

Topic Test: Applications of Trigonometry

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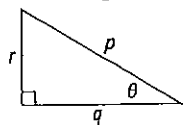
Remember: these are HSC-type questions.

Time allowed: 40 minutes Total marks: 25

Part A (Suggested time: 15 minutes)

Choose the correct answer (A, B, C or D) for each question. One mark each

1 Referring to the diagram, which statement is correct?



A $\sin \theta = \frac{p}{r}$

B $\cos \theta = \frac{r}{p}$

C $\tan \theta = \frac{q}{r}$

D none of these

2 $\cos \theta = ?$

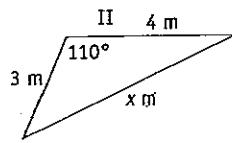
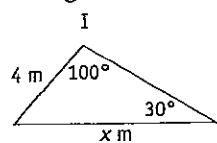
A $\frac{1}{7}$

B $\frac{5}{8}$

C $\frac{1}{2}$

D $\frac{11}{14}$

3 In order to find the value of x in each of these triangles, which rule should be used?



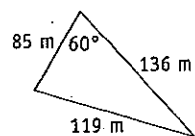
A sine rule in both

B sine rule in I, cosine rule in II

C cosine rule in I, sine rule in II

D cosine rule in both

4 Find the area of the paddock to the nearest square metre.



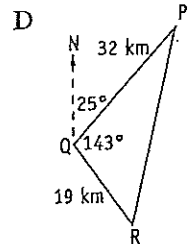
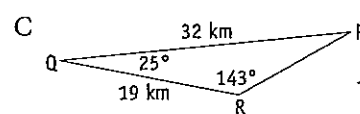
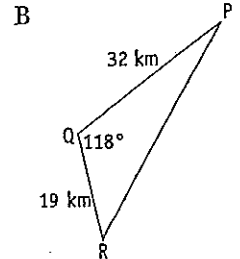
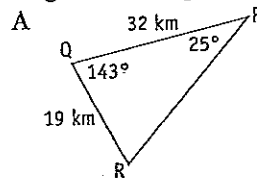
A 2890 m^2

B 7008 m^2

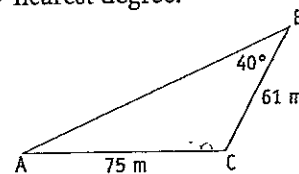
C 5006 m^2

D 4380 m^2

5 Town P is 32 km from Q on a bearing of 025° . Town R is 19 km from Q on a bearing of 143° . Which diagram best represents this information?



6 Use the sine rule to find the size of angle A to the nearest degree.



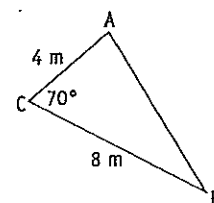
A 54°

B 52°

C 32°

D 27°

7 Use the cosine rule to find the length of side AB. Give the answer correct to one decimal place.



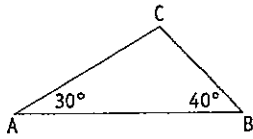
A 7.6 m

B 5.5 m

C 10.1 m

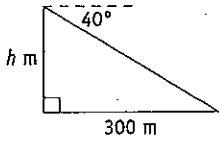
D 2.3 m

- 8** B is due east of A. $\angle CAB = 30^\circ$ and $\angle CBA = 40^\circ$. What is the bearing of B from C?



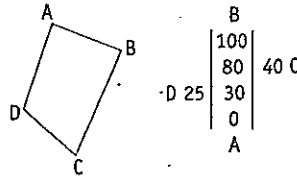
- A 110° B 120°
C 130° D 140°

- 9** From the top of a building, the angle of depression of an object on the ground, 300 metres from the base of the building on level ground, is 40° . Which expression will find the height (h m) of the building?



- A $300 \tan 40^\circ$ B $300 \tan 50^\circ$
C $\frac{300}{\tan 40^\circ}$ D $\frac{\tan 50^\circ}{300}$

- 10** A block of land has been surveyed and a rough sketch and notebook entries are given. What is the area of the land?

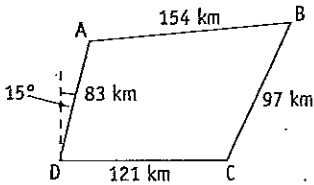


- A 1975 m^2 B 3250 m^2
C 3950 m^2 D 5225 m^2

Part B (Suggested time: 25 minutes)

Show all working. 15 marks

- 11** A, B, C and D are four towns. A is 83 km from D on a bearing of 015° and D is 121 km due west of C. B is 154 km from A and 97 km from C.



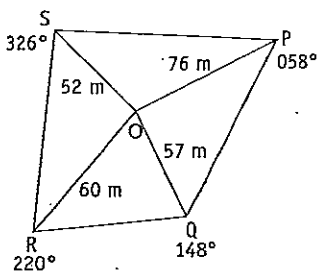
- a Find the size of $\angle ADC$. 1 mark
b Use the cosine rule in $\triangle ADC$ to find the distance from A to C to the nearest kilometre. 2 marks
c Find the size of $\angle ABC$ to the nearest degree. 2 marks

- c Find the size of $\angle SOP$. 1 mark
d Find the area of $\triangle SOP$. 1 mark

- 12** A plane leaves A and flies on a bearing of 125° to B, a distance of 320 km. From B it flies on a bearing of 205° to C, which is due south of A.

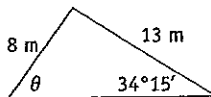
- a Draw a diagram showing the path of the plane. 1 mark
b What is the size of angle ABC? 1 mark
c Find the distance from B to C to the nearest kilometre. 2 marks

- 13** A compass radial survey has been completed of a block of land.



- a Explain why $\triangle OPQ$ is a right-angled triangle. 1 mark
b Find the length of PQ. 1 mark

- 14** Find θ to the nearest minute.

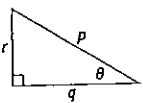


2 marks

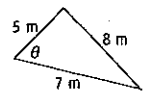
Solutions

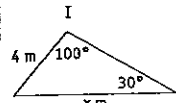
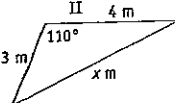
Topic Test p161

1. $\sin \theta = \frac{r}{p}$
 $\cos \theta = \frac{q}{p}$
 $\tan \theta = \frac{r}{q}$
 None are correct. D

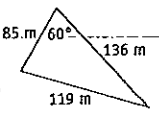


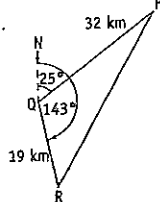
2. $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$
 $\cos \theta = \frac{5^2 + 7^2 - 8^2}{2 \times 5 \times 7}$
 $= \frac{10}{70}$
 $= \frac{1}{7}$ A

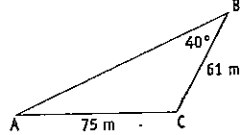


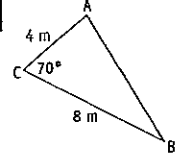
3.  
 sine rule in I; cosine rule in II B

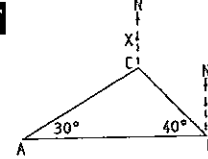
4. $A = \frac{1}{2} ab \sin C$
 $= \frac{1}{2} \times 85 \times 136 \times \sin 60^\circ$
 $= 5005.626\ 834 \dots$
 $= 5006$ (nearest unit)
 The area of the triangle is 5006 m^2 ,
 to the nearest square metre. C

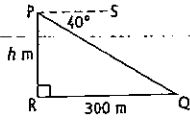


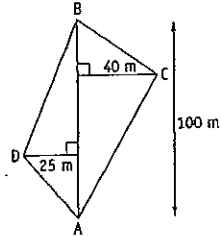
5. 
 $\angle PQR = 143^\circ - 25^\circ$
 $= 118^\circ$ B

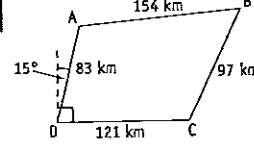
6. 
 $\frac{\sin A}{a} = \frac{\sin B}{b}$
 $\frac{\sin A}{61} = \frac{\sin 40^\circ}{75}$
 $\sin A = \frac{61 \sin 40^\circ}{75}$
 $\angle A = 31.520\ 297\ 67 \dots^\circ$
 $= 32^\circ$ (nearest degree) C

7. 
 $c^2 = a^2 + b^2 - 2ab \cos C$
 $= 8^2 + 4^2 - 2 \times 8 \times 4 \times \cos 70^\circ$
 $= 58.110\ 710\ 83 \dots$
 $c = \sqrt{58.110\ 710\ 83 \dots}$ ($c > 0$)
 $= 7.623\ 038\ 163 \dots$
 $= 7.6$ (1 d.p.)
 The length of AB is 7.6 metres,
 to one decimal place. A

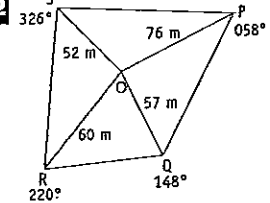
8. 
 Let X be a point due north of C
 and let Y be a point due north of B.
 $\angle YBA = 90^\circ$ (A is due west of B)
 $\angle YBC = 90^\circ - 40^\circ$
 $= 50^\circ$
 $\angle XCB + \angle YBC = 180^\circ$ (co-interior \angle s,
 parallel lines)
 $\angle XCB = 180^\circ - 50^\circ$
 $= 130^\circ$
 The bearing of B from C is 130° . C

9. 
 $\angle PQR = \angle SPQ$ (alternate angles,
 parallel lines)
 $\angle PQR = 40^\circ$
 $\tan 40^\circ = \frac{h}{300}$
 $h = 300 \tan 40^\circ$ A

10. 
 $A = \frac{1}{2} \times 100 \times 40 + \frac{1}{2} \times 100 \times 25$
 $= 3250$
 The area of the land is 3250 m^2 . B

11. 
 a C is due east of D
 $\angle ADC = 90^\circ - 15^\circ$
 $= 75^\circ$ ✓
 b $d^2 = a^2 + c^2 - 2ac \cos D$
 $= 121^2 + 83^2$
 $- 2 \times 121 \times 83 \times \cos 75^\circ$ ✓
 $= 16\ 331.360\ 66 \dots$
 $d = \sqrt{16\ 331.360\ 66 \dots}$ ($d > 0$)
 $= 127.794\ 2122 \dots$
 $= 128$ (nearest unit)
 The distance from A to C is
 128 km, to the nearest kilometre. ✓

c In $\triangle ABC$,
 $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$
 $= \frac{97^2 + 154^2 - (127.79 \dots)^2}{2 \times 97 \times 154}$ ✓
 $\angle B = 55.798\ 060\ 71 \dots^\circ$
 $= 56^\circ$ (nearest degree)
 $\angle ABC = 56^\circ$ to the nearest degree. ✓

12. 
 a $\angle POQ = 148^\circ - 58^\circ$
 $= 90^\circ$
 $\therefore \triangle OPQ$ is right-angled at O. ✓

$$\begin{aligned} \text{b } PQ^2 &= 76^2 + 57^2 \\ &= 9025 \end{aligned}$$

$$\begin{aligned} PQ &= \sqrt{9025} \quad (PQ > 0) \\ &= 95 \end{aligned}$$

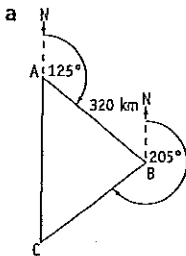
The length of PQ is 95 metres. ✓

$$\begin{aligned} \text{c } \angle SOP &= (360 - 326)^\circ + 58^\circ \\ &= 92^\circ \end{aligned} \quad \checkmark$$

$$\begin{aligned} \text{d } A &= \frac{1}{2} ps \sin O \\ &= \frac{1}{2} \times 52 \times 76 \times \sin 92^\circ \\ &= 1974.796\ 274 \dots \\ &= 1975 \quad (\text{nearest unit}) \end{aligned}$$

The area of triangle SOP is 1975 m², to the nearest square metre. ✓

13



b Let X be a point due north of A and let Y be a point due north of B.
 $\angle XAB + \angle ABY = 180^\circ$ (co-int. \angle s, parallel lines)

$$\begin{aligned} \angle ABY &= 180^\circ - 125^\circ \\ &= 55^\circ \end{aligned}$$

$$\angle ABC + 205^\circ + 55^\circ = 360^\circ \quad (\text{angles at a point})$$

$$\begin{aligned} \angle ABC &= 360^\circ - 260^\circ \\ &= 100^\circ \end{aligned} \quad \checkmark$$

c $\angle BAC = 180^\circ - 125^\circ$ (C is south of A)

$$= 55^\circ$$

$$\angle BCA + 55^\circ + 100^\circ = 180^\circ \quad (\text{angle sum of } \Delta)$$

$$\angle BCA = 180^\circ - 155^\circ$$

$$\angle BCA = 25^\circ \quad \checkmark$$

$$\begin{aligned} \frac{a}{\sin A} &= \frac{c}{\sin C} \\ \frac{a}{\sin 55^\circ} &= \frac{320}{\sin 25^\circ} \\ a &= \frac{320 \sin 55^\circ}{\sin 25^\circ} \end{aligned}$$

$$= 620.249\ 2365 \dots$$

$$= 620 \quad (\text{nearest unit})$$

The distance from B to C is 620 km, to the nearest kilometre. ✓

14

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin \theta}{13} = \frac{\sin 34^\circ 15'}{8}$$

$$\begin{aligned} \sin \theta &= \frac{13 \sin 34^\circ 15'}{8} \end{aligned} \quad \checkmark$$

$$\theta = 66.143\ 036\ 04 \dots^\circ$$

$$= 66^\circ 9' \quad (\text{nearest minute}) \quad \checkmark$$

