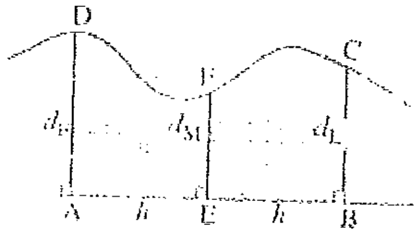
Simpsons Rule; Land and Time measurement

SIMPSON'S RULE

This method is used by surveyors to find the area between a curved boundary and a straight line.

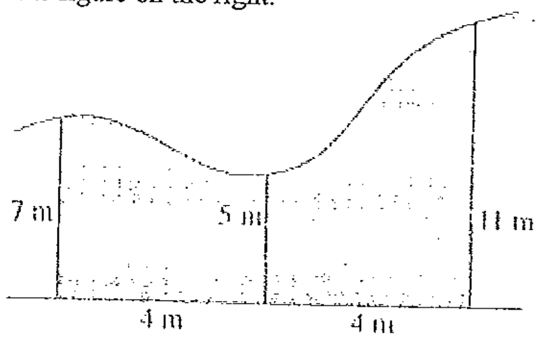


Let the vertical distances AD, EF and CB be d_F, d_M and d_L respectively. Let the lengths of the two equal divisions on the base line AB be h units.

Then by Simpson's Rule, the approximate shaded area enclosed by the region AEBCFD is given by:

$$\text{Area} = \frac{h}{3}\left(d_F + 4d_M + d_L\right)$$

Example: Find the area shaded in the figure on the right.



Solution:

$$\text{Area} = \frac{h}{3}\left(d_F + 4d_M + d_L\right)$$

$$= \frac{4}{3}\left(7 + 4 \times 5 + 11\right)$$

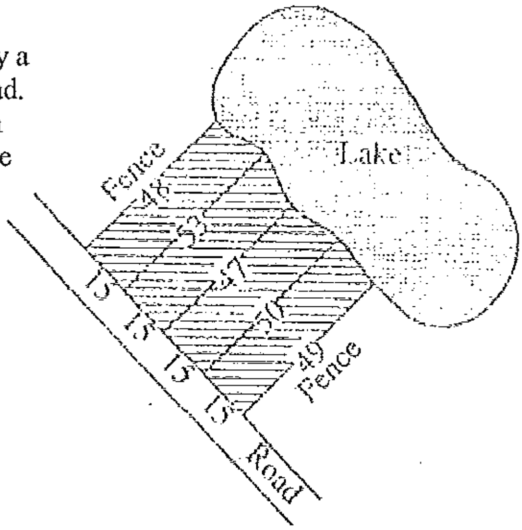
$$= 50.67 \text{ m}^2$$

The shaded area is approximately 50.67 m^2.

USING SIMPSON'S RULE TWICE

It often occurs that Simpson's Rule has to be applied more than once to find the area of a particular region. In fact, the more times it is applied, the more accurate the answer to the area becomes. It will be obvious in an exam question, how many times you should use it. Apply Simpson's Rule once when given 3 vertical lengths, and twice when given 5 vertical lengths.

Example: The field survey diagram shows an area bounded by a lake, two fences and a road. If all measurements are in metres, find the area of the region.



Solution: Using $A = \frac{h}{3}(d_F + 4d_M + d_L)$

$$A_1 = \frac{15}{3}(48 + 4\times 53 + 47)$$

$$= 1535 \text{ m}^2$$

$$A_2 = \frac{15}{3}(47 + 4\times 50 + 49)$$

$$= 1480 \text{ m}^2$$

Total area = A1 + A2

= 1535 + 1480

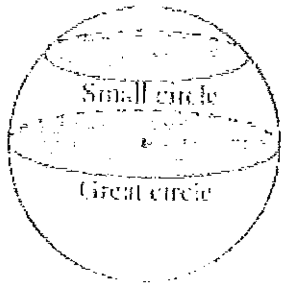
= 3015 m²

NOTE: The vertical length of 47 is used twice – d_L in first area calculation, and d_F in second area calculation.

THE EARTH

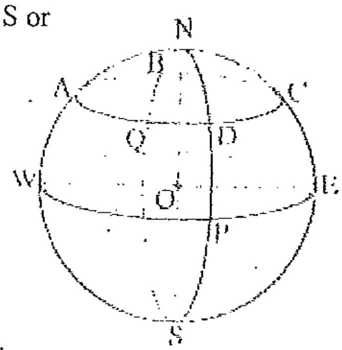
For this section of work, we shall consider the earth to be a perfect sphere, although in reality it is slightly more egg shaped. Firstly, it is important to understand the meaning of the following words:

GREAT CIRCLE - formed by any plane which passes through the centre of the earth & cuts the surface.



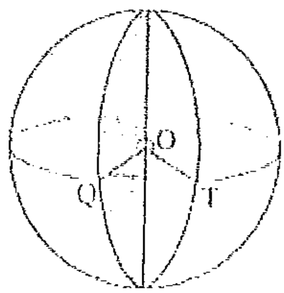
SMALL CIRCLE - formed by any plane cutting the surface which doesn't pass through the centre.

ANTIPODAL POINTS - the end points of any diameter on a great circle, i.e. N & S or W & E.



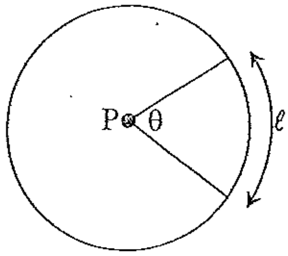
AXIS AND POLES - NOS represents the axis of the earth where N and S are the North and South poles respectively. WQEP represents the equator.

ANGLE BETWEEN TWO GREAT CIRCLES - this is the angle between the intersection of their planes. i.e. QOT in diagram.



ARC LENGTH OF A CIRCLE

The circumference (C) of a circle of radius (r) is given by the well known formula $C = 2\pi r$. Since there are 360° in a revolution, an angle of θ° at the centre of the circle will subtend a fraction $\frac{\theta}{360}$ of the circumference $2\pi r$.

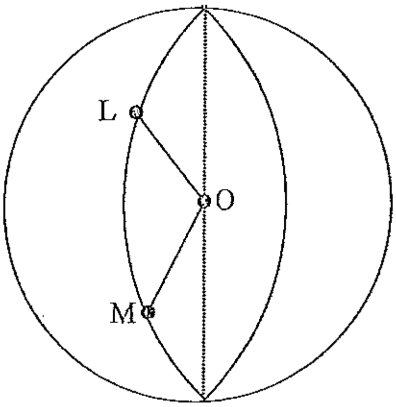


$$\ell = \frac{\theta}{360} \times 2\pi r$$

where ℓ = arc length
θ = angle at the centre

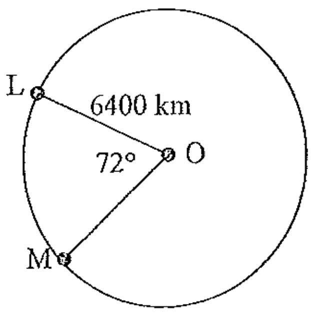
This formula is often used to find the distance between two points on the surface of the Earth.

Example:



If we consider the earth to be a sphere of radius 6400 km, find the spherical distance between two points L and M on the surface of the Earth which is subtended by an angle of 72° at the centre.

Solution:



The great circle containing the arc LM is more clearly shown in two dimensions

Using $\ell = \frac{\theta}{360} \times 2\pi r$

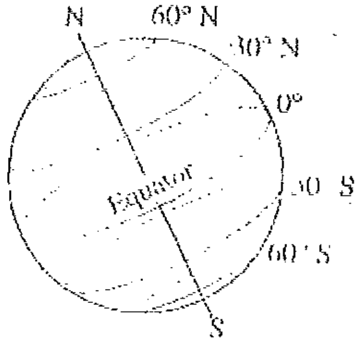
$$LM = \frac{72}{360} \times 2 \times \pi \times 6400$$

$$= 8042.48 \text{ km}$$

NOTE: In most exercises, the radius of the earth is usually given as 6400 km.

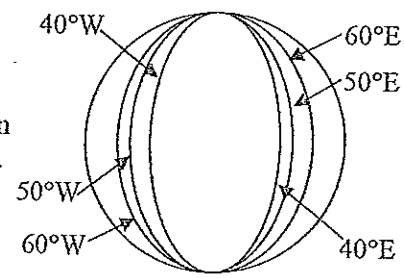
MAPPING POSITIONS ON THE EARTH

Parallels of latitude are the horizontal circles running East and West around the Earth. The equator is the only parallel of latitude which is a great circle. All the other parallels have an ever decreasing radius as they get further from the equator - going to a maximum of 90°N and maximum of 90° S.



> The latitude of a point is an angular measurement of its position north or south of the equator.

Meridians of longitude are the vertical circles running North and South around the Earth. The zero reference line of longitude, called the Greenwich meridian passes through Greenwich in London - also often called the '**prime meridian**'. All meridians lie ongreat circles and vary from 180° E of Greenwich to 180° W of Greenwich.



> The longitude of a point is an angular measurement of its position east or west of the Greenwich meridian.

If you recall, the position of any point on a number plane can be determined by an ordered pair (x, y), with the x coordinate always given first, and the y coordinate second.
Similarly, the position of any point on the earth's surface can be determined by the intersection of the circles of latitude and longitude. By convention, latitude is always given first and longitude second.

> Position on earth = (latitude, longitude)

EXAMPLES ON FINDING POSITIONS

Example: In the given figure, NABS is the Greenwich meridian.

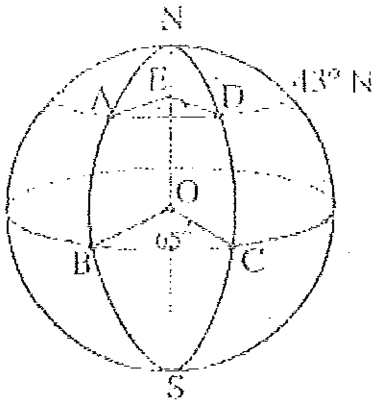
(a) What is the longitude of point C?

(b) What is the longitude of point D?

(c) Give the position coordinates of D.

(d) Give the position coordinates of A, B and C.

(e) Find the spherical distance between B and C.



Solutions:

(a) Longitude of point C is 65° E

(b) This point lies on same line of longitude as C, i.e. 65° E.

(c) Latitude of D is 43° N

Longitude of D is 65° E

∴ Position coordinates of D are (43°N, 65° E)

(d) Latitude of A is 43° N

Longitude of A is 0°

∴ Position coordinates are (43" N, 0°)

Latitude of B is 0°

Longitude of B is 0°

∴ Position coordinates of B are (0°, 0°)

Latitude of C is 0°

Longitude of C is 65° E

∴ Position coordinates of C are (0°, 65° E)

(e) BC lies on the equator which is a great circle.

Using $\ell = \frac{\theta}{360} \times 2\pi r$

$$BC = \frac{65}{360} \times 2 \times \pi \times 6400$$

$= 7260.57$ km

GREAT CIRCLE DISTANCES USING NAUTICAL MILES

The nautical mile, defined as 1852 m, has been agreed upon internationally as the unit of measurement to be used in navigation. Speeds in navigation are measured in knots where, **1 knot** is equivalent to a speed of **1 nautical mile per hour**.

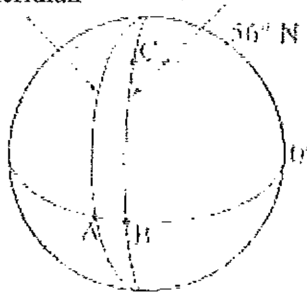
An important fact to memorise:

> An angle of 1 degree at the centre of the earth subtends an arc length of 60 nautical miles on a great circle.

Example: Using the information in the figure find:

(a) distance BC along the meridian

(b) distance AF along the equator



Solution:

(a) The arc BC on the great circle subtends an angle of 56° at the centre of the Earth.

Since 1° subtends an arc length of 60 n.m.

∴ 56° subtends an arc length of 56 x 60 n.m.

∴ BC = 3360 nautical miles

(b) The arc AB on the great circle subtends an angle of 14° at the centre of the earth.

$$\therefore \text{ Length of AB} = 14 \times 60\text{n.m.}$$
$$= 840 \text{ n.m.}$$

TIME

The Earth rotates from West to East 15° every hour. Therefore people living say 30°E of Greenwich will experience noon two hours earlier than the Greenwich time. It is important to be able to calculate times at different meridians around the world.

REMEMBER THE FOLLOWING:

1° longitude difference = 4 minutes time difference
Time in the **east** of Greenwich is least.

Example: If it is 2 p.m. at Greenwich, what is the local time in:

(a) Baghdad 45°E

(b) Los Angeles 120°W

Solution: (a)

$$1° \text{ longitude difference} = 4 \text{ minutes time difference}$$

$$45° \text{ longitude difference} = 45 \times 4 \text{ minutes time difference}$$

$$= 180 \text{ minutes}$$

$$= 3 \text{ hours}$$

Time in the east of Greenwich is least

∴ Time in Baghdad is 5 p.m.

i.e. 3 hours ahead of Greenwich time.

(b)

$$120° \text{ longitude difference} = 120 \times 4 \text{ minutes time difference}$$

$$= 480 \text{ minutes}$$

$$= 8 \text{ hours.}$$

Because Los Angeles is to the west of Greenwich, the time will be 8 hours behind.

i.e. Time in Los Angeles is 6 a.m.

NOTE: Think of time changes as a number line problem. To move east from one place to another, add the time difference. To move *west* from one place to another, ***subtract*** the time difference.

TIME ZONES AND STANDARD TIME

Even though local time varies by one hour for every 15° of longitude, in practice this would prove very confusing in places which are quite close together. For example, there could be time differences of up to 40 minutes in New South Wales alone. To prevent these problems, the Earth is divided into Standard Time Zones, and places within each zone use the same time. This is called STANDARD TIME.

In Australia, there are 3 times zones. The eastern states use time based on the 150°E meridian, called **Eastern Standard Time**. Adelaide is in the **Central Standard Time** zone which is 30 minutes behind Eastern Standard Time and is based on the 142.5°E meridian. Perth is in the **Western Standard Time** zone which is 2 hours behind Eastern Standard Time and based on the 120°E meridian.

Western Standard Time

Central Standard Time

Eastern Standard Time

NOTE: Broken Hill, in NSW, is an exception to these zones. It is isolated from population centres in NSW and is closer to towns in South Australia.

Example (i): A phone call is made at 5:45 p.m.(EST) from Sydney to Perth. What time would it be in Perth?

Solution: Perth is in the Western Standard Time zone which is 2 hours behind EST. i.e. subtract 2 hours

∴ Time in Perth is 2 hours before 5:45 p.m. i.e. 3:45 p.m.

Example (ii): A traveller leaves Sydney Airport at 11:15 a.m. After flying for 2½ hours, he lands in Adelaide. What is the time on the Adelaide Airport clock when he arrives?

Solution: The time on the traveller's watch when he arrives in Adelaide is

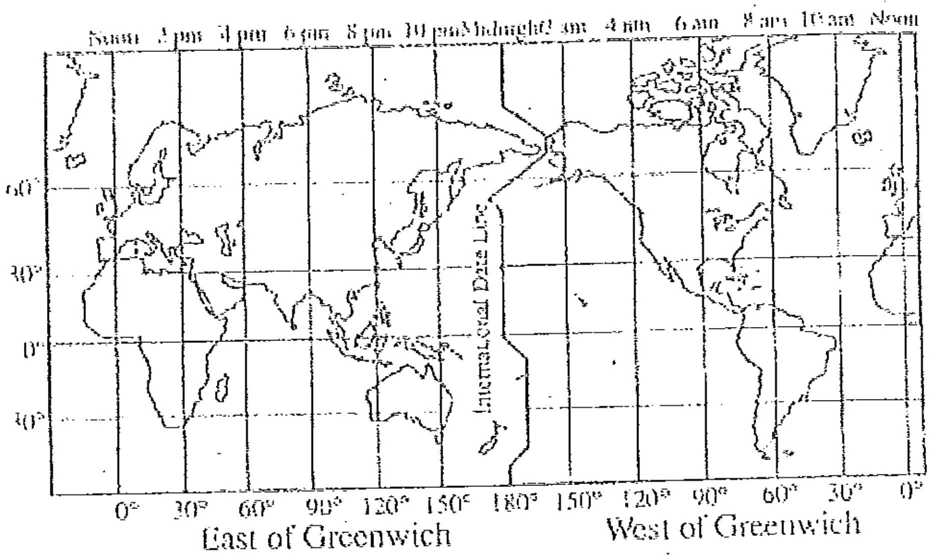
11:15 a.m. + 2h 30min = 13:45 = 1:45 p.m.

Adelaide is 30 minutes behind Sydney; need to subtract 30 minutes

∴ Time on Adelaide clock is 1:15

THE INTERNATIONAL DATE LINE

The International Date Line coincides closely with the 180° meridian. It is 'bent' to avoid confusion where it cuts across land masses.
When moving **easterly** across the International Date Line, the date is put **back** one day. When crossing the date line from the west, the date is put forward one day.



Example: New York standard time is based on a longitude of 75° W. If it is 7 a.m. in New York, what is the standard time in:
(a) London (b) Sydney?

Solution: (a) London is longitude of 0° (Greenwich meridian) and lies 75° to the east of New York.

$\therefore$ Time in London = 7 a.m. + (75 × 4) minutes
= 12 p.m.

(b) Sydney is on a latitude of 150° E of Greenwich.

$\therefore$ Angle difference = 15° + 150° = 225°

$\therefore$ Time difference = (225 × 4) minutes
= 15 hours ahead

$\therefore$ Time in Sydney = 10.00 p.m. at night.

REVIEW EXERCISE

1. A surveyor's notebook contained the following entry for a traverse survey of a field (at right). All measurements are in metres.

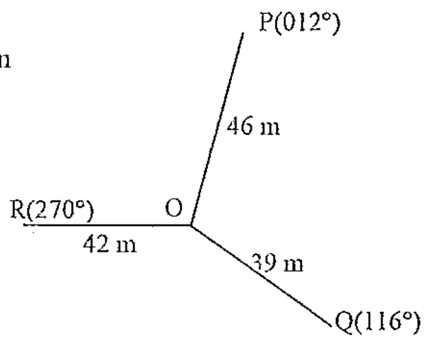
	B	
	84	
	67	17
22	41	
19	23	
	0	
	A	

   (a) Make an accurate scale drawing of the field. Use 1 mm = 0.5 m.

   (b) From your drawing find the perimeter of the field to the nearest 10 metres.

   (c) Calculate the area of the field to the nearest m^2.

2. A radial survey of a tract of land is shown (not to scale)

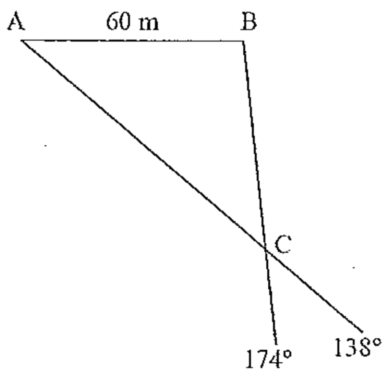


   (a) Find the size of $\angle POR$.

   (b) Find the area of ΔPOR to the nearest square metre.

   (c) Use the cosine rule to find the length of PR to the nearest metre.

3. AB is the baseline for a triangulation used to find the position of point C. AB is 60 metres long and runs East-West.

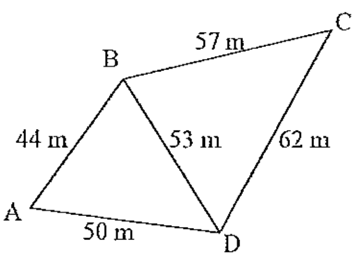


   (a) Find the angles in the triangle.

   (b) Use the sine rule to find distance BC.

   (c) What is the area of the triangle?

4. In a survey of a four-sided field, a student makes measurements as shown in the diagram at right. (Not drawn to scale.)



   (a) Make a scale diagram of the field, using a scale of 1 mm = 0.5 m.

   (b) Measure $\angle BCD$ and hence find the area of ΔBCD.

   (c) Use a similar method to find the area of ΔABD and hence the area of the field.

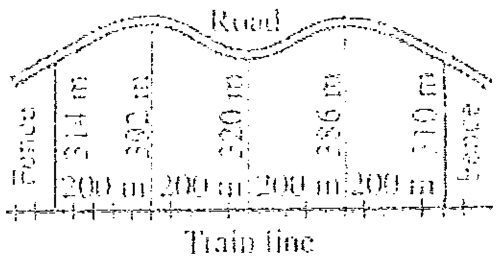
5. A geographer makes the following depth measurements across a creek. All measurements are in metres.

Distance across	0	4	8	12	16
Depth	0	2.2	2.1	1.7	0

(a) Use Simpson's Rule twice to estimate the cross-sectional area of the creek.

(b) The water in the creek at this point flows at a speed of 1.8 km/h. Approximately what volume of water flows past this point in a day? (Give answer to the nearest 100 m^3.)

6. The boundaries of the piece of land shown are road, the fences and a train line.



Find the area of the land in hectares.

7. In the following take the radius of earth as 6400 km.

(a) Port Villa (17.6°N, 168°E) is the capital of Vanuatu. The capital of Zimbabwe is Harare (17.6°N, 32°E).

(i) How far apart (in kiolmetres) are Port Villa and Harare?

(ii) Ignoring time zones, if it is 8 a.m. on Monday morning in Port Villa, write down the time and day in Harare.

(b) A plane leaves Cairo (30°N, 31°E) and flies due south to Durban (30.5°S, 31°E).

(i) Calculate the distance the plane flies.

(ii) If the plane flies at an average ground speed of 600 km/h, how long is the flight (to the nearest hour)?

(iii) The plane arrives in Durban at 6 a.m. on Thursday, February 13. Give the time and date when it left Cairo.

(c) New Caledonia is 11 hours ahead of Greenwich Mean Time. Venezuela is 4 hours behind GMT. If it is 9 a.m. on Saturday in New Caledonia, find the time and day in Venezuela.