19. A |
| 3. C | 9. D  | 15. B | 20. B |
| 4. B | 10. B | 16. C | 21. D |
| 5. A | 11. C | 17. B | 22. C |
| 6. A | 12. D |       |       |

1. (D) Range =  $54 - 23 = 31$ .
2. (B)  $8x^3 - 5x^3 = 3x^3$ .
3. (C)  $A = 230 \times 230$  (square base)  
 $= 52\,900 \text{ m}^2$ .  
 $V = \frac{1}{3}Ah$  (volume of a pyramid)  
 $= \frac{1}{3} \times 52\,900 \times 135$   
 $= 2\,380\,500 \text{ m}^3$ .
4. (B) Residential amount  
 $= 377\,000 \times 0.272\,950c$   
 $= 102\,902.15c$   
 $= \$1029.0215$   
 $\div \$1029.02$ .  
 Total payable =  $\$1029.02 + \$195.00$   
 $= \$1224.02$ .
5. (A) In each packet, 1 black + 5 yellow  
 $= 6$  jelly beans.



6. (A)  $y = 3x + 1$  has gradient 3,  $y$  intercept 1. The gradient is positive and steep, and the line cuts the  $y$  axis at 1.



7. (C) Market value of all shares =  $2000 \times \$4.80$   
 $= \$9600$ .

$$\text{Dividend yield} = \frac{240}{9600} \times 100\%$$

$$= 2.5\%$$

8. (D)  $z$ -score of 3 means 3 standard deviations above the mean.

$$\therefore \text{Di's mark} = 55 + 6 \times 3 = 73.$$

OR  $z = \frac{x - \bar{x}}{s}$

$$z = 3, \bar{x} = 55, s = 6,$$

$$\therefore 3 = \frac{x - 55}{6}$$

$$18 = x - 55$$

$$x = 73.$$

9. (D) From the table, the monthly repayment for a loan of \$200 000 for 30 years at 6.5% pa interest is \$1265.

$$\text{Total repayments} = \$1265 \times 12 \times 30$$

$$= \$455\,400.$$

10. (B) Radius of ball  $r = \frac{1}{2} \times 1.2 = 0.6 \text{ m}$ .

$$\text{Surface area of ball} = 4\pi r^2$$

$$= 4 \times \pi \times (0.6)^2$$

$$= 4.5238 \dots \text{ m}^2.$$

$$\text{Cost of vinyl} = 4.5238 \dots \times \$32$$

$$= \$144.764 \dots$$

$$\div \$145.$$

11. (C)  $V_0 = \$4999, r = 40\% = 0.4, n = 2$ .

$$S = V_0(1 - r)^n \text{ (Declining balance formula)}$$

$$= 4999(1 - 0.4)^2$$

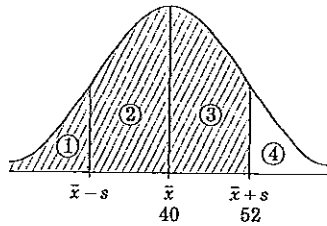
$$= 4999(0.6)^2$$

$$= 1799.64$$

$$\div \$1800.$$

12. (D) 52 is one standard deviation above the mean.

It is necessary to find the percentage represented by the shaded area in the following diagram.



**METHOD 1**

Areas ① + ② = 50% (half of scores are below the mean).

Areas ② + ③ = 68% (within 1 standard deviation of the mean).

$$\therefore \text{Area ③} = \frac{1}{2} \times 68\% = 34\%$$

$\therefore$  Shaded area = 50% + 34% = 84%.

**METHOD 2**

Areas ② + ③ = 68% (within 1 standard deviation of the mean).

Areas ① + ④ = 100% - 68% = 32%.

$$\therefore \text{Area ④} = \frac{1}{2} \times 32\% = 16\%$$

$\therefore$  Shaded area = 100% - Area ④ = 100% - 16% = 84%.

13. (D) Using the arc length of a circle formula with  $l = 12$ ,  $r = 10$ :

$$l = \frac{\theta}{360} \times 2\pi r$$

$$12 = \frac{\theta}{360} \times 2 \times \pi \times 10$$

$$12 \times 360 = \theta \times 20\pi$$

$$4320 = \theta \times 20\pi$$

$$\frac{4320}{20\pi} = \theta$$

$$\theta = 68.754 \dots^\circ \div 69^\circ$$

14. (A) The smallest numbers have the lowest powers of 10:

$$5.6 \times 10^{-2}, 7.2 \times 10^{-2}, 4.8 \times 10^{-1}$$

15. (B) Using the present value formula with  $M = \$1200$ ,  $r = 0.05$ ,  $n = 10$ :

$$N = M \left[ \frac{(1+r)^n - 1}{r(1+r)^n} \right]$$

$$= 1200 \left[ \frac{(1+0.05)^{10} - 1}{0.05(1+0.05)^{10}} \right]$$

$$= 1200 \left[ \frac{(1.05)^{10} - 1}{0.05(1.05)^{10}} \right]$$

$$= \$9266.0819 \dots \div \$9266.$$

16. (C)  $w = 2y^3 - 1$   
 $13 = 2y^3 - 1$   
 $14 = 2y^3$   
 $\frac{14}{2} = y^3$   
 $7 = y^3$   
 $y^3 = 7$   
 $\therefore y = \sqrt[3]{7}$

17. (B) From the graph, cumulative frequency for 3 movies = 25, cumulative frequency for 4 movies = 35.  
 $\therefore$  Frequency for 4 movies = 35 - 25 = 10.

18. (C) **METHOD 1**

$$P(\text{win 1st prize}) = \frac{1}{200}$$

$$\text{return} = \$100 - \$1 = \$99$$

$$P(\text{win 2nd prize}) = \frac{1}{200}$$

$$\text{return} = \$50 - \$1 = \$49$$

$$P(\text{not winning prize}) = \frac{198}{200}$$

$$\text{return} = \$0 - \$1 = -\$1$$

Financial expectation =  $\frac{1}{200}(\$99) + \frac{1}{200}(\$49) + \frac{198}{200}(-\$1)$   
 $= -\$0.25$

**METHOD 2**

Ticket sales =  $200 \times \$1 = \$200$ .

Total prizes =  $100 + 50 = \$150$ .

Raffle makes a profit of \$50.

Financial expectation for all tickets is -\$50.

Financial expectation for each ticket =  $\frac{-50}{200} = -\$0.25$ .

19. (A) **METHOD 1**

Let the original population be  $P$ .

After the first year, the population is  $P + 20\%P = P + 0.2P = 1.2P$ .

After the second year, the population

$$= 1.2P - 10\%(1.2P)$$

$$= 1.2P - 0.1(1.2P)$$

$$= 1.2P - 0.12P$$

$$= 1.08P$$

$\therefore P$  increasing to  $1.08P$  is a 0.08 increase or an 8% increase.

**METHOD 2**

Let the original population be, say, 10 000.

After the first year, the population is  $10\ 000 + 20\% \times 10\ 000 = 10\ 000 + 2000 = 12\ 000$ .

After the second year, the population is  $12\ 000 - 10\% \times 12\ 000 = 12\ 000 - 1200 = 10\ 800$ .

$$\therefore \text{Total increase} = \$10\ 800 - 10\ 000 = 800$$

$$\text{Hence \% increase} = \frac{800}{10\ 000} \times 100\% = 8\%$$

**METHOD 3**

Let the original population be  $P$ .

After the first year, the population is

$$P(1 + 0.2) = 1.2P$$

After the second year, the population is

$$1.2P(1 - 0.1) = 1.2P(0.9) = 1.08P$$

$\Rightarrow$  An increase of 0.08 or 8%.

20. (B) Rob (R) and Alex (A) each have twice the chance of Tan (T) to win the race.

Sample space is R, R, A, A, T.

$$\therefore P(\text{Tan winning}) = \frac{1}{5}$$

21. (D) Area of one sheet =  $0.21 \times 0.3 = 0.063 \text{ m}^2$ .

$$\therefore \text{Mass of one sheet} = 0.063 \times 80 \text{ g} = 5.04 \text{ g}$$

$$\text{Mass of pile of paper} = 25.2 \text{ kg} = 25.2 \times 1000 \text{ g} = 25\ 200 \text{ g}$$

$$\therefore \text{No. of sheets} = \frac{25\ 200 \text{ g}}{5.04 \text{ g}} = 5000$$

22. (C) Tax per dollar earned over \$30 000 is equal to the gradient of the line between (30 000, 1000) and (60 000, 7000).

$$\text{Gradient} = \frac{7000 - 1000}{60\ 000 - 30\ 000} = \frac{6000}{30\ 000} = \frac{1}{5}$$

$$= \$\frac{1}{5}$$

$$= 20 \text{ cents per dollar.}$$

**SECTION II**

**Question 23**

(a) (i) Fortnightly net pay =  $\$1500 - \$269.17 - \$7.88 - \$16 = \$1206.70$ .

(ii) 4 weeks' gross pay =  $2 \times \$1500 = \$3000$ .

Annual leave loading =  $17\frac{1}{2}\% \times \$3000 = \$525$ .

(iii) (1) **METHOD 1**

$$120\% \text{ of original price} = \text{€}180$$

$$1\% \text{ of original price} = \text{€}180 \div 120 = \text{€}1.5$$

$$\therefore \text{Original price (100\%)} = \text{€}1.5 \times 100 = \text{€}150$$

**METHOD 2**

$$1.2 \times \text{original price} = \text{€}180$$

$$\text{Original price} = \text{€}180 \div 1.2 = \text{€}150$$

(2) **METHOD 1**

$$\text{Let } \$Ax = \text{€}180$$

By comparing ratios of \$A amounts and € amounts:

$$\frac{x}{1} = \frac{180}{0.58}$$

$$x = 310.3448 \dots \div 310.34$$

$$\therefore \text{€}180 \div \$A310.34$$

**METHOD 2**

$$\$A1 = \text{€}0.58$$

Divide both sides by 0.58:

$$\$A1.7241 \dots = \text{€}1$$

Multiply both sides by 180:

$$\$A1.7241 \dots \times 180 = \text{€}180$$

$$\$A310.3448 \dots = \text{€}180$$

$$\therefore \text{€}180 \div \$A310.34$$

- (b) (i) **Katherine**

Using the compound interest formula with

$$P = \$50\ 000, n = 5, r = 3.1\% = \frac{3.1}{100} = 0.031$$

$$A = P(1+r)^n = \$50\ 000(1+0.031)^5 = \$50\ 000(1.031)^5 = \$58\ 245.627\ 81 \dots \div \$58\ 246 \text{ (to the nearest dollar)}$$

- (ii) **Liz**

Using the compound interest formula with

$$P = \$50\ 000,$$

$$r = \frac{0.03}{12} = 0.0025 \text{ (per month)},$$

$$n = 5 \times 12 = 60 \text{ (months)}$$

$$\begin{aligned}
 A &= P(1+r)^n \\
 &= \$50\,000(1+0.0025)^{60} \\
 &= \$50\,000(1.0025)^{60} \\
 &= \$58\,080.839 \dots \\
 &\div \$50\,081.
 \end{aligned}$$

∴ Katherine makes the better investment.

(c) (i) 10% deposit =  $10\% \times \$5000 = \$500$ .  
 Remaining balance =  $\$5000 - \$500 = \$4500$ .

Interest on balance =  $Prt$  (Simple interest formula)  
 =  $\$4500 \times 0.15 \times 3 = 2025$ .

Total owing =  $\$4500 + \$2025 = \$6525$ .

Monthly repayment =  $\$6525 \div 36 = \$181.25$ .

(ii) (1) 2 years = 24 months.  
 On the graph, each unit on the vertical axis represents \$100. The balance owing after 24 months = \$3400.

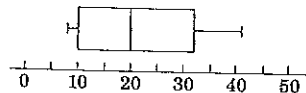
(2) Half-paid loan =  $\frac{1}{2} \times \$5000 = \$2500$ .

On the graph, each unit on the horizontal axis represents 1 month. The loan is half-paid at 35 months (or 2 years 11 months).

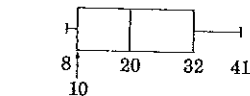
**Question 24**

(a) (i) Bags more than 30 kg = 3.  
 ∴  $P(\text{more than } 30) = \frac{3}{12} = \frac{1}{4}$ .

(ii) **METHOD 1**



**METHOD 2**



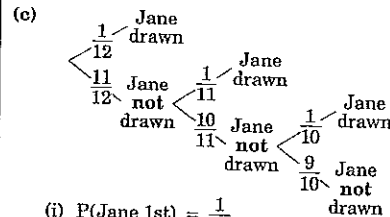
(iii)  $IQR = Q_3 - Q_1 = 32 - 10 = 22$ .

(b) (i)  $A = 53 - 10 = 43$  (or  $105 - 62 = 43$ )  
 $B = 10 + 8 = 18$  (or  $53 + 70 - 105 = 18$ ).

(ii)  $53 + 70 = 123$  cars (or  $18 + 105 = 123$ ).

(iii)  $\frac{\text{Number of female drivers}}{\text{Number of drivers}} = \frac{70}{123}$

(iv)  $\frac{\text{Number of female drivers, headlights on}}{\text{Number of female drivers}} = \frac{8}{70} = \frac{4}{35}$



(i)  $P(\text{Jane 1st}) = \frac{1}{12}$ .

(ii) **METHOD 1**  
 $P(\text{Jane 2nd}) = P(\text{not drawn first, then drawn 2nd}) = \frac{11}{12} \times \frac{1}{11} = \frac{1}{12}$

**METHOD 2**

Only one particular student can be drawn second.

∴  $P(\text{Jane 2nd}) = \frac{1}{12}$ .

(iii) **METHOD 1**

$P(\text{Jane not drawn}) = \frac{11}{12} \times \frac{10}{11} \times \frac{9}{10} = \frac{9}{12} = \frac{3}{4}$

**METHOD 2**

3 people are chosen,  
 ∴ 9 are not chosen.  
 ∴  $P(\text{Jane not chosen}) = \frac{9}{12} = \frac{3}{4}$ .

**Question 25**

(a) (i) (1)  $\triangle AED$  and  $\triangle ACB$ .

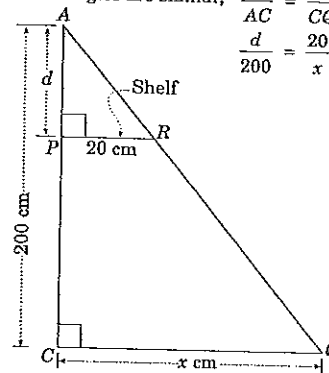
(2)  $AE = 50$  cm  
 $AC = 50$  cm +  $150$  cm =  $200$  cm.  
 ∴ Enlargement factor =  $\frac{AC}{AE} = \frac{200}{50} = 4$ .

(3) **METHOD 1**  
 $CB = 4$  times larger than  $ED = 4 \times 20 = 80$  cm.

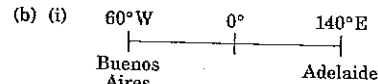
**METHOD 2**  
 By ratios of matching sides:  $\frac{CB}{ED} = \frac{AC}{AE} = \frac{200}{50} = 4$ .

∴  $CB = 4 \times 20 = 80$  cm.

(ii) As triangles are similar,  $\frac{AP}{AC} = \frac{PR}{CQ}$   
 $\frac{d}{200} = \frac{20}{x}$



(Or equivalent equations such as  $d = \frac{4000}{x}$  or  $dx = 4000$ .)



Difference in longitude =  $60^\circ + 140^\circ = 200^\circ$ .

**METHOD 1**  
 $1^\circ$  longitude = 4 minutes.  
 ∴ Difference in time =  $200 \times 4 = 800$  minutes = 13 h 20 min.

**METHOD 2**  
 $15^\circ$  longitude = 1 hour.  
 ∴ Difference in time =  $\frac{200^\circ}{15^\circ} = 13\frac{1}{3}$  hours = 13 h 20 min.

ie. Adelaide is 13 h 20 min ahead of Buenos Aires.

(ii) Roy must call 13 h 20 min after 7 pm Friday. That is, at 8:20 am Saturday (Adelaide time).

(c) (i) Using the formula for the area of an annulus:  
 Area =  $\pi(R^2 - r^2)$ , where  $R = 1.496 \times 10^8$   
 $r = 1.082 \times 10^8$   
 $= \pi \left[ (1.496 \times 10^8)^2 - (1.082 \times 10^8)^2 \right]$   
 $= 3.352 \dots \times 10^{16}$   
 $\div 3.4 \times 10^{16} \text{ km}^2$  (2 sig. figures).

(ii) **METHOD 1**  
 $A = \pi(R^2 - r^2)$   
 $A = \pi \times R^2 - \pi \times r^2$   
 $A + \pi \times r^2 = \pi \times R^2$

∴  $\pi \times R^2 = A + \pi \times r^2$   
 $R^2 = \frac{A + \pi \times r^2}{\pi}$   
 $R = \pm \sqrt{\frac{A + \pi r^2}{\pi}}$

but since  $R$  is the length of the radius, it must be positive,

∴  $R = \sqrt{\frac{A + \pi r^2}{\pi}}$ .

**METHOD 2**

$A = \pi(R^2 - r^2)$

$\frac{A}{\pi} = R^2 - r^2$

$\frac{A}{\pi} + r^2 = R^2$

$R^2 = \frac{A}{\pi} + r^2$

$R = \pm \sqrt{\frac{A}{\pi} + r^2}$ .

But since  $R$  is the length of the radius, it must be positive.

∴  $R = \sqrt{\frac{A}{\pi} + r^2}$ .

(iii)  $A = 6.79 \text{ mm}^2$ ,  $r = 0.75 \text{ mm}$ .

**METHOD 1**

Using the 1st formula from (ii),

$R = \sqrt{\frac{A + \pi r^2}{\pi}}$   
 $= \sqrt{\frac{6.79 + \pi \times (0.75)^2}{\pi}}$   
 $= 1.650\,401 \dots$   
 $\div 1.65$  (correct to 2 d.p.).

**METHOD 2**

Using the 2nd formula from (ii),

$R = \sqrt{\frac{A}{\pi} + r^2}$   
 $= \sqrt{\frac{6.79}{\pi} + 0.75^2}$   
 $= \sqrt{2.7238 \dots}$   
 $= 1.650\,401 \dots$   
 $\div 1.65 \text{ mm}$  (correct to 2 d.p.).

**METHOD 3**

Using  $A = \pi(R^2 - r^2)$ .  
 $6.79 = \pi(R^2 - 0.75^2)$   
 $= \pi(R^2 - 0.5625)$   
 $= \pi R^2 - 1.7671 \dots$   
 $6.79 + 1.7671 \dots = \pi R^2$   
 $8.557\,14 \dots = \pi R^2$



$$R^2 = \frac{8.55714 \dots}{\pi}$$

$$= 2.7238 \dots$$

$$R = \sqrt{2.7238 \dots}$$

$$= 1.650401 \dots$$

$$\approx 1.65 \text{ mm (correct to 2 d.p.)}$$

**Question 26**

(a) **METHOD 1**

For the average to be 6 after four quizzes, the total marks must be  $6 \times 4 = 24$ .  
Vicki's marks so far =  $3 \times 5 = 15$ .  
 $\therefore$  Vicki needs 9 marks in the next quiz.

**METHOD 2**

$$\bar{x} = \frac{\text{sum of scores}}{\text{no. of scores}}$$

After 3 quizzes,  $\bar{x} = 5$ ,

$$\therefore 5 = \frac{\text{sum of scores}}{3}$$

$\therefore$  sum of scores is 15.

Let the required mark be  $x$ .

After the next quiz, Vicki wants an average  $\bar{x} = 6$ .

$$\therefore 6 = \frac{\text{sum of scores}}{4}$$

$$6 = \frac{15 + x}{4}$$

$$24 = 15 + x$$

$$x = 24 - 15$$

$$= 9.$$

$\therefore$  Vicki needs 9 marks in the next quiz.

(b) (i)  $\bar{x} = 3.1\bar{3}$  (Sum of scores = 94, number of scores = 30.)  
 $\approx 3.13$  (correct to 2 d.p.)

(ii) Sample s.d.  $(\sigma_{n-1}) = 1.66$  (correct to 2 d.p.)

(iii) There are 30 scores.

$\therefore$  Median is average of 15th and 16th scores

$$= \frac{4+4}{2}$$

$$= 4.$$

(iv) As the scores are bunched at the higher end (4 or 5) the data are negatively skewed.

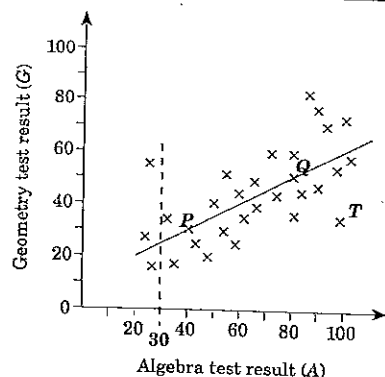
(v) In the sample,  $9 + 7 = 16$  students sent more than 3 text messages.

$$P(\text{student sending more than 3 messages}) = \frac{16}{30}$$

$$\text{Estimate} = \frac{16}{30} \times 150 = 80.$$

$\therefore$  The number is approximately 80 students.

(c)



(i) By counting the crosses on the left of the vertical line  $A = 30$ , three students scored less than 30 for algebra.

(ii) Using points  $P(40, 30)$  and  $Q(80, 50)$ :

• vertical change =  $50 - 30 = 20$ .

• horizontal change =  $80 - 40 = 40$ .

$$\therefore \text{Gradient} = \frac{20}{40} = \frac{1}{2}$$

(iii) **METHOD 1**

Extending the line,  $y$  intercept = 10.

$$\text{Equation } y = mx + b$$

$$\Rightarrow y = \frac{1}{2}x + 10.$$

**METHOD 2**

Equation of the line is  $y = mx + b$ ,

where  $m = \frac{1}{2}$ ,

$$\therefore y = \frac{1}{2}x + b.$$

Substitute  $P(40, 30)$  to find  $b$ :

$$30 = \frac{1}{2}(40) + b$$

$$= 20 + b$$

$$\therefore b = 10.$$

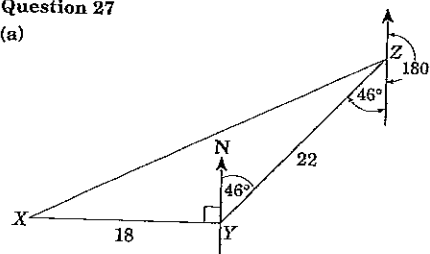
$$\therefore \text{Equation of the line is } y = \frac{1}{2}x + 10.$$

(iv) There is a strong positive correlation between the two sets of results.

(v) This statement is true for some students, but not all. For example, the student marked  $T$  on the graph came second (about 96) in algebra, but below half-way (about 32) in geometry.

**Question 27**

(a)



(i)  $\angle XYZ = 90 + 46 = 136^\circ$ .

(ii) Using the cosine rule:

$$c^2 = a^2 + b^2 - 2ab \cos c,$$

$$\text{with } a = 18, b = 22, c = 136^\circ.$$

$$XZ^2 = 18^2 + 22^2 - 2 \times 18 \times 22 \times \cos 136^\circ$$

$$XZ = 1377.71 \dots$$

$$X = \sqrt{1377.71 \dots}$$

$$= 37.117 \dots$$

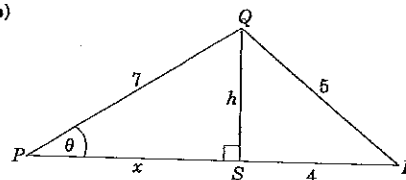
$$\approx 37.1 \text{ km (1 d.p.)}$$

(iii) See diagram in (i).

The marked angle at  $Z = 46^\circ$  by alternate angles between parallel lines.

Bearing of  $Y$  from  $Z = 180 + 46 = 226^\circ$ .

(b)



(i) Need to know  $PS$ . Let  $PS = x$  and  $QS = h$ . Use Pythagoras' theorem twice.

In  $\triangle QRS$ :  $h^2 + 4^2 = 5^2$

$$h^2 = 25 - 16$$

$$= 9$$

$$\therefore h = 3.$$

In  $\triangle QPS$ :  $x^2 + h^2 = 7^2$

$$x^2 = 49 - 9$$

$$= 40$$

$$\therefore x = \sqrt{40}$$

$$= 6.3 \text{ (1 d.p.)}$$

$$\therefore \text{Perimeter of } \triangle PQR = 7 + 5 + 4 + 6.3$$

$$= 22.3 \text{ cm (1 d.p.)}$$

(ii) Let  $\theta = \angle QPS$ .

Use  $\triangle QPS$ :  $\sin \theta = \frac{3}{7}$

$$\theta = 25.37 \dots$$

$$\therefore \angle QPS = 25^\circ \text{ (nearest degree)}$$

(c) (i) Shaded area (area  $ABFE$ ):

Using Simpson's rule,

$$A \approx \frac{h}{3}(d_f + 4d_m + d_l), \text{ with } h = 150$$

$$d_f = 120$$

$$d_m = 37$$

$$d_l = 40$$

$$= \frac{150}{3}(120 + 4 \times 37 + 40)$$

$$= 15\,400 \text{ m}^2.$$

(ii) Area of lake = area of rectangle

– (area  $ABFE$  + area  $DCFE$ )

$$\text{Area of rectangle} = 300 \times 200$$

$$= 60\,000 \text{ m}^2.$$

$$\text{Area } ABFE = 15\,400, \text{ from (i).}$$

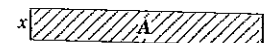
$$\text{Area } DCFE = \frac{150}{3}(80 + 4 \times 77 + 160)$$

$$= 27\,400. \text{ (Simpson's rule)}$$

$$\therefore \text{Area of lake} = 60\,000 - (15\,400 + 27\,400) = 17\,200 \text{ m}^2.$$

**Question 28**

(a) (i)



$A = bx$  (area of a rectangle).

Now  $b + 2x = 28$

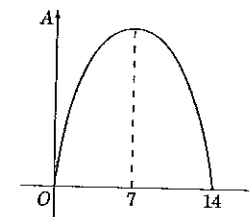
so  $b = 28 - 2x$ .

$$\therefore A = (28 - 2x)x$$

$$= 28x - 2x^2.$$

(ii)  $x$  must be greater than zero in order to have a gutter, and  $x$  must be less than 14 (half of 28) because when  $x = 14$  the base is zero.

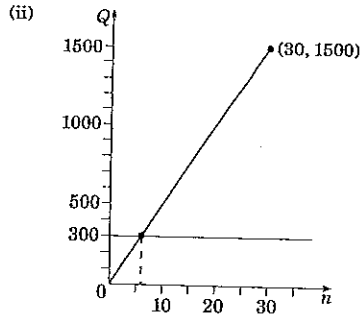
(iii)



Since the parabola is symmetrical, the maximum occurs at the middle, where  $x = 7$ .

$$\text{When } x = 7, A = 28 \times 7 - 2 \times 7^2 = 98 \text{ cm}^2.$$

(b) (i)  $Q = 50n$ .



(iii) On the graph, the point where the lines meet is the break-even point where Toby has earned the amount he spent on equipment. This occurs where  $n = 6$ , which means that after 6 parties he starts making a profit.

(iv) *Amount saved in each year*      \$

2001, Year 9:	900
2002, Year 10:	$900 + 30\% \times 900 = 1170$
2003, Year 11:	$1170 + 30\% \times 1170 = 1521$
2004, Year 12:	$1521 + 30\% \times 1510 = 1977.30$
$\therefore$ Total saved =	\$5568.30.

**METHOD 1**

Savings account

	Balance at start of year	Interest earned (4%)	Amount added	Balance at end of year
2005, 1st yr	5568.30	222.73	2500	8291.03
2006, 2nd yr	8291.03	331.64	2500	11 122.67
2007, 3rd yr	11 122.67	444.91	2500	14 067.58

With \$14 067.58, Toby does not quite reach his goal of \$15 000.

**METHOD 2**

After 2004, the investment can be thought of as a combination of:

- an annuity of \$2500 invested yearly for 3 years;
  - \$5568.30 invested for 3 years;
- These calculations are:
- \$2500 invested yearly at 4% pa compounded annually for 3 years.

Using the future value formula, with  $M = \$2500$ ,  $r = 4\% = 0.04$ ,  $n = 3$ :

$$A = \frac{M[(1+r)^n - 1]}{r}$$

$$= \frac{2500[(1+0.04)^3 - 1]}{0.04}$$

$$= \$7804;$$

- \$5568.30 invested for 3 years at 4% p.a. Using the compound interest formula, with  $P = \$5568.30$ ,  $r = 4\% = 0.04$ ,  $n = 3$ :

$$A = P(1+r)^n$$

$$= 5568.30(1+0.04)^3$$

$$= \$6263.58 \text{ (nearest cent).}$$

$$\therefore \text{Total value of investment} = \$7804 + \$6263.58 = \$14\,067.58.$$

With \$14 067.58, Toby will not reach his goal of \$15 000. (He will be short by \$932.42.)

END OF GENERAL MATHEMATICS SOLUTIONS

2001 HIGHER SCHOOL CERTIFICATE  
EXAMINATION PAPER  
GENERAL MATHEMATICS

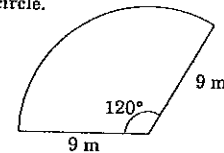
**Section I**

Total marks (22)

Attempt Questions 1 – 22

Allow about 30 minutes for this section

- Alex works in a shop where the normal weekday rate of pay is \$12 per hour. On Saturdays she is paid time-and-a-half.  
How much did Alex earn in a week in which she worked for seven hours on Thursday and three hours on Saturday?  
(A) \$84                      (B) \$120                      (C) \$138                      (D) \$180
- If  $w = \frac{15y}{y+12}$ , and  $y = 7$ , find the value of  $w$  (correct to two decimal places).  
(A) 5.53                      (B) 8.26                      (C) 15.75                      (D) 27.00
- This is a sketch of a sector of a circle.



Calculate the area of this sector (correct to one decimal place).

- (A) 9.4 m<sup>2</sup>                      (B) 18.8 m<sup>2</sup>                      (C) 36.8 m<sup>2</sup>                      (D) 84.8 m<sup>2</sup>
- Frank has a credit card with an interest rate of 0.05% per day and no interest-free period. Frank used the credit card to pay for car repairs costing \$480. He paid the credit card account 16 days later. What is the total amount (including interest) that he paid for the repairs?  
(A) \$480.24                      (B) \$483.84                      (C) \$504.00                      (D) \$864.00
  - Simplify  $3(x-2) - 2(x-1)$ .  
(A)  $x-4$                       (B)  $x-3$                       (C)  $x-1$                       (D)  $x-8$
  - The number represented by a 1 followed by one hundred zeros is called a googol. Which of the following is equal to a googol?  
(A)  $10^2$                       (B)  $10^{10}$                       (C)  $10^{99}$                       (D)  $10^{100}$