Further Practice: Applications of Probability

Remember: all questions match the numbered examples on pages 204–211.

1 a What is the probability of getting a three when a die is thrown?
b How many times would you expect to throw a three if a die is thrown sixty times?

2 How many times would you expect to get a tail if you toss a coin forty times?

3 The probability that the player who has the first turn wins a particular game is 55%. In 20 games, how many times would you expect the player who goes first to win?

4 a If a coin is tossed three times, what is the probability of getting:
i 3 heads
ii 2 heads and a tail?
b If three coins are tossed together a total of 24 times, how many times would you expect to get:
i 3 heads
ii 2 heads and a tail?

5 a How many unordered arrangements are there of four people from a group of nine?
b If there are three women and six men in the group, what is the probability that a particular arrangement has two women and two men?
c How many times would you expect to choose two women and two men in 28 random arrangements?

6 Six people, A, B, C, D, E and F are in a group that regularly plays tennis.
a In how many ways can two people be selected from six?
b List the possible selections of two people from the six above.
c How many times would you expect A to have played in 36 games, if the two players are chosen at random each time?

7 The digits from 0 to 9 are written on ten separate cards, one number on each card.
a If one of the cards is drawn at random, what is the probability that it is a multiple of 3?
b 'I chose a card at random, noted the result and then replaced the card and did this a number of times. I got a multiple of 3 nine times, which is exactly what I expected', said Courtney. How many cards did Courtney choose?

8 A spinner is equally likely to land on any of five scores. Kim spun the spinner a number of times and recorded the results in a table as shown below.

Score	1	2	3	4	5
Frequency	9	11	7	6	7

Which scores occurred less than the expected number of times?

9 a If a die is thrown, what is the probability of getting a five?
b Jose is playing a game and needs to throw a five in order to start. He has had eighteen throws without throwing a five. How many times would you have expected him to have thrown a five in eighteen throws?

10 Harley threw two dice together 24 times. He found that on nine occasions he got at least one six. Harley concluded that the dice were biased and more likely to show a six than any other number. Do you agree? Justify your answer with appropriate calculations. What do you suggest Harley should do to prove his claim?

11 A magazine gives away free entry coupons in a competition with one prize of $20 000. Natalie has eight coupons. If there are 50 000 coupons altogether, what is Natalie's financial expectation?

12 Seamus has an 80% chance of gaining $10 and a 20% chance of gaining $100 in a game. What is his financial expectation?

13 Kath has a 25% chance of winning $2000 and a 75% chance of losing $500. What is her financial expectation?

14 What is the financial expectation from a game in which the probability of winning $480 is $\frac{1}{12}$ and the probability of winning $240 is $\frac{1}{6}$, but if no amount is won you will lose $120?

15 Jade is considering playing a game. She has one chance in seven of winning $35 but six chances in seven of losing $7. Would you recommend that Jade should play the game? Justify your answer.

16 Daphne buys a ticket costing $5 in a lottery. 100 000 tickets are sold and there is one prize of $250 000. What is Daphne's financial expectation?

17 Terri buys ten tickets in a raffle in which 500 tickets are sold. Two prizes are drawn, one after the other without replacement.

a What is the probability that Terri wins first prize?

b What is the probability that Terri wins second prize?

c What is the probability that Terri does not win a prize?

d If the tickets cost $5 each and if first prize is worth $1000 and second prize $500, find Terri's financial expectation.

18 Logan decides to play a game involving throwing two dice. The numbers on the uppermost faces are subtracted to form the score. (The smaller number is always taken from the larger number so that the score is never negative.) Logan will win $60 if the score is 5, and will win $36 if the score is zero, but will lose $12 otherwise. What is Logan's financial expectation from playing the game?

19 Sheridan is considering playing a game where she chooses a card at random from five cards. With one particular card she will win $50, with another she will win $25, and with a third she will get nothing. If she chooses either of the other two cards she will lose $30. Sheridan suggests only having four cards, one winning $50, one winning $25 and two losing $30. 'The amounts that can be won or lost are still the same, so the expectation is still the same', Sheridan claims. Is she correct? Justify your answer with appropriate calculations.

20 Dylan has a business that operates six days a week, He wants to prepare a weekly plan and wants to arbitrarily choose different days for certain activities to occur. How could he simulate the possible choices?

21 Gabrielle is examining the possible sequence of switch settings in the network she controls. Every switch is equally likely to be either off or on. Gabrielle wanted to know how many switches she would need to examine on average before finding one that was off. She tossed a coin to simulate the possibilities with a head to represent a switch that was on and a tail a switch that was off. She completed the simulation for 30 trials and the results are shown below.

Trial	Results	Trial	Results
1	H, H, H, T	16	H, T
2	H, H, T	17	H, H, T
3	T	18	T
4	T	19	H, T
5	T	20	H, T
6	H, H, T	21	T
7	T	22	H, H, H, T
8	H, T	23	T
9	H, H, T	24	H, H, T
10	T	25	T
11	T	26	T
12	T	27	H, T
13	H, T	28	H, H, H, H, T
14	T	29	T
15	T	30	T

a Are more switches on or off? Is this what you would expect?

b What is the average number of switches per trial?

c What is the largest number of switches per trial?

d What percentage of trials have just one switch?

22 Certain packets of breakfast cereal each contain one of three cards (showing soccer, cricket and basketball) and one of two plastic toys (a kangaroo or a platypus).

a How many different arrangements of cards and toys are there?

b Verify the answer to part a by listing the possibilities, numbering each arrangement.

c What is the least number of packets that can be bought to have all three cards and both toys?

d It is not possible to give the most number of packs needed. Why not?

e Edie wants to know how likely it is to be able to have all the cards and both toys by buying just three packets. She decides to simulate the possibilities by throwing a die and recording the results. She makes twenty-five trials, each time throwing the die and using the results matching her listed arrangements to determine the number of throws necessary in each trial to have all the cards and toys. Complete a trial, like Edie's.

f Using the results of the simulation what is the probability that only three packets will be needed?

23 The following table has been drawn up to show the results of a survey of high school students and driving licences.

	Licence	No licence
Year 11	17	72
Year 12	58	40

a How many year 11 students were surveyed?
b How many of the students surveyed had a licence?
c What percentage of year 12 students had no licence?
d What percentage of those without a licence were year 12 students?
e If a student is selected at random from the group, what is the probability that he or she is a year 11 driver?

24 Ellen conducted a survey of 95 men and 80 women to find out whether or not they agreed with a particular policy. She found that 59 men agreed but 48 women did not agree.
a Draw up a two-way table to show the given information and complete the table.
b Is the percentage of men who agree greater or smaller than the percentage of women who do not agree? Justify your answer with appropriate calculations.
c What is the probability that a person chosen at random from the surveyed group agrees with the policy?

25 A woman claimed to be able to read minds. As a simple test a person was asked to choose a card (either red or blue) and concentrate on the colour. The woman was then asked to 'read' the colour. This was done a number of times. The results are given in the table.

	Judged red	Judged blue
Card red	15	10
Card blue	12	18

a What percentage of cards did the woman correctly judge?
b What is the probability that a red card was judged to be red?
c What is the probability that a card judged blue was actually blue?

26 Laboratory tests were carried out to determine whether people had a particular disease. Not all test results were accurate and some were later proven to be false. The two-way table shows the results.

	Accurate	False
Positive	23	5
Negative	408	64

a How many tests were carried out?
b What is the probability that a particular test was false?
c How many people had the disease?
d What is the probability, as a percentage, that a person chosen at random from the group did not have the disease?
e What is the probability that an accurate test was positive?

Go to p 291 for **Quick Answers**
or to pp 347–50 for **Worked Solutions**

Challenge: Applications of Probability

1 Josh considers playing a game involving throwing two dice together. He would win or lose different amounts depending on the sum of the two uppermost faces of the dice.

Sum	Result
less than 7	lose $1800
7 or 8	gain $600
9, 10 or 11	gain $1000
12	gain $6000

What is Josh's financial expectation if he plays the game? *Hint 1*

2 Stan is considering playing a game in which the probability of winning is 15%. It costs $20 to play the game and Stan has calculated that his financial expectation is –$8. How much could Stan win if he played the game? *Hint 2*

3 A two-way table has been drawn up to show the results when a lie-detector was used on statements the truth of which was known.

	Detected True	Detected false
True Statement	17	8
False statement	11	19

a What percentage of statements were accurately judged by the lie-detector?

b Is the lie-detector more likely to show a true statement false, or a false statement true? Justify your answer. *Hint 3*

4 Two parents both carry genes for a particular disease. They are equally likely to pass, or not pass the gene onto their offspring. If a child receives the gene from both parents he or she will have the disease. If the child receives the gene from one parent only, he or she will be a carrier of the disease but will not have it. If the child does not receive the gene from either parent, he or she will not have the disease nor be a carrier. If these parents have 12 children together, how many would you expect to have the disease? *Hint 4*

5 Jody plays a game. It costs $5 to play. He chooses a card at random, from the four shown below.

Pay extra $5	Win nothing	Money back	Win $10

a What is Jody's financial expectation?

b Jody thinks that the second card 'win nothing' should be left out because it doesn't change anything. Is he right? Justify your answer. *Hint 5*

6 A diagnostic test has been used on people to test for a particular disease. Some of the test results were later proved to be inaccurate. The results are shown in the two-way table below.

	Accurate	Inaccurate
Positive	17	3
Negative	36	4

Based on these results, if 200 people were selected, how many would you expect to have the disease? *Hint 6*

Go to p 291 for **Quick Answers**
or to p 350 for **Worked Solutions**

Hint 1: First find the probability of each of the possible scores. Draw up a table.
Hint 2: Let $x be the amount Stan could win.
Hint 3: Calculate the probability of each as a percentage.
Hint 4: Use a tree diagram to determine the probability.
Hint 5: Winning nothing and getting the money back are not the same outcomes. You must consider the cost to play.
Hint 6: Find the probability that a person has the disease. Remember if a negative result is inaccurate, it should have been positive.

Solutions

Ch 11: Applications of Probability

Unless the question specifically asks for the answer in a certain form (as a percentage, for example), the answer to any question requiring the probability of an event could be given as a fraction, decimal or percentage.

Further Practice p212

1 a $P(3) = \frac{1}{6}$

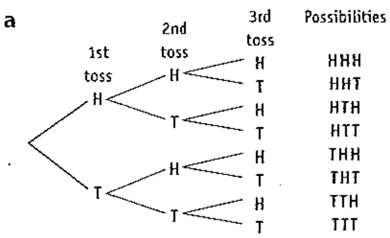
b Expected outcomes $= \frac{1}{6} \times 60$
$= 10$
You would expect to throw a three ten times.

2 $P(\text{tail}) = \frac{1}{2}$

Expected outcomes $= \frac{1}{2} \times 40$
$= 20$

3 Expected number $= 55\% \times 20$
$= 11$
The player who goes first would be expected to win 11 games.

4 a



i $P(\text{three heads}) = \frac{1}{8}$

ii $P(\text{two heads, one tail}) = \frac{3}{8}$

b i Expected number with three heads
$= \frac{1}{8} \times 24$
$= 3$

ii Expected number with two heads and a tail
$= \frac{3}{8} \times 24$
$= 9$

5 a Unordered selections $= \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1}$
$= 126$

Solutions continue next page.

b Women:

Unordered selection of 2 from 3

$= \frac{3\times 2}{2\times 1}$

$= 3$

Men:

Unordered selection of 2 from 6

$= \frac{6\times 5}{2\times 1}$

$= 15$

$P(\text{2 women, 2 men}) = \frac{3\times 15}{126} = \frac{5}{14}$

c Expected number $= \frac{5}{14}\times 28 = 10$

You would expect to choose two women and two men ten times.

6 **a** Unordered selections of 2 from 6

$= \frac{6\times 5}{2\times 1}$

$= 15$

Two people can be selected from six in 15 ways.

b

AB	AC	AD	AE	AF
BC	BD	BE	BF	
CD	CE	CF		
DE	DF			
EF				

c $P(\text{A plays}) = \frac{5}{15} = \frac{1}{3}$

Expected number $= \frac{1}{3}\times 36 = 12$

7 **a** Multiples of three are 3, 6 and 9.

$P(\text{multiple of 3}) = \frac{3}{10}$

b Let n be the number of cards.

$9 = \frac{3}{10}\times n$

$n = 30 \quad [9 \div \frac{3}{10}]$

Courtney chose thirty cards.

8

Score	1	2	3	4	5
Frequency	9	11	7	6	7

$P(\text{each score}) = \frac{1}{5}$

Total tosses $= 9 + 11 + 7 + 6 + 7 = 40$

In 40 tosses, the expected number of each outcome $= \frac{1}{5}\times 40 = 8$

Scores occurring less than 8 times are 3, 4 and 5.

9 **a** $P(\text{five}) = \frac{1}{6}$

b Expected number $= \frac{1}{6}\times 18 = 3$

You would expect Jose to have thrown a 5 three times.

10

1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

$P(\text{at least one six}) = \frac{11}{36}$

Expected number $= \frac{11}{36}\times 24 = 7\frac{1}{3}$

Harley is not correct. The number of times at least one 6 occurs is only a little more than the expected number. He could repeat the experiment and see if he gets the same result.

11 $P(\text{Natalie wins}) = \frac{8}{50\,000} = \frac{1}{6250}$

Financial expectation

$= \frac{1}{6250}\times \$20\,000$

$= \$3.20$

Natalie's financial expectation is \$3.20

12 Financial expectation

$= 80\% \times \$10 + 20\% \times \$100$

$= \$28$

Seamus' financial expectation is \$28.

13 Financial expectation

$= 25\% \times \$2000 - 75\% \times \$500$

$= \$500 - \$375$

$= \$125$

Kath's financial expectation is \$125.

14 $P(\text{winning a prize}) = \frac{1}{12} + \frac{1}{6} = \frac{1}{4}$

$P(\text{losing}) = 1 - \frac{1}{4} = \frac{3}{4}$

Financial expectation

$= \frac{1}{12}\times \$480 + \frac{1}{6}\times \$240 - \frac{3}{4}\times \$120$

$= -\$10$

The financial expectation is a loss of \$10.

15 Financial expectation

$= \frac{1}{7}\times \$35 - \frac{6}{7}\times \$7$

$= \$5 - \$6$

$= -\$1$

Jade can expect to lose one dollar. Jade should not play because she can expect to lose money. (Although the amounts are small so it might be a risk worth taking.)

16 $P(\text{Daphne wins}) = \frac{1}{100\,000}$

Financial expectation

$= \frac{1}{100\,000}\times \$250\,000 - \$5$

$= -\$2.50$

17 **a** $P(\text{win first prize}) = \frac{10}{500} = \frac{1}{50}$

b $P(\text{win second prize}) = \frac{10}{500} = \frac{1}{50}$

c $P(\text{not winning a prize})$

$= 1 - \left(\frac{1}{50} + \frac{1}{50}\right)$

$= \frac{24}{25}$

d The tickets cost \$5 each, so Terri spends \$50. If Terri wins first prize she gains \$950 and if she wins second prize she gains \$450. Otherwise she loses \$50.

Financial expectation

$= \frac{1}{50}\times \$950 + \frac{1}{50}\times \$450 - \frac{24}{25}\times \$50$

$= -\$20$

Terri's financial expectation is a loss of \$20.

18

−	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

$P(\text{score is }5) = \frac{2}{36} = \frac{1}{18}$

$P(\text{score is }0) = \frac{6}{36} = \frac{1}{6}$

$P(\text{another score}) = \frac{28}{36} = \frac{7}{9}$

Financial expectation

$= \frac{1}{18}\times\$60 + \frac{1}{6}\times\$36 - \frac{7}{9}\times\$12$

$= \$0$

Logan's financial expectation is $0.

19 Original financial expectation

$= \frac{1}{5}\times\$50 + \frac{1}{5}\times\$25 + \frac{1}{5}\times\$0 - \frac{2}{5}\times\$30$

$= \$3$

New financial expectation

$= \frac{1}{4}\times\$50 + \frac{1}{4}\times\$25 - \frac{1}{2}\times\$30$

$= \$3.75$

Sheridan is not correct. The financial expectation actually increases by 75 cents.

20 He could throw a die; one score to represent each of the different days of the working week.

21

Trial	Results	Trial	Results
1	H, H, H, T	16	H, T
2	H, H, T	17	H, H, T
3	T	18	T
4	T	19	H, T
5	T	20	H, T
6	H, H, T	21	T
7	T	22	H, H, H, T
8	H, T	23	T
9	H, H, T	24	H, H, T
10	T	25	T
11	T	26	T
12	T	27	H, T
13	H, T	28	H, H, H, H, T
14	T	29	T
15	T	30	T

a There are 26 switches on and 30 switches off.
You would expect more switches to be off, because if the first one is off no more are examined.

b Total switches $= 26 + 30$
$= 56$

Average $= 56 \div 30$
$= 1.866\,666\ldots$
$= 1.9$ (1 d.p.)

c The largest number of switches per trial is 5.

d There are sixteen trials with just one switch.

Percentage with 1 switch $= \frac{16}{30}\times 100\%$
$= 53\frac{1}{3}\%$

22 **a** There are 3 cards and two toys.
Number of arrangements $= 3 \times 2$
$= 6$

b [The arrangements could be listed in different orders, with different numbering.]
1 Soccer and Kangaroo
2 Cricket and Kangaroo
3 Basketball and Kangaroo
4 Soccer and Platypus
5 Cricket and Platypus
6 Basketball and Platypus

c Least number = 3

d It is possible (but not likely) that a particular card or a particular toy doesn't get picked.

e [This is just one simulation. Any number of different simulations could exist.]

Trial	Scores
1	1, 3, 2, 5
2	4, 1, 1, 3, 2
3	6, 2, 4
4	4, 6, 6, 2
5	5, 3, 1
6	2, 5, 1, 3
7	5, 5, 4, 1, 3
8	1, 1, 1, 5, 1, 4, 5, 2, 1, 2, 4, 3
9	6, 3, 3, 6, 5, 5, 1
10	2, 1, 3, 2, 6
11	4, 2, 6
12	3, 2, 1, 5
13	1, 6, 3, 4, 1, 3, 4, 2
14	5, 5, 5, 2, 6, 4
15	4, 6, 5, 5, 1
16	2, 6, 4
17	4, 1, 5, 4, 1, 6
18	4, 3, 6, 1, 3, 6, 3, 1, 2
19	6, 4, 1, 2
20	3, 5, 1
21	3, 6, 2, 3, 6, 3, 6, 5, 4
22	2, 3, 6, 6, 1
23	3, 2, 6, 3, 2, 5, 3, 1
24	6, 1, 1, 2
25	6, 5, 1

f Using these results:

$P(3\text{ packets}) = \frac{6}{25}$

23

	Licence	No licence
Year 11	17	72
Year 12	58	40

a Total year 11 $= 17 + 72$
$= 89$
89 year 11 students were surveyed.

b Total with licence $= 17 + 58$
$= 75$
75 of the students surveyed had a licence.

c Total year 12 $= 58 + 40$
$= 98$

Percentage without licence

$= \frac{40}{98}\times 100\%$
$= 40.816\,326\,53\ldots\%$
$= 41\%$ (nearest per cent)

Approximately 41% of year 12 students did not have a licence.

d Total without licence $= 72 + 40$
$= 112$

Percentage of year 12

$= \frac{40}{112}\times 100\%$
$= 35.714\,285\,71\ldots\%$
$= 36\%$ (nearest per cent)

Approximately 36% of those without a licence are in year 12.

e Total surveyed $= 89 + 98$
$= 187$

$P(\text{year 11 driver}) = \frac{17}{187} = \frac{1}{11}$

24 **a** Men not agreeing $= 95 - 59$
$= 36$

Women agreeing $= 80 - 48$
$= 32$

	Agree	Don't agree	Total
Men	59	36	95
Women	32	48	80
Total	91	84	175

b Percentage of men who agree

$= \frac{59}{95}\times 100\%$
$= 62.105\,263\,16\ldots\%$

Percentage of women who do not agree

$= \frac{48}{80}\times 100\%$
$= 60\%$

The percentage of men who agree is slightly higher than the percentage of women who don't agree.

c Total surveyed $= 95 + 80$
$= 175$

Total who agree $= 59 + 32$
$= 91$

$$P(\text{person agrees}) = \frac{91}{175} = \frac{13}{25}$$

25 **a** Correctly judged $= 15 + 18$
$= 33$

Total cards $= 33 + 10 + 12$
$= 55$

	Judged red	Judged blue
Card red	15	10
Card blue	12	18

$$\text{Percentage correctly judged} = \frac{33}{55} = 60\%$$

b Red cards $= 15 + 10$
$= 25$

$$P(\text{red card judged red}) = \frac{15}{25} = \frac{3}{5}$$

c Judged blue $= 10 + 18$
$= 28$

$$P(\text{actually blue}) = \frac{18}{28} = \frac{9}{14}$$

26 **a** Total tests $= 23 + 5 + 408 + 64$
$= 500$

500 tests were carried out.

	Accurate	False
Positive	23	5
Negative	408	64

b False tests $= 5 + 64$
$= 69$

$$P(\text{false test}) = \frac{69}{500}$$

c Total with disease $= 23 + 64$
$= 87$

87 people had the disease.
[If a negative test is false, it should have been positive.]

d Total without disease $= 5 + 408$
$= 413$

$$P(\text{no disease}) = \frac{413}{500} \times 100\% = 82.6\%$$

e Accurate tests $= 23 + 408$
$= 431$

$$P(\text{positive}) = \frac{23}{431}$$

Challenge p215

1

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$$P(\text{less than } 7) = \frac{15}{36} = \frac{5}{12}$$

$$P(7 \text{ or } 8) = \frac{11}{36}$$

$$P(9, 10 \text{ or } 11) = \frac{9}{36} = \frac{1}{4}$$

$$P(12) = \frac{1}{36}$$

Josh's financial expectation

$$= \frac{1}{36} \times \$6000 + \frac{1}{4} \times \$1000 + \frac{11}{36} \times \$600 - \frac{5}{12} \times \$1800$$
$$= -\$150$$

2 Financial expectation $= -\$8$

Let $\$x$ be the amount Stan could win.

Financial expectation

$= 15\% \times \$x - \$20$

$-8 = 0.15x - 20$

$12 = 0.15x$

$x = 12 \div 0.15$

$= 80$

The amount of money Stan could win is \$80.

3

	Detected True	Detected false
True Statement	17	8
False statement	11	19

a Total statements $= 17 + 8 + 11 + 19$
$= 55$

Accurately judged $= 17 + 19$
$= 36$

Percentage accurately judged

$$= \frac{36}{55} \times 100\%$$
$= 65.454\,545\ldots\,\%$
$= 65\%$ (nearest per cent)

b Total true statements $= 17 + 8$
$= 25$

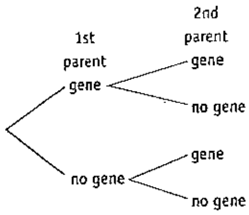
$$P(\text{true statement shown false}) = \frac{8}{25} = 32\%$$

Total false statements $= 11 + 19$
$= 30$

$P(\text{false statement shown true})$
$$= \frac{11}{30}$$
$= 36.6666\ldots\,\%$

Based on these results, the lie-detector is more likely to show a false statement as true.

4



$$P(\text{disease}) = \frac{1}{4}$$

In 12 children, expected number

$$= \frac{1}{4} \times 12$$
$= 3$

It would be expected that 3 children would have the disease.

5 **a** If Jody plays then he will either lose \$10, lose \$5, win \$0 or win \$5 given that it costs \$5 to play.

Financial expectation

$$= -\frac{1}{4} \times \$10 - \frac{1}{4} \times \$5 + \frac{1}{4} \times \$0 + \frac{1}{4} \times \$5$$
$= -\$2.50$

b No, Jody is not right. Although the card says win nothing, it actually means that he will lose \$5 because of the cost of playing the game. If any of the cards were left out, then the probability of selecting any card would change to one in three and so this would also change the expectation.

6 Number with disease $= 17 + 4$
$= 21$

Total tested $= 17 + 3 + 36 + 4$
$= 60$

$$P(\text{having disease}) = \frac{21}{60} = \frac{7}{20}$$

$$\text{Expected number} = \frac{7}{20} \times 200 = 70$$