

# Topic Test: Modelling Linear and Non-linear Relationships

Remember: these are HSC-type questions.

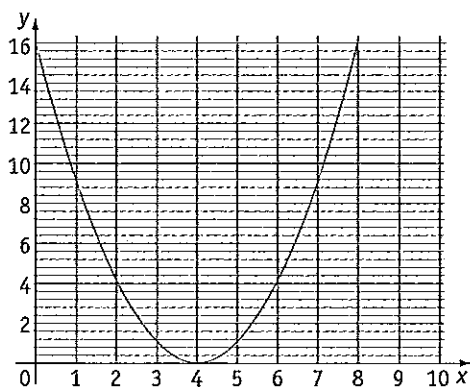
Time allowed: 40 minutes Total marks: 25

## Part A (Suggested time: 15 minutes)

Choose the correct answer (A, B, C or D) for each question. One mark each

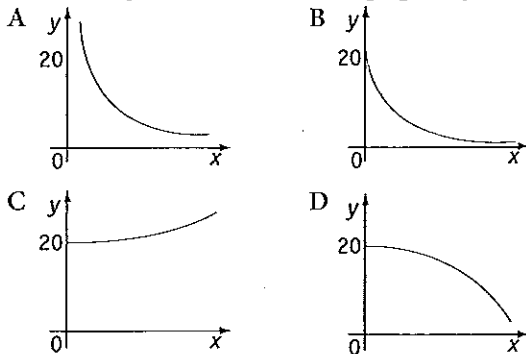
- 1 The equation  $y = \frac{x}{3}$  is an example of what type of relationship?  
 A linear B quadratic  
 C cubic D hyperbolic

- 2 Which could be the equation of the curve shown?



- A  $y = (x - 4)^2$  B  $y = 16 - 8x + x^2$   
 C  $y = x(x - 8) + 16$  D any of the above

- 3 Which diagram could show the graph of  $y = 20(0.8)^x$ ?



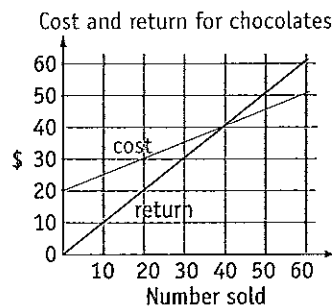
- 4 Under ideal conditions, the height above sea level (in metres) varies directly with the square of the distance (in kilometres) to the horizon. It is 20 km to the horizon from a place 32 m above sea level. How far above sea level is a place from where the distance to the horizon is 40 km?  
 A 64 m B 45 m  
 C 128 m D 102 m

- 5 Which rule describes the relationship between the variables in the table of values?

x	0	2	6
y	3	7	111

- A  $y = 2x + 3$  B  $y = x^2 + 3$   
 C  $y = \frac{1}{2}x^3 + 3$  D  $y = 18x + 3$

- 6 Ellen makes home-made chocolates, which she sells at her local market. The diagram shows the cost of producing the packets of chocolates and the return from their sale.

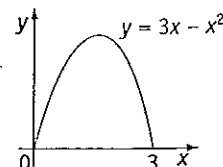


One Saturday, Ellen sells fifty packets. Which statement is correct?

- A Ellen makes a profit of \$50  
 B Ellen breaks even  
 C Ellen loses \$15  
 D Ellen makes a profit of \$5

- 7  $y$  varies inversely with  $x$ . When  $x = 12$ ,  $y = 15$ . What is the value of  $x$  when  $y = 9$ ?  
 A 20 B 7.2  
 C 11.25 D 18

- 8 The diagram shows a sketch of  $y = 3x - x^2$ . What is the maximum value of  $y$ ?

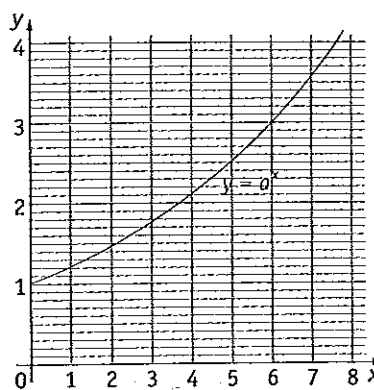


- A 1.5  
 B 2.25  
 C 3  
 D There is not enough information to find the maximum value.

9 Which graph will pass through the point (4, 9)?

- A  $y = 2x + 1$                       B  $y = x^2 - 2x + 1$   
 C  $y = \frac{36}{x}$                               D all of these

10 The diagram shows the graph of  $y = a^x$ .



What would be the approximate value of  $2(a^6)$ ?

- A 3                                      B 5  
 C 6                                      D 64

## Part B

(Suggested time: 25 minutes)

Show all working.

15 marks



a Complete the table of values for  $y = \frac{x}{2} + 5$ .

x	0	2	4	6
y				

1 mark

b Sketch the graph of  $y = \frac{x}{2} + 5$ .

1 mark

c On the same diagram sketch  $y = 8 - x$ .

1 mark

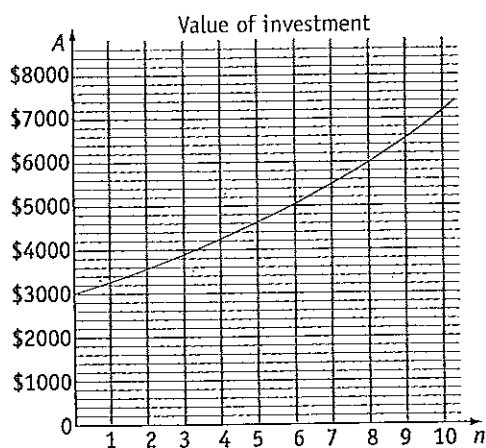
d What is the point of intersection of

$$y = \frac{x}{2} + 5 \text{ and } y = 8 - x$$

1 mark



The diagram shows the amount of money (\$A) in an account after  $n$  years.



a What is the value of A when  $n = 3$ ?                      1 mark

b After how many years will the sum of money have doubled?                      1 mark

c Jessie said: 'Based on this graph there will be about \$12 000 in the account after 16 years.'  
 Do you agree? Justify your answer.                      2 marks



The volume of any of a certain special type of cone varies with the cube of the radius so that  $V = kr^3$ .

a If one of these cones with radius 6 cm has volume  $972 \text{ cm}^3$ , find the value of  $k$ .                      1 mark

b Find the volume of one of these cones that has radius 16 cm.                      1 mark

c Find the radius of one of these cones if its volume is  $7776 \text{ cm}^3$ .                      1 mark



a Complete the table of values for  $y = 4x^2 - 28x + 59$ .

x	0	1	2	3	4	5	6	7	8
y									

1 mark

b Sketch the graph of  $y = 4x^2 - 28x + 59$  for values of  $x$  between 0 and 8.                      1 mark

c What is the minimum value of  $y$ ?                      2 marks

Go to p 294 for **Quick Answers**  
 or to pp 361–2 for **Worked Solutions**

# Solutions

## Topic Test ..... p268

1  $y = \frac{x}{3}$  is a linear relationship. A  
[It is a straight line when graphed.]

2 All of the equations represent the same graph. D

3  $y = 20(0.8)^x$  is the equation of an exponential graph. Because 0.8 is less than 1, it is an example of exponential decay. When  $x = 0$ ,  $y = 20$ . B

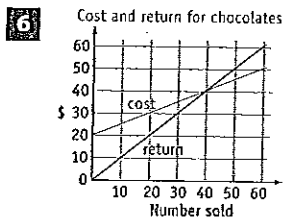
4  $h = kd^2$   
When  $d = 20$ ,  $h = 32$   
 $32 = k \times 20^2$   
 $32 = 400k$   
 $k = 0.08$   
 $\therefore h = 0.08d^2$   
When  $d = 40$ ,  
 $h = 0.08 \times 40^2$   
 $= 128$  C

Solutions continued

next page

x	0	2	6
y	3	7	111

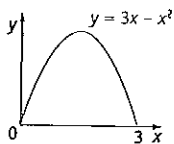
The only rule that works for all three pairs of variables is  $y = \frac{1}{2}x^3 + 3$ . C



If 50 packets are sold, the return is \$50 and the cost is \$45. Ellen makes a profit of \$5. D

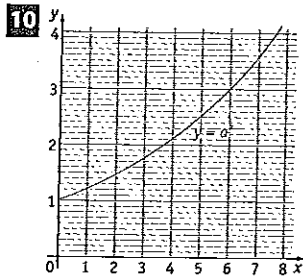
**7**  $y = \frac{a}{x}$   
 When  $x = 12$ ,  $y = 15$ .  
 $15 = \frac{a}{12}$   
 $a = 15 \times 12$   
 $= 180$   
 $\therefore y = \frac{180}{x}$   
 When  $y = 9$ ,  
 $9 = \frac{180}{x}$   
 $9x = 180$   
 $x = 20$  A

**8** The maximum value occurs when  $x = 1.5$ .



When  $x = 1.5$ ,  
 $y = 3 \times 1.5 - (1.5)^2$   
 $= 2.25$  B

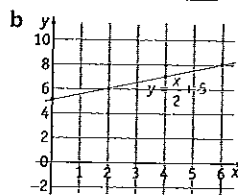
**9**  $y = 2x + 1$   
 When  $x = 4$ ,  
 $y = 2 \times 4 + 1$   
 $= 9$   
 $y = x^2 - 2x + 1$   
 When  $x = 4$ ,  
 $y = 4^2 - 2 \times 4 + 1$   
 $= 9$   
 $y = \frac{36}{x}$   
 When  $x = 4$ ,  
 $y = \frac{36}{4}$   
 $= 9$   
 All pass through the point (4, 9). D



$y = a^x$   
 When  $x = 6$ ,  $y = 3$   
 $\therefore a^6 = 3$   
 $2(a^6) = 2 \times 3$   
 $= 6$  C

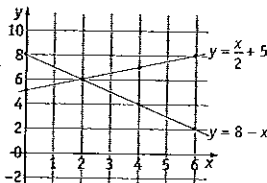
**11** a  $y = \frac{x}{2} + 5$

x	0	2	4	6
y	5	6	7	8

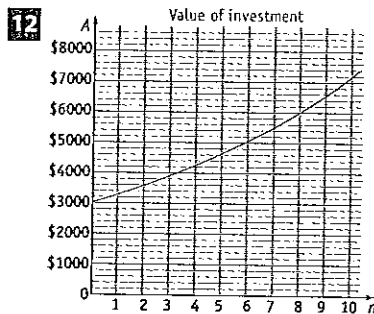


c  $y = 8 - x$

x	0	2	4	6
y	8	6	4	2



d The point of intersection is at (2, 6). ✓



- a When  $n = 3$ ,  $A \approx 3850$  ✓  
 b When  $A = 6000$ ,  $n \approx 8$   
 The sum of money will have doubled in a little over eight years. ✓  
 c Yes, Jessie is right. ✓  
 The sum of money doubles roughly every 8 years. There is approximately \$6000 in the account after 8 years so there will be about twice that amount in another eight years. ✓

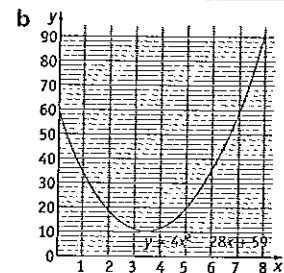
**13** a  $V = kr^3$   
 When  $r = 6$ ,  $V = 972$   
 $972 = k \times 6^3$   
 $972 = 216k$   
 $k = 972 \div 216$   
 $= 4.5$  ✓

b  $V = 4.5r^3$   
 When  $r = 16$ ,  
 $V = 4.5 \times 16^3$   
 $= 18\,432$   
 The volume of a cone with radius 16 cm would be  $18\,432 \text{ cm}^3$ . ✓

c  $V = 4.5r^3$   
 When  $V = 7776$ ,  
 $7776 = 4.5r^3$   
 $r^3 = 1728$   
 $r = \sqrt[3]{1728}$   
 $= 12$   
 The radius of a cone with volume  $7776 \text{ cm}^3$  would be 12 cm. ✓

**14** a  $y = 4x^2 - 28x + 59$

x	0	1	2	3	4	5	6	7	8
y	59	35	19	11	19	35	59	91	



- c The minimum value will occur when  $x = 3.5$ . ✓  
 $y = 4x^2 - 28x + 59$   
 When  $x = 3.5$ ,  
 $y = 4 \times (3.5)^2 - 28 \times 3.5 + 59$   
 $= 10$   
 The minimum value of  $y$  is 10. ✓