Probability

TYPICAL EXAM QUESTIONS

Example (i): Two soldiers fire one round at the same target. Soldier A has a success rate of 75% and soldier B has a success rate of 60%. Find the probability that the target will be hit at least once.

Solution: P(soldier A hitting target) $= 0.75 = \frac{3}{4}$

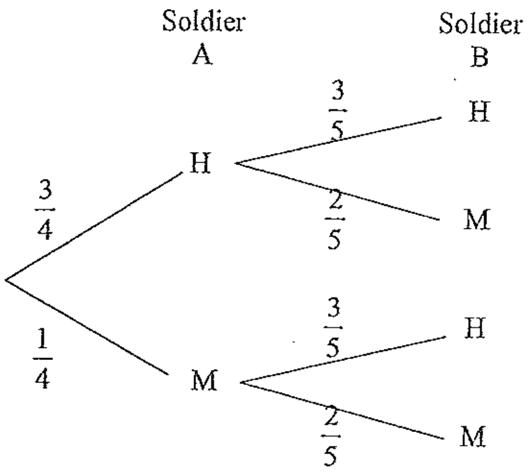
P(soldier B hitting target) $= 0.60 = \frac{3}{5}$

$\therefore$ P(soldier A missing) $= 0.25 = \frac{1}{4}$

$\therefore$ P(soldier B missing) $= 0.40 = \frac{2}{5}$

The problem can easily be solved with the aid of a PROBABILITY TREE.

Solution:



Sample space	Probability
HH	$\frac{3}{4}\times\frac{3}{5}=\frac{9}{20}$
HM	$\frac{3}{4}\times\frac{2}{5}=\frac{6}{20}$
MH	$\frac{1}{4}\times\frac{3}{5}=\frac{3}{20}$
MM	$\frac{1}{4}\times\frac{2}{5}=\frac{2}{20}$

$\therefore$ P(target will be hit at least once) $= \frac{9}{20}+\frac{6}{20}+\frac{3}{20}$

$= \frac{18}{20} = \frac{9}{10}$

ALTERNATIVELY:

P(target will be hit at least once) $= 1 -$ P(both soldiers miss)

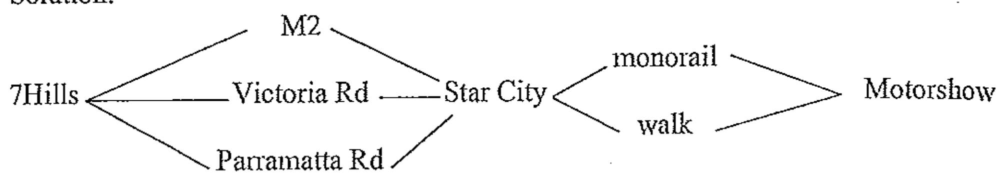
$= 1 - \frac{2}{20}$

$= \frac{9}{10}$

SYSTEMATIC COUNTING

Example (i): Darren and his mates live in Seven Hills and plan to visit the Motorshow. They plan to drive in and park at Star City Casino. There are 3 main roads they can use to get to Star City – the M2, Victoria Rd or Parramatta Rd. After parking, they can either catch the monorail to the Motorshow or walk. How many different routes can Darren and his mates take between Seven Hills and the Motorshow?

Solution:



On leaving Seven Hills, there are 3 alternatives to get to Star City.
Once at Star City, there are 2 alternatives to get to the Motorshow.
Using the **fundamental counting principle** there are:

$3 \times 2 = 6$ different routes.

Example (ii): For theme day, Claire intends to dress in 70's dress. Claire has 3 shirts, 4 pairs of pants, 2 hats and 2 pairs of shoes that would be suitable. How many different sets of "70's clothes" can Claire make?

Solution: A tree diagram could be used, but would be rather large with 4 sets of branches. Alternatively, Claire could multiply the number of choices for each item of clothing required.

Shirts		pants		hats		shoes		
3	×	4	×	2	×	2	=	48

Therefore, Claire has 48 different sets of 70's clothes.

Example (iii): The school canteen stocks the following items: 4 different fruits, 6 different sandwiches, 5 different drinks and 4 different snacks. Each school day Rebecca orders 1 piece of fruit, 1 sandwich, 1 drink and 1 snack. How many different selections can Rebecca make before she repeats a selection?

Solution:

fruit		sandwich		drink		snack		
4	×	6	×	5	×	4	=	480

∴ Rebecca can make 480 different selections.

PERMUTATIONS AS ORDERED ARRANGEMENTS

Example (i): 5 students: Mark, Alex, Nick, Dean and Stuart are competing for the positions of house captain and vice captain.
In how many ways can the two positions be filled?

Solution:

captain		vice captain		
5	×	4	=	20 possibilities

NOTE: The arrangement Mark (captain) and Alex (vice captain) is different from Alex (captain) and Mark (vice captain).

> When **order is important,** the number of possible arrangements is called a PERMUTATION.

Example (ii): Daniel has 9 cards, numbered 1, 2, 3, 4, 5, 6, 7, 8 and 9. How many different 3-digit numbers can he form by taking 3 cards at a time?

Solution:

1^{st} digit		2^{nd} digit		3^{rd} digit		
9 ways	×	8 ways	×	7 ways	=	504

So, there are 504 different 3-digit numbers that can be formed.

A common application of a permutation is selecting a 'trifecta' in horse racing.
A trifecta involves selecting the first, second and third horse in that order.

Example (iii): In a twelve horse race, if each trifecta is equally likely, what is the probability of picking the winning trifecta?

Solution: First place can be filled by any of the 12 horses.

Second place can be filled by any of the remaining 11 horses.

Third place can be filled by any of the remaining 10 horses.

$\therefore$ Number of trifectas $= 12 \times 11 \times 10 = 1320$

$$\text{Probability (picking the winning trifecta)} = \frac{1}{1320}$$

COMBINATIONS – ORDER IS NOT IMPORTANT

Consider the number of different ways of choosing two letters from the three letters A, B, C **if order is not important**. It is easy to see that there are only 3 such possible arrangements, i.e. AB, AC and BC.

Note that the arrangement AB is the same as BA because order is not important.

> When the order in which items are arranged **is not** important, it is called a COMBINATION.

There are many instances when order of selection plays no part.
For example:

- Selecting a committee of 4 from a group of 12 people;
- Selecting a *quinella* (choosing the first two horses past the post in a race, in either order);
- Selecting the correct group of numbers in lotto.

To work out the number of combinations:

> Find the number of ordered selections and then divide by the number of rearrangements possible within the group.

The [^nC_r] key on a calculator can also be used.

Example (i): In how many ways can a committee of two be selected from a group of 5?

Solution: Number of ways to choose a pair = $5 \times 4 = 20$

However order is not important, so divide by the number of ways of arranging the two people (= 2)

$\therefore$ the number of ways of selecting a committee of two people from five is: $\frac{5\times4}{2}=10$

or using a calculator with a [^nC_r] key: [5] [^nC_r] [2] [=] 10

FURTHER EXAMPLES ON COMBINATIONS

Example (ii): In how many ways can a team of 5 basketball players be selected from a squad of 8 players?

Solution: This question involves COMBINATIONS because it makes no difference to the team the order in which the 5 players are selected.

$$\therefore \quad \text{number of combinations} = \frac{8\times7\times6\times5\times4}{5\times4\times3\times2\times1} = 56$$

or on a calculator: [8] [^nC_r] [5] [=] 56

Example (iii): In the gambling game of Ozlotto the punter must choose 6 numbers out of a possible 45 numbers. Calculate the number of combinations of 6 numbers and the probability of selecting the correct combination.

Solution: $\therefore$ number of combinations $= \frac{45\times44\times43\times42\times41\times40}{6\times5\times4\times3\times2\times1} = 8{,}145{,}060$

$$\text{P(selecting correct 6 numbers)} = \frac{1}{8145060}$$

QUINELLA – choosing the first 2 placegetters in a horse race in any order. To find out how many possible quinella combinations in a race involving n horses, it is necessary to calculate $^nC_2$.

Example (iv): If 10 horses are in a race, how many possible quinellas are there? If all the horses have the same chance of winning, what is the probability of picking the correct quinella?

Solution: number of quinellas $= \frac{10\times9}{2\times1} = 45$ or $^{10}C_2 = 45$

Probability of picking the correct quinella $= \frac{1}{45}$.

TWO-WAY TABLES AND TEST RESULTS

Two-way tables are used to compare statistics and probabilities of different categories.

Example: As part of the road safety campaign, a number of road users were tested for drugs in their system. The results were presented to the committee in the following two-way table.

the test correctly identified 16 people had drugs in their system

the test said 3 drug users had no drugs present

19 people had drugs in their system when tested

	Accurate	Not Accurate	Totals
Number of road users with drugs	16	3	$16 + 3 = 19$
Number of road users with no drugs	224	7	$224 + 7 = 231$
	240	10	

the test correctly identified 224 people with no drugs in their system

the test was accurate for 240 people

the test was inaccurate for 10 people

231 people had no drugs in their system

the test said 7 non-drug users had drugs in their system when tested

NOTE: There are 3 ways to calculate the number of people tested:

(i) Add the 4 numbers in the centre of the table $(16 + 3 + 224 + 7)$ to get 250.

(ii) Add the total results for accurate and not accurate $(240 + 10)$ to get 250.

(iii) Add the total results for people with drugs to the total number of people without drugs $(19 + 231)$ to get 250.

(a) What percentage of road users had drugs in their system?

(b) What percent of test results are accurate?

(c) What is the probability that a drug user was tested accurately?

(d) What is the probability that a non-drug user was tested inaccurately?

(e) What is the probability that a person chosen at random has drugs in their system?

Solution: (a) $$\% \text{ road users with drugs} = \frac{\text{number of drug users}}{\text{number of people tested}} \times 100\%$$

$$= \frac{19}{250} \times 100\% = 7.6\%$$

(b) $$\% \text{ test results accurate} = \frac{\text{number of accurate results}}{\text{number of people tested}} \times 100\%$$

$$= \frac{240}{250} \times 100\% = 96\%$$

(c) $$P(\text{drug user test accurate}) = \frac{\text{no.of accurate tests on drug users}}{\text{number of drug users}}$$

$$= \frac{16}{19}$$

(d) P(non-drug user tested inaccurately)

$$= \frac{\text{no.of inaccurate tests on non drug users}}{\text{number of non drug users}}$$

$$= \frac{7}{231}$$

(e) $$P(\text{person has drugs in system}) = \frac{\text{number of drug users}}{\text{number of people tested}}$$

$$= \frac{19}{250}$$

$$= 0.076$$

REVIEW EXERCISE – LEVEL 1

1. A die has faces numbered 1, 1, 2, 3, 4 and 5. When the die is rolled, what is the probability of getting:

   (a) a 4 (b) a 1 (c) an odd number

2. In a bag, there are 5 red, 2 blue and 5 white balls. One ball is drawn at random. What is the probability that the ball is:

   (a) red (b) red or blue (c) not red

3. Two regular dice are rolled, find the probability that:

   (a) the dice have the same number on them.

   (b) the total on the dice is 12.

   (c) the total is less than 7.

4. One card is drawn from a normal deck of cards. It is replaced, then another is drawn. What is the probability that:

   (a) both cards are red (b) the cards are different in colour

   [Hint: there are really only two outcomes here – red and black]

5. A drawer contains 8 black socks and 8 white socks. Two socks are pulled out at random.

   (a) What is the probability that they are the same colour?

   (b) How many socks would need to be pulled out to ensure a matching pair?

6. A bag contains twelve jelly beans. Three are red, four are black and 5 are green. Peter eats three jelly beans chosen randomly from the bag. What is the probability that:

   (a) the first jelly bean is black?

   (b) all three jelly beans are black?

   (c) exactly one of the jelly beans is black?

7. In a new game called MULTO, one normal die with faces numbered 1, 2, 3, 4, 5 and 6 and another die with faces numbered 0, 1, 2, 3, 4 and 4 are rolled. The product of the uppermost numbers is then found.

   (a) Draw up a table of possibilities.

   (b) What is the probability of obtaining: (i) zero (ii) an odd product?

   (c) In sixty trials, what is the expected number of eights?

REVIEW EXERCISE – LEVEL 2

1. In a family of 4 children what is the probability of:

   (a) 4 sons? (b) 3 sons and 1 daughter? (c) at least 1 daughter?

2. A card is drawn from a normal deck of cards. What is the probability that:

   (a) a red card is drawn?

   (b) an even numbered card (2, 4, 6, 8, 10) is drawn?

   (c) a red, even numbered card is drawn?

   (d) a red or an even numbered card is drawn?

3. Gwen buys 3 tickets in a raffle in which 500 tickets are sold. What is the probability of her winning:

   (a) 1^{st} prize? (b) 1^{st} and 3^{rd} prizes? (c) no prize?

   (d) all 3 prizes? (e) at least 1 prize?

4. There are two sets of cards. The first set has cards numbered 1, 2, 3 and 4. The second set has cards numbered 4, 6, 7 and 8. A card is drawn from the first set, then a card is drawn from the second set and placed to the right of the first to form a two-digit number. Find the probability that the number is:

   (a) odd (b) divisible by 3.

5. A three digit number is formed from the digits 3, 4, 5 and 6. If each digit can only be used once, what is the probability of the number being:

   (a) even? (b) divisible by 5? (c) greater than 500?

6. A bag contains 5 red and 10 green discs. One disc is drawn out, replaced, and then another disc is drawn. Find the probability that:

   (a) the discs are the same colour

   (b) the first disc is red and the second one is green.

7. Repeat Question 6, but the first disc is not replaced.

8. There are 8 runners in a race. In how many different ways can the first four places be filled?

9. When putting on the green, Ima Wonder has a 48% chance of making her first putt. Each putt after that has a 95% chance of being successful.

   (a) What is the probability of two putting a green?

   (b) What is the probability she three putts the green?

   (c) In a 72-hole tournament will Ima have more one-putt greens than two-putt greens? Justify your answer.

10. To win in Lotto, 6 numbers out of 44 must be correctly chosen.

    (a) How many different combinations are possible?

    (b) A System 8 entry allows gamblers to choose 8 numbers, any of which can be used to form an entry.

       (i) How many different combinations of 6 from 8 numbers are there?

       (ii) If a single Lotto entry costs 50 cents, how much should a Systems 8 entry cost?

11. From an SRC of 12 members, 2 from each of Years 7 to 12, a committee of 4 must be chosen.

    (a) In how many different ways can this be done?

    (b) If the school captain (from Year 12) must be on the committee and the two Year 7 students are not considered for the committee,

       (i) how many different committees are possible?

       (ii) What is the probability that John, a Year 7 student, is on the committee?

       (iii) What is the probability that Jane, a Year 9 student, is on the committee?

12. A horse race has 16 starters.

    (a) What is the probability of selecting the quinella (first two placegetters, order not important)?

    (b) What is the probability of selecting the trifecta (first three placegetters, order important)?

13. In a dice game, if you throw a 1 or a 2, you win $2. If you throw a 3 or a 4, you win $3. Otherwise you lose.

    (a) Over a large number of games, what is the expected return per game?

    (b) If it costs $2 per game to play, is it worth playing?

