Question 28

(a) (i) $2.50 = $2.00 + 50c

$\quad = $1 + $1 + 50c$

$\quad = $1 + 50c + 50c + 50c$

$\quad = 50c + 50c + 50c + 50c + 50c.$

(ii) $P($2\cdot50) = P($2 \text{ in the 1st draw and } 50c$

$\qquad\qquad\qquad \text{in the 2nd draw})$

$\qquad\qquad + P(50c \text{ in the 1st draw and}$

$\qquad\qquad\qquad\qquad $2 \text{ in the 2nd draw})$

$\quad = \dfrac{3}{11} \times \dfrac{2}{10} + \dfrac{2}{11} \times \dfrac{3}{10}$

$\quad = \dfrac{6}{55}.$

(iii) Total value $= $(2x + 1y + 0\cdot5w)$

$\qquad\qquad\quad = $(2x + y + 0\cdot5w).$

(b) (i) Since a $1 increase in toll will lead to 500 fewer vehicles using the tunnel, then a $12 increase in the toll will lead to $12 \times 500 = 6000$ fewer vehicles using the tunnel. Hence the lowest toll for which no vehicles will use the tunnel is $12.

(ii) The number of vehicles that will use the tunnel $= 6000 - 5 \times 500$

$\qquad\qquad\qquad\qquad\qquad = 3500$ vehicles.

Income $= 3500 \times $5 = $17\,500.$

(iii) If the toll is $1 then $6000 - 1 \times 500$ will use the tunnel.

If the toll is $2 then $6000 - 2 \times 500$ will use the tunnel.

Hence if the toll is d (dollars), then $6000 - d \times 500$ will use the tunnel.
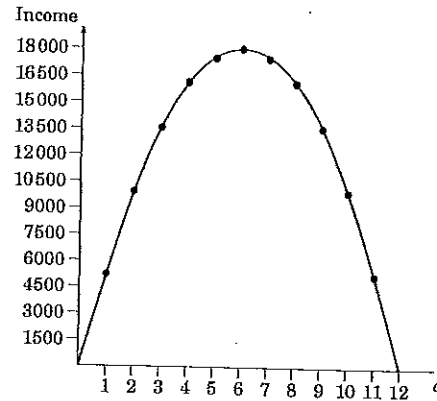
So, $V = 6000 - 500d.$

(b) (iv) *METHOD 1*

Toll (d)	No. vehicles (v)	Income ($d \times v$)
0	6000	0
1	5500	5500
2	5000	10 000
3	4500	13 500
4	4000	16 000
5	3500	17 500
6	3000	18 000
7	2500	17 500
8	2000	16 000
9	1500	13 500
10	1000	10 000
11	500	5500
12	0	0

From the table, the maximum daily income is $18 000.

Anne is incorrect because a higher toll also means fewer vehicles, so lower income.

METHOD 2

From the graph, the maximum income is $180 000. Anne is incorrect for tolls bigger than 6 because more tolls leads to less income.

END OF GENERAL MATHEMATICS SOLUTIONS

2005 HIGHER SCHOOL CERTIFICATE EXAMINATION PAPER

GENERAL MATHEMATICS

## Section I

22 marks

Attempt Questions 1 – 22

Allow about 30 minutes for this section

1 What is the mean of the set of scores?

$\quad$ 3, 4, 5, 6, 6, 8, 8, 8, 15

(A) 6 $\qquad$ (B) 7 $\qquad$ (C) 8 $\qquad$ (D) 9

2 What is the value of $\dfrac{a-b}{4}$, if $a = 240$ and $b = 56$?

(A) 4 $\qquad$ (B) 46 $\qquad$ (C) 226 $\qquad$ (D) 736

3 Four radio stations reported the probability of rain as shown in the table.

Which radio station reported the highest probability of rain?
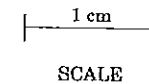
(A) 2AT $\qquad$ (B) 2BW

(C) 2CZ $\qquad$ (D) 2DL

Radio station	Probability of rain
2AT	0·53
2BW	17%
2CZ	$\dfrac{13}{25}$
2DL	0·6

4 The diagram is a scale drawing of a butterfly.
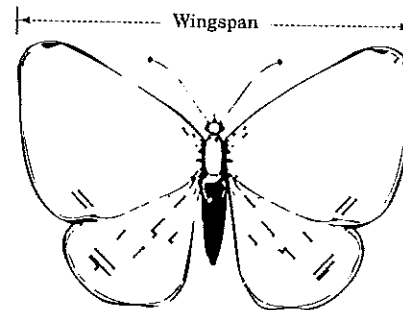
What is the actual wingspan of the butterfly?

(A) 2·5 cm $\qquad$ (B) 3 cm

(C) 15 cm $\qquad$ (D) 18·75 cm

SCALE

5 Which formula should be used to calculate the distance between Toby and Frankie?

(A) $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$ $\qquad$ (B) $c^2 = a^2 + b^2$

(C) $A = \dfrac{1}{2} ab \sin C$ $\qquad$ (D) $c^2 = a^2 + b^2 - 2ab \cos C$

NOT TO SCALE
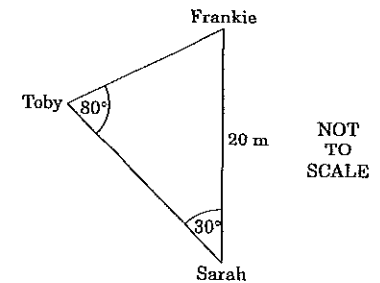
6 Janet's gross income last year was $60 000. She had allowable tax deductions of $5000. Janet paid 1·5% of her taxable income for the Medicare levy.

How much was Janet's Medicare levy?

(A) $750 (B) $825 (C) $900 (D) $975

7 Simplify $2m^2 \times 3mp^2$.

(A) $5m^2p^2$ (B) $5m^3p^2$ (C) $6m^2p^2$ (D) $6m^3p^2$

8 If $\tan\theta = 85$, what is the value of θ, correct to the nearest minute?

(A) $11°25'$ (B) $11°26'$ (C) $89°19'$ (D) $89°20'$

9 A set of data is represented by the cumulative frequency histogram and ogive.

What is the best approximation for the interquartile range for this set of data?

(A) 25

(B) 30

(C) 35

(D) 40
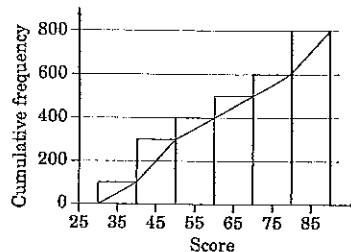
10 The table is used to calculate monthly loan repayments.

Samantha has borrowed $70 000 at 8% per annum for 15 years.

What is her monthly loan repayment?

(A) $143·40 (B) $669·20

(C) $8 030·40 (D) $10 038·00

Monthly loan repayments (in dollars) per $1000 borrowed

Interest rate % pa	5 years	10 years	15 years	20 years
5%	18·87	10·61	7·91	6·60
6%	19·33	11·10	8·44	7·16
7%	19·80	11·61	8·99	7·75
8%	20·28	12·13	9·56	8·36
9%	20·76	12·67	10·14	9·00

11 The diagram shows a spinner.

The arrow is spun and will stop in one of the six sections.

What is the probability that the arrow will stop in a section containing a number greater than 4?

(A) $\frac{2}{5}$ (B) $\frac{2}{3}$

(C) $\frac{1}{3}$ (D) $\frac{1}{2}$
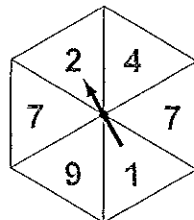
12 The shaded region represents a block of land bounded on one side by a road.

What is the approximate area of the block of land, using Simpson's rule?

(A) 680 m² (B) 760 m²

(C) 840 m² (D) 1360 m²
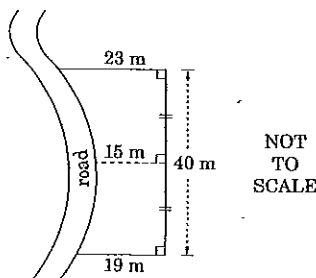
13 Last year, Helen bought 150 shares at $2·00 per share. They are now worth $2·50 per share. Helen receives a dividend of $0·10 per share.

What is the dividend yield?

(A) 4% (B) 20% (C) $15 (D) $75

14 Using the formula $d = 5t^3 - 2$, Marcia tried to find the value of t when $d = 137$.

Here is her solution. She made one mistake. Which line does NOT follow correctly from the previous line?

(A) Line A (B) Line B

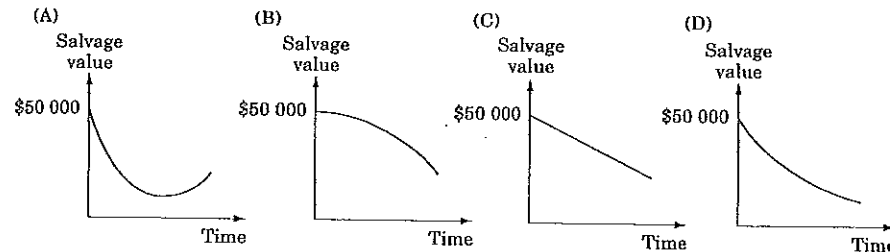(C) Line C (D) Line D

$$d = 5t^3 - 2$$
$$137 = 5t^3 - 2 \quad \dots\dots\dots \text{Line } A$$
$$135 = 5t^3 \quad \dots\dots\dots \text{Line } B$$
$$27 = t^3 \quad \dots\dots\dots \text{Line } C$$
$$t = 3 \quad \dots\dots\dots \text{Line } D$$

15 A car bought for $50 000 is depreciated using the declining balance method.

Which graph best represents the salvage value of the car over time?

16 On a television game show, viewers voted for their favourite contestant. The results were recorded in the two-way table.

|	Male viewers	Female viewers
Contestant 1	1372	3915
Contestant 2	2054	3269

One male viewer was selected at random from all of the male viewers.

What is the probability that he voted for Contestant 1?

(A) $\frac{1372}{10\,610}$ (B) $\frac{1372}{5287}$ (C) $\frac{1372}{3426}$ (D) $\frac{1372}{2054}$

17 The total cost, C, of a school excursion is given by $C = 2n + 5$, where n is the number of students. If three extra students go on the excursion, by how much does the total cost increase?

(A) $6 (B) $11 (C) $15 (D) $16

18 A model yacht has two triangular sails. These triangles are similar to each other. Some dimensions of the sails, in centimetres, are shown on the diagram.
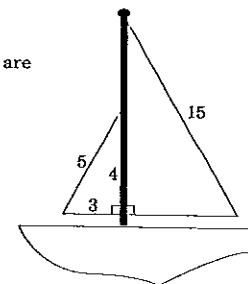
What is the total area of both sails?

(A) 24 cm² (B) 27 cm²

(C) 60 cm² (D) 97 cm²

19 The location of Town A is 25°N 45°E. The location of Town B is 10°N 105°E.

Which of the following is true? (Ignore time zones.)

(A) Town A is four hours behind Town B.

(B) Town A is four hours ahead of Town B.

(C) Town A is one hour behind Town B.

(D) Town A is one hour ahead of Town B.

20 Dave's school has computer security codes made up of four digits (eg 0773). Juanita's school has computer security codes made up of five digits (eg 30568).

How many more codes are available at Juanita's school than at Dave's school?

(A) 10   (B) 50   (C) 90 000   (D) 100 000

21 Yousef used the 'capture-recapture' technique to estimate the number of kangaroos living in a particular area.

• He caught, tagged and released 50 kangaroos.

• Later, he caught 200 kangaroos at random from the same area.

• He found that 5 of these 200 kangaroos had been tagged.

What is the correct estimate for the total number of kangaroos living in this area, using the 'capture-recapture' technique?

(A) 245   (B) 250   (C) 2000   (D) 10 000

22 Two groups of people were surveyed about their weekly wages. The results are shown in the box-and-whisker plots.
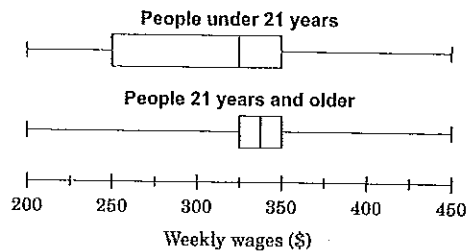
Which of the following statements is true for the people surveyed?

(A) The same percentage of people in each group earned more than $325 per week.

(B) Approximately 75% of people under 21 years earned less than $350 per week.

(C) Approximately 75% of people 21 years and older earned more than $350 per week.

(D) Approximately 50% of people in each group earned between $325 and $350 per week.

Section II

78 marks

Attempt Questions 23 – 28

Allow about 2 hours for this section

Question 23 (13 marks)

(a) There are 100 tickets sold in a raffle. Justine sold all 100 tickets to five of her friends. The number of tickets she sold to each friend is shown in the table.

| Friend	Number of tickets
Danielle	45
Khalid	5
Nancy	10
Shani	14
Herman	26
Total	100

Marks

(i) Justine claims that each of her friends is equally likely to win first prize. Give a reason why Justine's statement is NOT correct.   1

(ii) What is the probability that first prize is NOT won by Khalid or Herman?   2
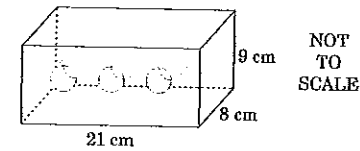
(b) A clay brick is made in the shape of a rectangular prism with dimensions as shown.
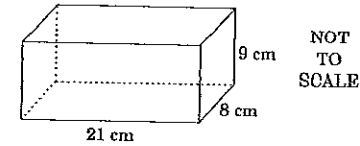
(i) Calculate the volume of the clay brick.   1

Three identical cylindrical holes are made through the brick as shown. Each hole has a radius of 1·4 cm.

(ii) What is the volume of clay remaining in the brick after the holes have been made? (Give your answer to the nearest cubic centimetre.)   3

(iii) What percentage of clay is removed by making the holes through the brick? (Give your answer correct to one decimal place.)   1

(c) Moheb owns five red and seven blue ties. He chooses a tie at random for himself and puts it on. He then chooses another tie at random, from the remaining ties, and gives it to his brother.

(i) What is the probability that Moheb chooses a red tie for himself?   1

(ii) Copy the tree diagram. Complete your tree diagram by writing the correct probability on each branch.   2

(iii) Calculate the probability that both of the ties are the same colour.   2

Question 24 (13 marks)

(a) (i) Draw a stem-and-leaf plot for the following set of scores.   2

21   45   29   27   19   35   23   58   34   27

(ii) What is the median of the set of scores?   1

(iii) Comment on the skewness of the set of scores?   1

(b) The formula $D = \dfrac{2A}{15}$ is used to calculate the dosage of Hackalot cough medicine to be given to a child.

• D is the dosage of Hackalot cough medicine in millilitres (mL).

• A is the age of the child in months.

(i) If George is nine months old, what dosage of Hackalot cough medicine should he be given?   1

(ii) The correct dosage of Hackalot cough medicine for Sam is 4 mL. What is the difference in the ages of Sam and George, in months?   3
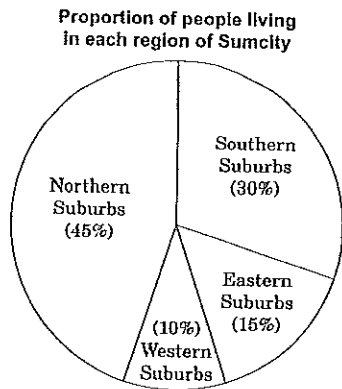
(c) Make L the subject of the equation $T = 2\pi L^2$.   **2**

(d) The sector graph shows the proportion of people, as a percentage, living in each region of Sumcity. There are 24 000 people living in the Eastern Suburbs.

Proportion of people living in each region of Sumcity

(i) Show that the total number of people living in Sumcity is 160 000.   **1**

Jake used the information above to draw a column graph.
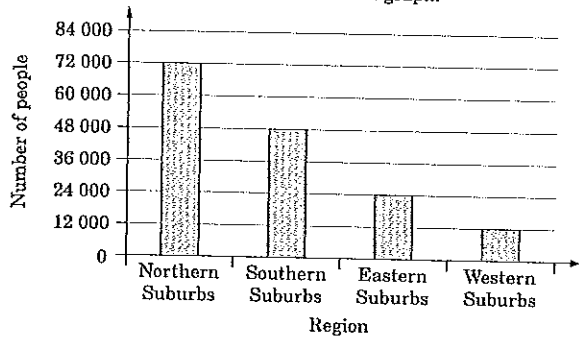
(ii) The column graph height is incorrect for one region. Identify this region and justify your answer.   **2**

Question 25 (13 marks)

(a) Reece is preparing his annual budget for 2006. His expected income is:
* $90 every week as a swimming coach
* Interest earned from an investment of $5000 at a rate of 4% per annum.
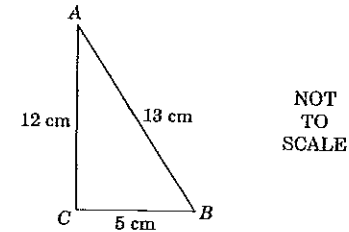
His planned expenses are:
* $30 every week on transport
* $12 every week on lunches
* $48 every month on entertainment.

Reece will save his remaining income. He uses the spreadsheet [on page 7] for his budget.

(i) Determine the values of X, Y and Z. (Assume there are exactly 52 weeks in a year.)   **3**

(ii) At the beginning of 2006, Reece starts saving.   **2**
Will Reece have saved enough money during 2006 for a deposit of $2100 on a car if he keeps to his budget? Justify your answer with suitable calculations.

(b)

(i) Use Pythagoras' theorem to show that $\triangle ABC$ is a right-angled triangle.   **1**

(ii) Calculate the size of $\angle ABC$ to the nearest degree.   **2**

(c) Robyn plays a game in which she randomly chooses one of these five cards. She plays the game 60 times, replacing the card after each game.
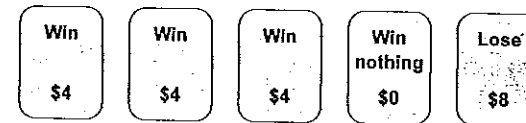
(i) How many times would she expect to win $4?   **1**

(ii) What is the financial expectation of the game?   **2**

(iii) Another card is added to the game with 'Win nothing $0' written on it.   **2**
Robyn claims that the financial expectation will not change.
Do you agree? Justify your answer with suitable calculations.

Question 26 (13 marks)

(a) A printing machine worth $150 000 is bought in December 2005.   **2**
In December each year, beginning in 2006, the value of the printing machine is depreciated by 10% using the declining balance method of depreciation.
In which year will the depreciated value first fall below $120 000?

(b) Rod is saving for a holiday. He deposits $3600 into an account at the end of every year for four years. The account pays 5% per annum interest, compounding annually.
The table shows future values of an annuity of $1.

Future values of an annuity of $1

End of year	Interest rate				
	1%	2%	3%	4%	5%
1	1·0000	1·0000	1·0000	1·0000	1·0000
2	2·0100	2·0200	2·0300	2·0400	2·0500
3	3·0301	3·0604	3·0909	3·1216	3·1525
4	4·0604	4·1216	4·1836	4·2465	4·3101
5	5·1010	5·2040	5·3091	5·4163	5·5256
6	6·1520	6·3081	6·4684	6·6330	6·8019
7	7·2135	7·4343	7·6625	7·8983	8·1420
8	8·2857	8·5830	8·8923	9·2142	9·5491

   (i) Use the table to find the value of Rod's investment at the end of four years.   2

   (ii) How much interest does Rod earn on his investment over the four years?   2

(c) The weights of boxes of Brekky Bicks are normally distributed. The mean is 754 grams and the standard deviation is 2 grams.

   (i) What is the z-score of a box of Brekky Bicks with a weight of 754 g?   1

   (ii) What is the weight of a box that has a z-score of –1?   1

   (iii) Brekky Bicks boxes are labelled as having a weight of 750 g. What percentage of boxes will have a weight less than 750 g?   2

(d) Peta borrows $28 000 from a credit union at 6% per annum compounding monthly. She will repay the money over nine years.

Peta uses the formula:

$$28\,000 = M \left\{ \frac{(1+r)^n - 1}{r(1+r)^n} \right\}$$

to calculate her monthly repayment, M.

   (i) Rewrite the formula, showing the correct substitutions for r and n.   2

   (ii) Calculate Peta's monthly repayment.   1

Question 27 (13 marks)

(a) The area graph shows sales figures for Shoey's shoe store.
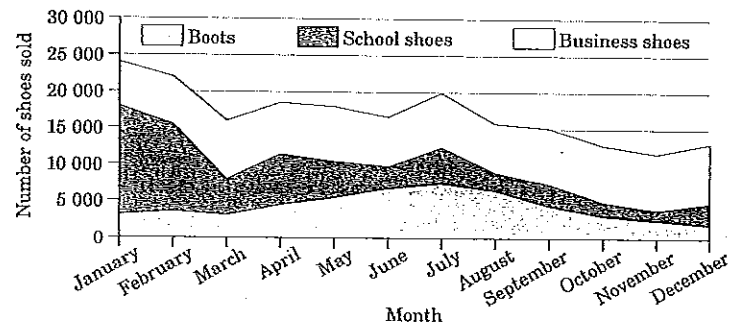
   (i) Approximately how many school shoes were sold in January?   1

   (ii) For which month does the graph indicate that the same number of school shoes and business shoes was sold?   1

   (iii) Identify ONE trend in this graph, and suggest a valid reason for this trend.   2

(b) This diagram represents Earth. O is at the centre, and A and B are points on the surface.   2

Calculate the distance from A to B along the great circle through A and B. Give your answer in nautical miles.

(Radius of Earth is 6400 km. 1·852 km = 1 nautical mile)

A: 35°N 20°E

B: 8°S 20°E

(c)
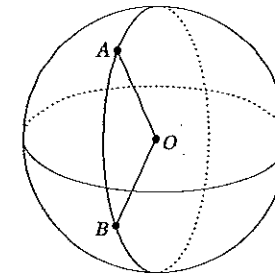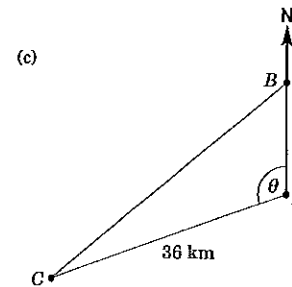
NOT TO SCALE

The bearing of C from A is 250° and the distance of C from A is 36 km.

   (i) Explain why θ is 110°.   1

   (ii) If B is 15 km due north of A, calculate the distance of C from B, correct to the nearest kilometre.   3

(d) Nine students were selected at random from a school, and their ages were recorded.

Ages		
12	11	16
14	16	15
14	15	14

   (i) What is the sample standard deviation, correct to two decimal places?   2

   (ii) Briefly explain what is meant by the term *standard deviation*.   1

Question 28 (13 marks)

(a) The Mitchell family has moved to a new house which has an empty swimming pool. The base of the pool is in the shape of a rectangle, with a semicircle on each end.

Base of pool
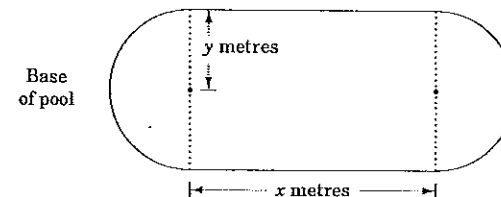
y metres

x metres

   (i) Explain why the expression for the area of the base of the pool is $2xy + \pi y^2$.   1

   (ii)   4

The pool is 1·1 metres deep.

1·1 m
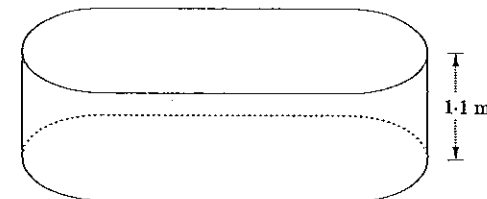
The sides and base of the pool are covered in tiles. If $x = 6$ and $y = 2 \cdot 5$, find the total area covered by tiles. (Give your answer correct to the nearest square metre.)

(iii) Before filling the pool, the Mitchells need to install a new shower head, which saves 6 litres of water per minute.   **2**

The shower is used 5 times every day, for 3 minutes each time.

If the charge for water is $\$1 \cdot 013$ per kilolitre, how much money would be saved in one year by using this shower head? (Assume there are 365 days in a year.)

(b) Sue and Mikey are planning a fund-raising dance. They can hire a hall for $400 and a band for $300. Refreshments will cost them $12 per person.

(i) Write a formula for the cost (C) of running the dance for x people.   **1**

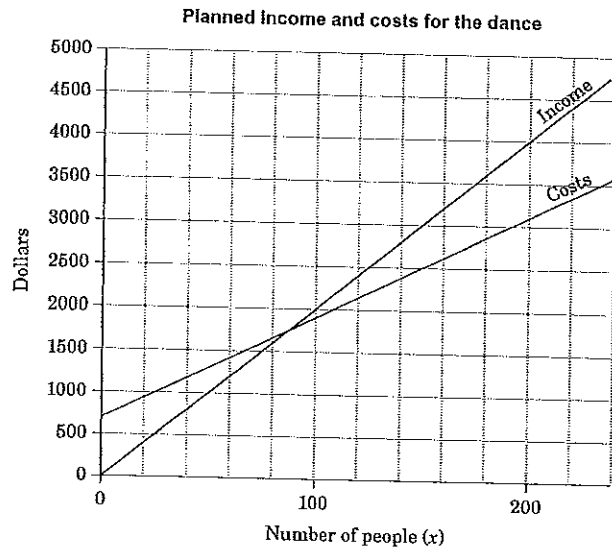The graph shows planned income and costs when the ticket price is $20.

Planned income and costs for the dance

(ii) Estimate the minimum number of people needed at the dance to cover the costs.   **1**

(iii) How much profit will be made if 150 people attend the dance?   **1**

(iv) Sue and Mikey plan to sell 200 tickets. They want to make a profit of $1500.   **3**
What should be the price of a ticket, assuming all 200 tickets will be sold?

End of paper

2005 HIGHER SCHOOL CERTIFICATE SOLUTIONS

GENERAL MATHEMATICS

SECTION I
SUMMARY

1. B	7. D	13. A	18. C
2. B	8. D	14. B	19. A
3. D	9. C	15. D	20. C
4. B	10. B	16. C	21. C
5. A	11. D	17. A	22. B
6. B	12. A		

1. (B) $\text{Mean} = \dfrac{\text{sum of scores}}{\text{number of scores}}$

$\qquad\qquad = \dfrac{63}{9}$

$\qquad\qquad = 7.$

2. (B) $\dfrac{a - b}{4} = \dfrac{240 - 56}{4}$

$\qquad\quad = \dfrac{184}{4}$

$\qquad\quad = 46.$

3. (D) Converting probabilities to decimals:

$0 \cdot 53 = 0 \cdot 53$

$17\% = 0 \cdot 17$

$\dfrac{13}{25} = 0 \cdot 52$

$0 \cdot 6 = 0 \cdot 6.$

$0 \cdot 6$ is the highest.

4. (B) Scale: $2 \cdot 5$ cm represents 1 cm

Measured wingspan $= 7 \cdot 5$ cm.

Actual wingspan $= 7 \cdot 5 \div 2 \cdot 5$

$\qquad\qquad\qquad = 3$ cm.

5. (A) Let distance between Toby and Frankie be x (in metres), then the sine rule is needed:

$$\dfrac{x}{\sin 30°} = \dfrac{20}{\sin 80°}.$$

6. (B) Taxable income $= \$60\,000 - \$5000$

$\qquad\qquad\qquad = \$55\,000.$

Medicare levy $= 1 \cdot 5\% \times \$55\,000$

$\qquad\qquad\qquad = \$825.$

7. (D) $2m^2 \times 3mp^2 = 6m^3 p^2.$

8. (D) $\tan \theta = 85$

$\qquad \theta = 89 \cdot 325\,9631 \ldots$

$\qquad\quad \doteq 89° 19' 33 \cdot 47''$

$\qquad\quad \doteq 89° 20'.$

9. (C) Read the graph at cumulative frequency of 600 (to find the upper quartile) and 200 (to find the lower quartile).

Interquartile range $= 80 - 45$

$\qquad\qquad\qquad\quad = 35.$

10. (B) From table, monthly loan repayment is $70 \times \$9 \cdot 56 = \$669 \cdot 20.$

11. (D) Numbers greater than 4 are 7, 7, 9.

$P(\text{number greater than } 4) = \dfrac{3}{6} = \dfrac{1}{2}.$

12. (A) *Note:* In this case, $h = \dfrac{1}{2} \times 40 = 20.$

$A \doteq \dfrac{h}{3}\left(d_f + 4d_m + d_\ell\right)$

$\quad = \dfrac{20}{3}\left(19 + 4 \times 15 + 23\right)$

$\quad = 680 \text{ m}^2.$

13. (A) Dividend yield $= \dfrac{\text{dividend}}{\text{market price}} \times 100\%$

$\qquad\qquad\qquad = \dfrac{\$0 \cdot 10}{\$2 \cdot 50} \times 100\%$

$\qquad\qquad\qquad = 4\%.$

14. (B) $\qquad d = 5t^3 - 2$

$\qquad 137 = 5t^3 - 2 \qquad \text{Line } A$

Line B should read

$\qquad 139 = 5t^3.$

15. (D) Correct graph must be a hyperbolic function, with the graph approaching but not touching the Time axis.

16. (C) Total male viewers $= 1372 + 2054$

$\qquad\qquad\qquad\qquad = 3426.$

$P(\textit{male}$ viewer, selected from *male* viewers, selects Contestant 1$)$

$\qquad = \dfrac{1372}{3426}.$

17. (A) **METHOD 1**

$5 is a fixed cost, and for each additional student, the cost is increased by $2.

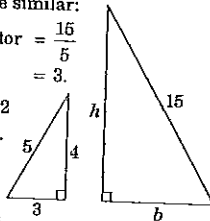*Extra* cost for 3 students $= 2n$
$$= 2 \times 3$$
$$= \$6.$$

METHOD 2

$5 is a fixed cost. For no students,
$$C = 2 \times 0 + 5 = \$5.$$
For three students,
$$C = 2 \times 3 + 5 = \$11.$$
∴ Additional cost $= \$6$.

18. (C) The triangles are similar:

Enlargement factor $= \dfrac{15}{5}$
$$= 3.$$

∴ $h = 4 \times 3 = 12$
$b = 3 \times 3 = 9.$

∴ Total area
$$= \frac{1}{2} \times 3 \times 4 + \frac{1}{2} \times 9 \times 12$$
$$= 6 + 54$$
$$= 60 \text{ cm}^2.$$

19. (A) Difference in longitude $= 105° - 45° = 60°$.

Time difference $= 60 \times 4$ min
$$= 240 \text{ min}$$
$$= 4 \text{ hours.}$$

Town A is west of Town B.

∴ Town A is 4 hours behind Town B.

20. (C) Number of 4 digit codes
$$= 10 \times 10 \times 10 \times 10$$
$$= 10\,000.$$

Number of 5 digit codes
$$= 10 \times 10 \times 10 \times 10 \times 10$$
$$= 100\,000.$$

∴ Extra codes $= 100\,000 - 10\,000$
$$= 90\,000.$$

21. (C) Let the estimate for the number of kangaroos be E.

$$\frac{5}{200} = \frac{50}{E}$$
$$5E = 50 \times 200$$
$$= 10\,000.$$
∴ $E = 2000.$

22. (B) The only correct statement is (B).

The other statements are false because:

(A) 50% of people under 21 years earned more than $325 per week, but 75% of people 21 years and older earned more than $325 per week;

(C) 75% of people 21 years and older earned *less* than $350 per week;

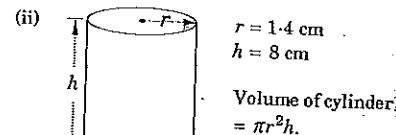(D) statement only true for 21 years and older.

SECTION II

Question 23

(a) (i) Justine's claim is incorrect because each friend bought a different number of tickets. The friends who have more tickets will have a higher chance of winning.

(ii) P(Khalid or Herman win) $= \dfrac{1}{20} + \dfrac{13}{50}$
$$= \frac{31}{100}.$$

P(not won by Khalid or Herman) $= 1 - \dfrac{31}{100}$
$$= \frac{69}{100}.$$

(b) (i) Volume of brick $= \ell b h$
$$= 21 \times 8 \times 9$$
$$= 1512 \text{ cm}^3.$$

(ii)

$r = 1 \cdot 4$ cm
$h = 8$ cm

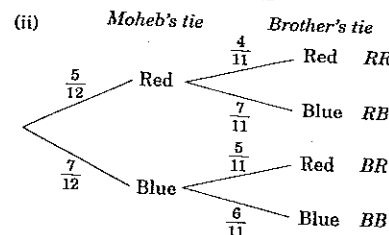Volume of cylinder
$= \pi r^2 h.$

Total volume of three holes
$= 3 \times \pi \times 1 \cdot 4^2 \times 8$
$= 147 \cdot 780\,518\,4 \ldots$

The volume of the remaining clay
$= 1512 - 147 \cdot 780\,518\,4 \ldots$
$= 1364 \cdot 219\,482 \ldots$
$\doteqdot 1364 \text{ cm}^3$ (nearest cm^3).

(iii) Percentage of clay removed
$$= \frac{\text{volume of clay removed}}{\text{volume of the brick}} \times 100\%$$
$$\doteqdot \frac{147 \cdot 7805 \ldots}{1512} \times 100\%$$
$$\doteqdot 9 \cdot 7738 \ldots$$
$$\doteqdot 9 \cdot 8\% \text{ (correct to 1 d.p.).}$$

(c) (i) P(choosing a red tie) $= \dfrac{5}{12}.$

(ii)

Moheb's tie	Brother's tie

$\dfrac{5}{12}$ Red $\dfrac{4}{11}$ Red RR

$\dfrac{7}{11}$ Blue RB

$\dfrac{7}{12}$ Blue $\dfrac{5}{11}$ Red BR

$\dfrac{6}{11}$ Blue BB

(iii) P(same colour) $= P(RR) + P(BB)$
$$= \frac{5}{12} \times \frac{4}{11} + \frac{7}{12} \times \frac{6}{11}$$
$$= \frac{31}{66}.$$

Question 24

(a) (i) Ordered stem-and-leaf plot.

Key: $2 \,|\, 1 = 21$

Stem	Leaf
1	9
2	1 3 7 7 9
3	4 5
4	5
5	8

(ii) Median $=$ average of the two middle scores
$$= \frac{27 + 29}{2}$$
$$= 28.$$

(iii) The scores are positively skewed as there are more scores at the lower end of the range (ie. the long tail points to the higher scores).

(b) (i) $D = \dfrac{2A}{15}$. In this case, $A = 9$.

∴ $D = \dfrac{2 \times 9}{15}$
$$= 1 \cdot 2 \text{ mL.}$$

(ii) In this case, $D = 4$, $A = ?$

∴ $4 = \dfrac{2A}{15}$
$$60 = 2A$$
$$A = 30.$$
∴ Sam is 30 months old.

The difference in ages $= 30 - 9$
$$= 21 \text{ months.}$$

(c) $T = 2\pi L^2$
$$\frac{T}{2\pi} = L^2$$
$$L = \pm \sqrt{\frac{T}{2\pi}}.$$

(d) (i) Eastern Suburbs $= 15\%$.

∴ 15% of population $= 24\,000$
1% of population $= 24\,000 \div 15$
$$= 1600.$$
∴ Total population (100%) $= 1600 \times 100$
$$= 160\,000.$$

(ii) Northern Suburbs $= 45\%$ of 160 000
$$= 0 \cdot 45 \times 160\,000$$
$$= 72\,000.$$

Northern Suburbs $= 30\%$ of 160 000
$$= 0 \cdot 3 \times 160\,000$$
$$= 48\,000.$$

Eastern Suburbs $= 24\,000$ (given).

Western Suburbs $= 10\%$ of 160 000
$$= 16\,000.$$

∴ The Western Suburbs column height is incorrect on the graph as it is displayed as 12 000.

Question 25

(a) (i) $X = 4\% \times \$5000 = \$200.$
$Y = \$30 \times 52 = \$1560.$
$Z = \$48 \times 12 = \$576.$

(ii) Total income for year $= \$4680 + \$200$
$$= \$4880.$$
Total expenses for year
$$= \$1560 + \$624 + \$576$$
$$= \$2760.$$
Savings $=$ total income $-$ total expenses
$$= \$4880 - \$2760$$
$$= \$2120.$$

Yes. Reece will have saved enough money for a deposit of $2100 on a car.

(b) (i)

$\triangle ABC$ is right-angled if
$$AB^2 = AC^2 + BC^2.$$
$$AB^2 = 13^2$$
$$= 169.$$
$$AC^2 + BC^2 = 12^2 + 5^2$$
$$= 144 + 25$$
$$= 169$$
$$= AB^2.$$
∴ $\triangle ABC$ is right-angled.

(ii) Let $\angle ABC = \theta$
$$\tan \theta = \frac{12}{5}$$
∴ $\theta = 67 \cdot 380\,135\,05 \ldots$
$$\doteqdot 67° \text{ (nearest degree).}$$

Note: The same result could be found using $\sin \theta = \dfrac{12}{13}$ or $\cos \theta = \dfrac{5}{13}$.

(c) (i) P(win $4) $= \dfrac{3}{5}.$

∴ Expected number of wins $= \dfrac{3}{5} \times 60$
$$= 36.$$

(ii) Financial expectation
$$= \frac{3}{5} \times \$4 + \frac{1}{5} \times 0 + \frac{1}{5} \times (-\$8)$$
$$= \$0 \cdot 80.$$

(iii) Robyn is not correct because there will be six cards to choose from which will change the probabilities and the financial expectation.

New financial expectation

$$= \frac{3}{6} \times \$4 + \frac{2}{6} \times 0 + \frac{1}{6} \times (-\$8)$$

$$= \$\left(\frac{2}{3}\right)$$

$$= \$0.67.$$

Question 26

(a) $S = V_0 (1-r)^n.$ $S < 120\,000$

                            $V_0 = 150\,000$

                            $r = 10\% = 0.1$

                            $n = ?$

$S = 150\,000(1 - 0.1)^n$

$\;\;= 150\,000(0.9)^n.$

Using 'Guess, check and improve':

$n = 1$ (ie. value in December 2006):

    $S = 150\,000 \times 0.9^1 = 135\,000.$

$n = 2$ (value in December 2007):

    $S = 150\,000 \times 0.9^2 = 121\,500.$

$n = 3$ (value in December 2008):

    $S = 150\,000 \times 0.9^3 = 109\,350.$

$\therefore$ The depreciated value of the machine will first fall below \$120 000 in 2008.

Note: These calculations can be done quickly on a calculator by continuing to multiply the value of the machine by 0.9.

(b) (i) From the table, an annuity of \$1 at 5% for 4 years has a future value of \$4.3101.

     $\therefore$ Future value of Rod's investment

        $= \$3600 \times 4.3101$

        $= \$15\,516.36.$

    (ii) Total payments over 4 years $= \$3600 \times 4$

                                  $= \$14\,400.$

     $\therefore$ Interest earned $= \$15\,516.36 - \$14\,400$

                               $= \$1116.36.$

(c) (i)   Mean $(\bar{x}) = 754$

      Standard deviation $(s) = 2$

      When $x = 754,$ $z = \dfrac{x - \bar{x}}{s}$

                             $= \dfrac{754 - 754}{2}$
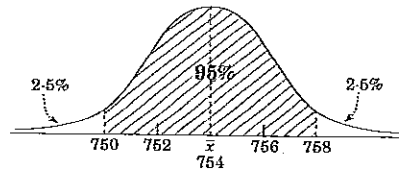
                             $= 0.$

    (ii) When $z = -1,$ $-1 = \dfrac{x - 754}{2}$

                           $-2 = x - 754$

                             $x = 752.$

     $\therefore$ Weight is 752 g.

    (iii) When $x = 750,$ $z = \dfrac{750 - 754}{2}$

                                $= -2.$

     $\therefore$ 750 g is 2 standard deviations below the mean.

For a normal distribution, 95% of the scores lie within 2 standard deviations of the mean.

$\therefore$ Percentage of boxes with weight less than 750 g $= 5\% \div 2$

                              $= 2.5\%.$

(d) (i) $28\,000 = M\left\{ \dfrac{(1+r)^n - 1}{r(1+r)^n} \right\}$

     $r = 6\%$ per annum

       $= \dfrac{6}{12}\%$ per month

       $= 0.5\%$ per month.

     $n = 9$ years

       $= (9 \times 12)$ months

       $= 108$ months.

     So the formula becomes

$$28\,000 = M\left\{ \dfrac{(1+0.005)^{108} - 1}{0.005(1+0.005)^{108}} \right\}.$$

    (ii) Need to find the value of $M.$

$$28\,000 = M\left\{ \dfrac{(1.005)^{108} - 1}{0.005 \times 1.005^{108}} \right\}$$

               $= M \times \dfrac{0.713\,699\,498\dots}{0.005 \times 1.005^{108}}$

    $28\,000 = M \times 83.293\,424\,46\dots$

         $M = \dfrac{28\,000}{83.293\,424\,46\dots}$

              $= 336.160\,9897\dots$

              $\doteqdot \$336.16.$

Question 27

(a) (i) Number of school shoes sold

      $\doteqdot 18\,000 - 3000$

      $= 15\,000.$

    (ii) School shoes and business shoes sales were equal in April (since the bands representing school shoes and business shoes are both the same height).

    (iii) Suggested possible answers are:

     • sales of school shoes are the highest in January and February, at the start of the school year; OR

     • sales of boots are the highest in July,

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     during the colder winter period; OR

     • sales of business shoes are constant through the year, as business needs do not change.

(b) Difference in latitude $= 35° + 8° = 43°.$

METHOD 1

Since an angular distance of 1° on a great circle equals 60 nautical miles,

     Distance $AB = 43 \times 60$

                    $= 2580$ nautical miles.
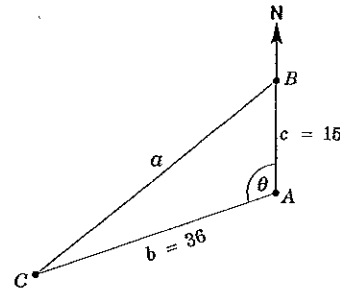
METHOD 2

Using the arc length of a circle formula

$$\ell = \frac{\theta}{360} \times 2\pi r,$$

Distance AB

$= \dfrac{43}{360} \times 2\pi \times 6400$ km

$= 4803.146\,101\dots$ km

$= 4803.146\,101\dots \div 1.852$ nautical miles

$= 2593.491\,415\dots$ nautical miles

$\doteqdot 2593$ nautical miles.

(c) (i) True north is 0° (or 360°).

     Reflex angle $\angle BAC = 250°$

     $\therefore$ $\theta = 360° - 250°$

                    $= 110°.$

    (ii)

Using the cosine rule:

$a^2 = b^2 + c^2 - 2ab\cos A$

$a^2 = 36^2 + 15^2 - 2 \times 36 \times 15 \times \cos 110°$

    $= 1890.381\,755\dots$

$a = \sqrt{1890.381\,755\dots}$

    $= 43.478\,520\,61\dots$

$\therefore$ Distance of C from B is 43 km (nearest km).

(d) (i) For *sample* standard deviation, use σ_{n-1} on a calculator. From the calculator,

     $\sigma_{n-1} = 1.691\,481\,928\dots$

           $\doteqdot 1.69$ (correct to 2 d.p.).

    (ii) Standard deviation measures the spread of scores about the mean.

Question 28

(a) (i) Area of rectangle $= x \times 2y = 2xy.$

     Area of 2 semicircles

        (each with radius y metres)

        $= 2 \times \dfrac{1}{2} \times \pi \times y^2$

        $= \pi y^2.$

     $\therefore$ Area of base $= 2xy + \pi y^2.$

    (ii) Total area $=$ (area of base)

                        $+$ (area of straight sides)

                        $+$ (area of curved ends).

     Area of base $= 2xy + \pi y^2$

                  $= 2 \times 6 \times 2.5 + \pi \times 2.5^2$

                  $= 49.634\,954\,08\dots$ m$^2.$

     Area of straight sides $= 2 \times (xh)$

                          $= 2 \times 6 \times 1.1$

                          $= 13.2$ m$^2.$

     Area of curved ends $= 2\pi rh$

                        $= 2 \times \pi \times 2.5 \times 1.1$

                        $= 17.278\,759\,59\dots$ m$^2.$

     Total area

     $= 49.634\,954\,08\dots + 13.2 + 17.278\,759\,59\dots$

     $= 80.113\,713\,68\dots$

     $\doteqdot 80$ m^2 (nearest m^2).

    (iii) Water savings $= 6 \text{ L} \times 5 \times 3 \times 365$

                       $= 32\,850$ L/year

                       $= (32\,850 \div 1000)$ kL/year

                       $= 32.85$ kL/year.

     $\therefore$ Money saved $= 32.85 \times \$1.013$

                     $= \$33.277\,05$

                     $\doteqdot \$33.28$ (nearest cent).

(b) (i) Cost (\$$C$) for running the dance for x people:

     $C = 12x + 400 + 300$

     $\therefore \;\; C = 12x + 700.$

    (ii) From the graph, minimum number of people is approximately 88.

     Note: This is found by reading from the graph the point of intersection of the 2 lines.

    (iii) From the graph, when $x = 150,$

     income $= \$3000$ (or $\$20 \times 150$).

     $\therefore \;\;\;\; C = 12x + 700$

                $= 12 \times 150 + 700$

                $= 2500.$

     Costs $= \$2500$ (can also be found from the graph).

     $\therefore$ Profit $= \$3000 - \$2500$

                     $= \$500.$

    (iv) *METHOD 1*

     Total cost for 200 people $= 12 \times 200 + 700$

                                $= \$3100.$

For profit to be $1500,

$$income = \$3100 + \$1500$$
$$= \$4600.$$
$$\therefore \text{Ticket price} = \$4600 \div 200$$
$$= \$23.$$

METHOD 2

From the graph, the current profit
for 200 people $= \$4000 - \$3100$
$$= \$900.$$

Need $600 more overall, so extra charge
per ticket $= \$600 \div 200$
$$= \$3.$$
$\therefore$ Ticket price $= \$20 + \$3$
$$= \$23.$$

METHOD 3

$$\text{Fixed costs per person} = \left(\frac{1500 + 700}{200} \right)$$
$$= \$11.$$

Refreshment cost per person $= \$12.$

$\therefore$ Total cost per person $= \$11 + \$12$
$$= \$23.$$

END OF GENERAL MATHEMATICS SOLUTIONS

2004 HIGHER SCHOOL CERTIFICATE EXAMINATION PAPER

GENERAL MATHEMATICS

Section I

22 marks
Attempt Questions 1 – 22
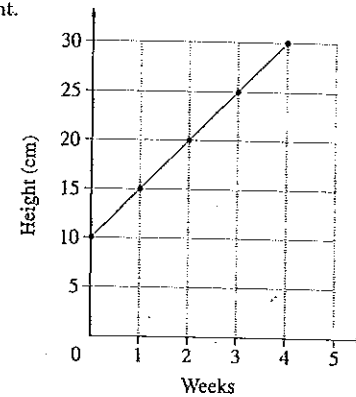**Allow about 30 minutes for this section**

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1 Which fraction is equal to a probability of 25%?

(A) $\dfrac{1}{25}$ (B) $\dfrac{1}{4}$ (C) $\dfrac{1}{3}$ (D) $\dfrac{1}{2}$

2 Susan drew a graph of the height of a plant.
What is the gradient of the line?

(A) 1
(B) 5
(C) 7·5
(D) 10

3 If $K = Ft^3$, $F = 5$ and $t = 0.715$, what is the value of K correct to three significant figures?
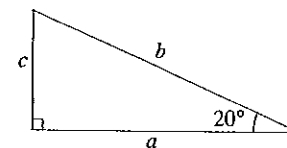
(A) 1·82   (B) 1·827   (C) 1·828   (D) 1·83

4 A real estate agent sells a house for $400 000. From the selling price he earns $10 000 for his services.
Which term is used to describe the money he earns?

(A) Commission   (B) Income tax   (C) Royalty   (D) Superannuation

5 What is the correct expression for $\tan 20°$ in this triangle?

(A) $\dfrac{a}{b}$

(B) $\dfrac{a}{c}$

(C) $\dfrac{c}{b}$

(D) $\dfrac{c}{a}$

NOT TO SCALE