Further Practice: Multi-stage Events

Remember: all questions match the numbered examples on pages 185–195.

1 If a bag holds 7 blue, 2 white and 4 yellow pegs, what is the probability that a peg chosen at random is yellow?

2 In a factory, dials tell whether particular parts of the machinery are hot or cold. Three dials are inspected at random, and they all are equally likely to be hot or cold.

a Draw a tree diagram showing the possible results.
b What is the probability that the three dials are all hot?
c What is the probability that exactly two are hot?

3 A drawer holds 1 pink, 1 grey and 1 white T-shirt. One T-shirt is taken at random from the drawer, its colour noted and the shirt replaced. A second T-shirt is then chosen at random.

a Draw a tree diagram to show the possible outcomes.
b What is the probability that the pink T-shirt was chosen both times?
c What is the probability that one shirt was grey and one white?
d Draw another tree diagram to show the possible outcomes, if the first shirt is not replaced before the second one is drawn.
e What is the new probability that one is grey and one is white?

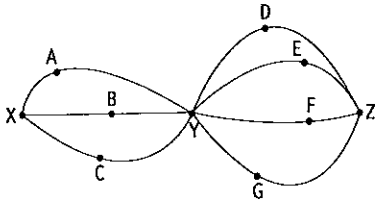
4 A coin is tossed five times. What is the total number of possible outcomes?

5 At a seminar there are three choices of lectures for the morning session, four for the afternoon session and two for the evening session. How many different sets of three lectures are possible?

6 Upon leaving a party, the guests are all given a pack to take home. Each pack holds a chocolate (plain or mint), a piece of fruit (apple, orange or banana), and a mug (white, yellow, blue or mauve).

a How many different packs are possible?
b Marissa is one of the guests. If there is one of each possible pack being given at random, what is the probability that Marissa receives a pack with a mint chocolate, an apple and a blue mug?

7 To travel from X to Y there are three possible routes, one passing through A, one through B and one through C. From Y to Z there are four routes, through D, E, F or through G.



a How many different possible routes are there from X to Z?
b Verify the answer to part a by listing the possible routes.
c What is the probability that if a route is chosen at random, it passes through both B and D?

8 a If a car number plate is made up of two letters followed by four numbers, how many different plates are possible?
b If a number plate is made up of four letters followed by two numbers, how many more number plates are possible?

9 Delish's Desserts consist of a base of apple pie (P), lemon tart (T) or apricot Danish (D), a jelly (raspberry (R), strawberry (S) or lime(L)) and a topping of either custard (C) or ice-cream (I).

a How many different desserts are possible?
b List all the different desserts.
c How many different desserts contain strawberry jelly?
d What is the probability that if you receive a dessert at random, it will have apricot Danish as a base?

10 In how many ways can 5 people be arranged in a row?

11 Four people are standing for an election. In how many different ways can their names appear on the ballot paper?

12 A concert will be made up of eight different acts. In how many different ways could the acts be arranged?

13 Six cards are identical except that each has a different letter written on it. The letters are A, G, H, L, S and U. The cards are shuffled and laid out on a table with the letters face up next to one another.
   a How many different arrangements are there?
   b What is the probability that the letters spell the word 'laughs'?

14 a In how many ways can the digits 6, 7, 8 and 9 be arranged to form a four-digit number if each digit can only be used once?
   b Verify by listing all the possible arrangements.
   c If one number is chosen at random from the list, what is the probability that it:
      i is 9876,
      ii begins with 7?

15 In how many ways can the three winning places be filled if seven cyclists race?

16 From a group of nine members of a committee, a secretary and treasurer are to be selected. How many different selections are possible?

17 Pam can't remember the last three digits of a friend's phone number. She knows it is made up of an 8, a 6 and a 0. What is the probability that the telephone number ends in 608?

18 From a team of 17 players, one is chosen at random to be captain and another to be goalkeeper. Mark and Harry are two players in the team. What is the probability that Harry is chosen to be captain and Mark the goalkeeper?

19 Three people are to be selected from four people (A, B, C and D) to be president, vice-president and deputy vice-president of a club.
   a How many different selections are possible?
   b List the possible selections.
   c What is the probability that B is the vice-president?
   d What is the probability that A and D are both chosen?

20 Three people are chosen from six people, A, B, C, D, E and F. List the possible unordered selections and verify that there are 20.

21 How many unordered selections are possible of four people from six?

22 How many unordered selections of three names from eight are possible?

23 Lachlan has five different coloured pencils (blue, green, orange, red and yellow). He chooses two pencils at random to complete a design.
   a How many different choices are there?
   b Verify by listing the possible selections.
   c What is the probability that he chooses green and orange?
   d What is the probability that one of the colours is blue?

24 In a particular lottery type game, a player must choose six numbers from thirty. If Chelsea makes one choice, what is the probability that she will win the lottery?

25 How many selections of four people are possible from five, if the selections are:
   a ordered
   b unordered?

26 From a group of seven players, how many different pairs can be chosen to play squash?

27 If eight athletes take part in a race, in how many different ways can the first three places be filled?

28 A type of car comes in four different colours (white, red, silver and blue), three interior trims (leather, black cloth or beige cloth), and in either manual or automatic transmission. If a dealer orders one at random, what is the probability that it is a silver manual with black cloth trim?

29 Five students from eleven are chosen to form a committee.
   a How many different committees are possible?
   b Ben is one of the eleven students. What is the probability that Ben is in the committee?

30 A chairperson and deputy are chosen from a group of twenty people.
   a How many different selections are possible?
   b It is known that Lynsey was chosen to be chairperson. How many selections are now possible?

31 There are four dogs (A, B, C and D) and two cats (E and F) in a local show. Two animals are chosen at random to represent the local area at the regional show.
   a How many different representative groups are possible?
   b List the possible groups.
   c What is the probability that the representative group consists of a dog and a cat?

32 There are four men and six women in a committee. Four are chosen at random to attend a function. What is the probability that the chosen group consists of two men and two women?

33 Five blue and three green balloons are in a packet. One balloon is drawn at random, its colour noted, and then replaced before a second balloon is drawn.

a Draw a probability tree diagram to show the possible outcomes.

b What is the probability that both balloons are green?

c What is the probability that one balloon is blue and one is green?

34 An unusual die has six faces but three faces show 1, and two faces show 2. The other face shows 3. The die is thrown twice.

a Draw a probability tree diagram to show the possible results.

b What is the probability that the sum of the two throws is 4?

35 A bag holds 8 red and 4 white discs. Two discs are drawn at random, one after the other without replacement.

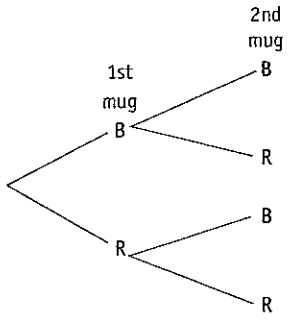
a Draw a probability tree diagram to show the possible outcomes.

b What is the probability that both discs are red?

36 Three checked, four plain and five striped ties are in a drawer. Two are chosen at random without replacement. What is the probability that the ties have the same pattern type?

37 There are seven blue and three red mugs on a shelf. Two are selected at random, one after the other without replacement.

a Copy and complete the tree diagram by writing the probabilities on the branches.



b Amber comments that there is a slightly less than fifty per cent chance that the mugs will be different colours. Do you agree? Justify your answer.

38 A group of twenty-one students go on a three-day excursion. Before the excursion, three students are chosen, one for each day, to be group captain. The names of all the students were placed in a hat and three were drawn, one after the other without replacement. Jane is one of the students.

a What is the probability that Jane is group captain on the first day?

b What is the probability that Jane is captain on the last day?

c Jane does not want to be captain. She calculates the probability and is pleased to realise that she has a less than 15% chance of being chosen. Is Jane correct? Justify your answer.

39 A biased coin is three times as likely to show tails as heads.

a What is the probability that it shows tails?

b Erin plays a game in which she tosses two coins, the biased one and a fair one. List all the possible results.

c What is the probability that Erin throws two heads?

Go to pp 289–90 for **Quick Answers**
or to pp 344–6 for **Worked Solutions**

Challenge: Multi-stage Events

1 Alice, Ben, Charlie, Dan and Erica line up for a photo.
- a How many different ways can they form a straight line?
- b What is the probability that Ben is furthest to the left? *Hint 1*

2 One green, four blue and five red pegs are in a box. Two pegs are taken, one after the other, to hang up a shirt. It is known that the first peg was blue. What is the probability that the pegs are both blue? *Hint 2*

3 A die is thrown three times. What is the probability that the die showed an even number on exactly two of those tosses? *Hint 3*

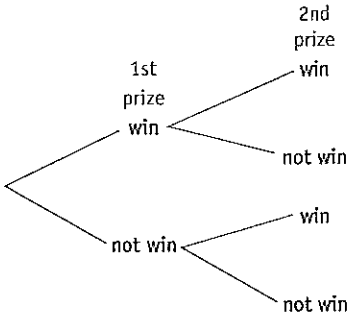
4 Twelve horses run in a race.
- a In how many ways can the first three places be filled?
- b Ted believes that he knows which three horses will fill the first three places, but he is not sure of the order. In how many ways can three horses fill the first three places?
- c Ted intends to place large bets on each of the ways that his chosen three horses can fill the first three places of the race. Joel said: 'You're mad, Ted! The probability of winning is less than half of one percent.' Is Joel correct? Discuss. *Hint 4*

5 Three people are to be chosen from a group of nine people to form a sub-committee.
- a How many different sub-committees are possible?
- b If the original group consists of seven women and two men, what is the probability that the two men are on the sub-committee? *Hint 5*

6
- a How many unordered selections of two people are possible from four (A, B, C and D)?
- b Verify the answer to part a by listing all the possible arrangements.
- c At a tennis tournament there are four players from the home club and four players from the visiting club. How many games of tennis will be played if every possible arrangement of two people from the home club plays doubles against every possible arrangement from the visiting club? *Hint 6*

7 Three paintings are randomly chosen from five (A, B, C, D and E), to be put on display.
- a How many different choices are there?
- b What is the probability that A and E are both chosen? *Hint 7*

8 Neil has five tickets in a raffle in which 100 tickets have been sold and there are two prizes.
- a What is the probability that Neil wins first prize?
- b Copy the tree diagram and write the probabilities on each of the branches.



- c Find the probability that Neil wins a prize. *Hint 8*

Go to p 290 for **Quick Answers**
or to pp 346–7 for **Worked Solutions**

Hint 1: It doesn't matter how many arrangements there are. How many possibilities are there for this position?
Hint 2: After the first peg has been drawn, how many pegs are there to choose from? How many of these are blue?
Hint 3: Draw a tree diagram. Only record whether the toss is odd or even.
Hint 4: Find the probability if each of the outcomes is equally likely. Are they equally likely?
Hint 5: How many different sub-committees are possible with both men and one woman? How many women could be the third member?
Hint 6: For each arrangement from part a, how many games will they need to play?
Hint 7: There are only a small number of choices altogether. List them.
Hint 8: Find the probability that Neil does not win a prize.

UNIT 4: PROBABILITY

Solutions

Ch 10: Multi-stage Events

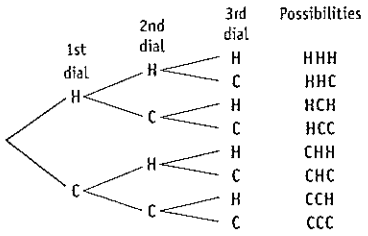
Unless the question specifically asks for the answer in a certain form (as a percentage, for example), the answer to any question requiring the probability of an event could be given as a fraction, decimal or percentage.

Further Practice p196

1 Number of pegs $= 7 + 2 + 4$
$= 13$

$P(\text{yellow}) = \frac{4}{13}$

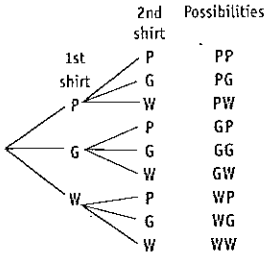
2 a



b $P(\text{HHH}) = \frac{1}{8}$

c $P(\text{exactly two hot}) = \frac{3}{8}$

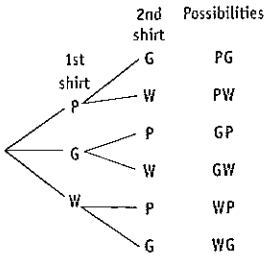
3 a



b $P(\text{PP}) = \frac{1}{9}$

c $P(\text{one grey, one white}) = \frac{2}{9}$

d



e $P(\text{one grey, one white}) = \frac{2}{6}$
$= \frac{1}{3}$

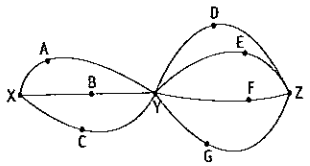
4 There are two possible outcomes for each toss.
Total outcomes
$= 2 \times 2 \times 2 \times 2 \times 2$ [or 2^5]
$= 32$
There are 32 possible outcomes when a coin is tossed five times.

5 Possible sets $= 3 \times 4 \times 2$
$= 24$
24 different sets of lectures are possible.

6 a Number of packs $= 2 \times 3 \times 4$
$= 24$

b $P(\text{mint chocolate, apple, blue mug}) = \frac{1}{24}$

7 a Number of routes $= 3 \times 4$
$= 12$



b Routes pass through:

AD	AE	AF	AG
BD	BE	BF	BG
CD	CE	CF	CG

c $P(\text{BD}) = \frac{1}{12}$

8 a Number of plates
$= 26 \times 26 \times 10 \times 10 \times 10 \times 10$
$= 6\,760\,000$

b Number of different plates
$= 26 \times 26 \times 26 \times 26 \times 10 \times 10$
$= 45\,697\,600$
Difference $= 45\,697\,600 - 6\,760\,000$
$= 38\,937\,600$

9 a Number of desserts $= 3 \times 3 \times 2$
$= 18$

b

PRC	PRI	PSC	PSI	PLC	PLI
TRC	TRI	TSC	TSI	TLC	TLI
DRC	DRI	DSC	DSI	DLC	DLI

c There are six different desserts with strawberry jelly.

d $P(\text{apricot Danish}) = \frac{1}{3}$

10 Number of arrangements
$= 5 \times 4 \times 3 \times 2 \times 1$
$= 120$
Five people can be arranged in a row in 120 ways.

11 Number of arrangements
$= 4 \times 3 \times 2 \times 1$
$= 24$
The names could appear in 24 different arrangements.

12 Number of arrangements
$= 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$
$= 40\,320$
The acts could be arranged in 40 320 ways.

13 a Number of arrangements
$= 6 \times 5 \times 4 \times 3 \times 2 \times 1$
$= 720$

b $P(\text{laughs}) = \frac{1}{720}$

14 a Arrangements $= 4 \times 3 \times 2 \times 1$
$= 24$

b

6789	7689	8679	9678
6798	7698	8697	9687
6879	7869	8769	9768
6897	7896	8796	9786
6978	7968	8967	9867
6987	7986	8976	9876

c i $P(9876) = \frac{1}{24}$

ii $P(\text{begins with 7}) = \frac{6}{24}$
$= \frac{1}{4}$

15 Ordered selections $= 7 \times 6 \times 5$
$= 210$
The places can be filled in 210 different ways.

16 Ordered selections $= 9 \times 8$
$= 72$
The positions could be filled in 72 different ways.

17 Ordered selections $= 3 \times 2 \times 1$
$= 6$

$P(\text{ends in 608}) = \frac{1}{6}$

18 Ordered selections $= 17 \times 16$
$= 272$

$P(\text{Harry, Mark}) = \frac{1}{272}$

19 a Ordered selections $= 4 \times 3 \times 2$
$= 24$

b

ABC	BAC	CAB	DAB
ABD	BAD	CAD	DAC
ACB	BCA	CBA	DBA
ACD	BCD	CBD	DBC
ADB	BDA	CDA	DCA
ADC	BDC	CDB	DCB

c $P(\text{B is vice-president}) = \frac{6}{24}$
$= \frac{1}{4}$

d $P(\text{A, D selected}) = \dfrac{12}{24}$
$= \dfrac{1}{2}$

20 ABC ABD ABE ABF
ACD ACE ACF
ADE ADF
AEF
BCD BCE BCF
BDE BDF
BEF
CDE CDF
CEF
DEF
There are twenty possible selections.

21 Unordered selections $= \dfrac{6\times5\times4\times3}{4\times3\times2\times1}$
$= 15$
There are fifteen unordered selections of four people from six.

22 Unordered selections $= \dfrac{8\times7\times6}{3\times2\times1}$
$= 56$
There are 56 unordered selections of three names from eight names.

23 **a** Unordered selections $= \dfrac{5\times4}{2\times1}$
$= 10$
There are ten possible selections.

b BG BO BR BY
GO GR GY
OR OY
RY

c $P(\text{green and orange}) = \dfrac{1}{10}$

d $P(\text{blue}) = \dfrac{4}{10}$
$= \dfrac{2}{5}$

24 Unordered selections
$= \dfrac{30\times29\times28\times27\times26\times25}{6\times5\times4\times3\times2\times1}$
$= 593\,775$
$P(\text{winning}) = \dfrac{1}{593\,775}$

25 **a** Ordered selections $= 5\times4\times3\times2$
$= 120$
120 ordered selections are possible.

b Arrangements of four $= 4\times3\times2\times1$
$= 24$
Unordered selections $= 120 \div 24$
$= 5$
5 unordered selections are possible.

26 Unordered selections $= \dfrac{7\times6}{2\times1}$
$= 21$
Twenty-one different pairs can be selected.

27 Ordered selections $= 8\times7\times6$
$= 336$
The places could be filled in 336 different ways.

28 Number of arrangements $= 4\times3\times2$
$= 24$
$P(\text{silver manual with black cloth trim}) = \dfrac{1}{24}$

29 **a** Unordered selections
$= \dfrac{11\times10\times9\times8\times7}{5\times4\times3\times2\times1}$
$= 462$
Four hundred and sixty-two different committees are possible.

b Five of the eleven students will be in the committee.
$P(\text{Ben}) = \dfrac{5}{11}$

30 **a** Ordered selections $= 20\times19$
$= 380$

b Any one of the remaining 19 could be deputy. There are 19 selections.

31 **a** Unordered selections $= \dfrac{6\times5}{2\times1}$
$= 15$

b AB AC AD AE AF
BC BD BE BF
CD CE CF
DE DF
EF

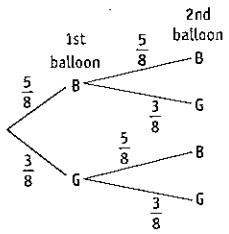
c $P(\text{a dog and a cat}) = \dfrac{8}{15}$

32 Unordered selections of 4 from 10
$= \dfrac{10\times9\times8\times7}{4\times3\times2\times1}$
$= 210$
There are 210 possible groups.

continued ...

Men:
Unordered selections of 2 from 4
$= \dfrac{4\times3}{2\times1}$
$= 6$
There are 6 possible selections of 2 men.
Women:
Unordered selections of 2 from 6
$= \dfrac{6\times5}{2\times1}$
$= 15$
There are 15 possible selections of 2 women.
Number of selections of 2 men and 2 women
$= 6\times15$
$= 90$
$P(\text{2 men and 2 women}) = \dfrac{90}{210}$
$= \dfrac{3}{7}$

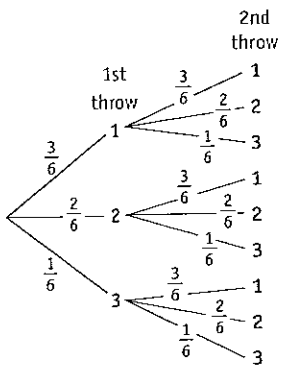
33 **a**



b $P(\text{GG}) = \dfrac{3}{8}\times\dfrac{3}{8}$
$= \dfrac{9}{64}$

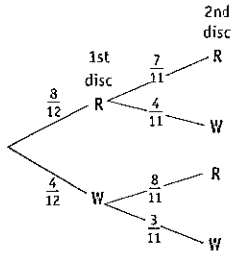
c $P(\text{one blue, one green})$
$= P(\text{BG}) + P(\text{GB})$
$= \dfrac{5}{8}\times\dfrac{3}{8} + \dfrac{3}{8}\times\dfrac{5}{8}$
$= \dfrac{15}{32}$

34 **a**



b $P(\text{sum is 4})$
$= P(1 \text{ and } 3) + P(2 \text{ and } 2)$
$+ P(3 \text{ and } 1)$
$= \dfrac{3}{6}\times\dfrac{1}{6} + \dfrac{2}{6}\times\dfrac{2}{6} + \dfrac{1}{6}\times\dfrac{3}{6}$
$= \dfrac{5}{18}$

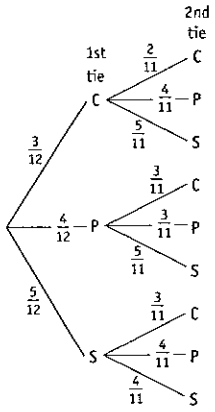
35 a



b $P(\text{both red}) = \frac{8}{12} \times \frac{7}{11}$

$= \frac{14}{33}$

36



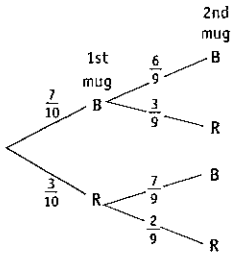
$P(\text{same pattern type})$

$= P(\text{CC}) + P(\text{PP}) + P(\text{SS})$

$= \frac{3}{12} \times \frac{2}{11} + \frac{4}{12} \times \frac{3}{11} + \frac{5}{12} \times \frac{4}{11}$

$= \frac{19}{66}$

37 a



b $P(\text{different colours})$

$= P(\text{BR}) + P(\text{RB})$

$= \frac{7}{10} \times \frac{3}{9} + \frac{3}{10} \times \frac{7}{9}$

$= \frac{7}{15}$

$= \frac{7}{15} \times 100\%$

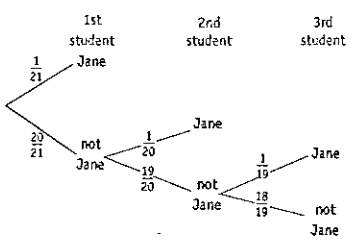
$= 46\frac{2}{3}\%$

Amber is correct. The probability that the mugs are different colours is just under 50%.

38 a $P(\text{Jane captain first}) = \frac{1}{21}$

b $P(\text{Jane captain for last day}) = \frac{1}{21}$

c



$P(\text{Jane not chosen}) = \frac{20}{21} \times \frac{19}{20} \times \frac{18}{19}$

$= \frac{6}{7}$

$P(\text{Jane is chosen}) = \frac{1}{7}$

$= 14\frac{2}{7}\%$

Jane is correct.

39 a There are three chances of tossing tails and one of heads.

$P(\text{tails}) = \frac{3}{4}$

b HH HT TH TT

c $P(\text{HH}) = \frac{1}{4} \times \frac{1}{2}$

$= \frac{1}{8}$

Challenge p199

1 a Number of ways 5 people can line up

$= 5 \times 4 \times 3 \times 2 \times 1$

$= 120$

b $P(\text{Ben is furthest left}) = \frac{1}{5}$

2 If the first peg is blue there are three blue pegs left.

Total pegs left $= 1 + 3 + 5$

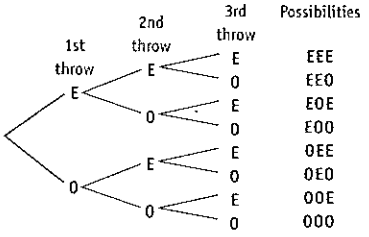
$= 9$

$P(\text{second peg is blue}) = \frac{3}{9}$

$= \frac{1}{3}$

The probability that the pegs are both blue is $\frac{1}{3}$.

3



$P(\text{2 even tosses}) = \frac{3}{8}$

4 a Number of selections $= 12 \times 11 \times 10$

$= 1320$

b Number of ways $= 3 \times 2 \times 1$

$= 6$

c $P(\text{Ted wins}) = \frac{6}{1320}$

$= \frac{1}{220}$

$= 0.454\,545\,45\ldots\,\%$

Joel would be correct if the outcomes were equally likely. In a horse race it might be true that some horses have a better chance of winning than others so the outcomes may not be equally likely.

5 a Number of sub-committees

$= \frac{9 \times 8 \times 7}{3 \times 2 \times 1}$

$= 84$

There are 84 different possible sub-committees.

b If the two men are both on the sub-committee, then the other person is one of the seven women. There are seven different sub-committees with both men.

$P(\text{both men}) = \frac{7}{84}$

$= \frac{1}{12}$

6 a Unordered selections $= \frac{4 \times 3}{2 \times 1}$

$= 6$

There are 6 unordered selections of two people from four.

b AB AC AD BC BD CD

c Each of the six possible selections from the home club play each of the possible selections from the visiting club.

Number of games $= 6 \times 6$

$= 36$

7 a Unordered selections $= \frac{5 \times 4 \times 3}{3 \times 2 \times 1}$

$= 10$

There are ten possible choices.

b The choices are:

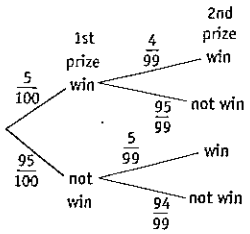
ABC ABD ABE ACD ACE
ADE BCD BCE BDE CDE

$P(\text{A and E}) = \frac{3}{10}$

8 a $P(\text{Neil wins}) = \frac{5}{100}$

$= \frac{1}{20}$

b



c $P(\text{Neil does not win a prize})$

$$= \frac{95}{100} \times \frac{94}{99}$$

$$= \frac{893}{990}$$

$$P(\text{Neil wins a prize}) = 1 - \frac{893}{990}$$

$$= \frac{97}{990}$$