

Unit 5 Test: Algebraic Modelling

Algebraic Skills and Techniques and Modelling Linear and Non-linear Relationships

Remember: these are HSC-type questions.

Time allowed: 1 hour 10 minutes Total marks: 50

Part A

(Suggested time: 30 minutes)

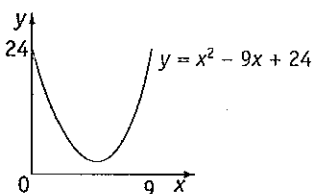
Choose the correct answer (A, B, C or D) for each question.

One mark each

- 1 $2x^2 + 3x \times 4x = ?$
 A $20x$
 B $20x^2$
 C $14x^2$
 D $2x^2 + 12x$

- 2 If $a = 3$, what is the value of $4a^2 - a$?
 A 141
 B 46
 C 1846
 D 33

- 3 The diagram shows the graph of $y = x^2 - 9x + 24$.



What is the minimum value of y ?

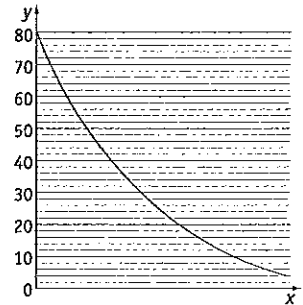
- A 0
 B 3.75
 C 4.5
 D There is not enough information to find the minimum value.

- 4 $y = kt^3$. When $t = 6$, $y = 27$.
 What is the value of t for which $y = 64$?
 A 4
 B 8
 C 16
 D 64

- 5 The population of one country is 3.2×10^7 and that of another country is 4.8×10^8 . How many times larger is the population of the second country?
 A 15
 B 44.8
 C 1.6 million
 D 448 million

- 6 The values have been mistakenly erased from the x -axis. The equation of the relationship shown in the graph could be

- A $y = \frac{80}{x}$
 B $y = 80(0.7)^x$
 C $y = 80(1.2)^x$
 D $y = 80 - x^2$



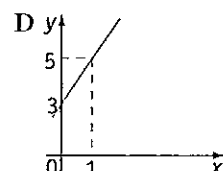
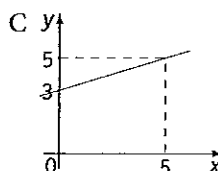
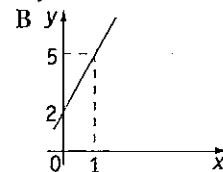
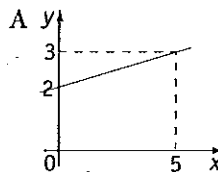
- 7 Given that $y = ax^3$, which statement is correct?

- A $x = \sqrt[3]{\frac{y}{a}}$
 B $x = \frac{\sqrt[3]{y}}{a}$
 C $x = \sqrt[3]{\frac{a}{y}}$
 D $x = \frac{\sqrt[3]{a}}{y}$

- 8 The total cost to rent a large house is \$480 per week. What is the least number of people who would need to share the house so that the rent per person is less than \$75?

- A 5
 B 6
 C 7
 D 8

- 9 Which could be the graph of $y = 2x + 3$?



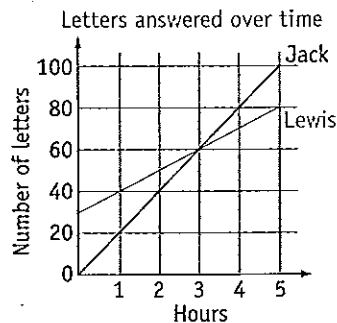
19 Given that $V = \frac{1}{3}\pi r^2 h$, find r , to one decimal place,

- if $V = 1020$ when $h = 4$.
- A 5.2 B 20.8
C 15.6 D 62.4

15 If $5000 = 27\,500(0.88)^n$, between what two values must n lie?

- A 11 and 12 B 12 and 13
C 13 and 14 D 14 and 15

12 Jack and Lewis work in an office answering letters. The graph shows the number of letters from a particular batch that both have answered at different times over a five-hour shift.



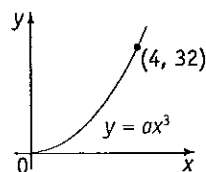
During this shift, how many more letters did Jack answer than Lewis?

- A 20 B 30
C 50 D 100

18 If $p = 72$, $q = 18$ and $r = 12$, what is the value of $\frac{p}{q+r}$?

- A 16 B 24
C 3 D 2.4

14 The diagram shows the graph of $y = ax^3$. What is the value of a ?



- A 2 B 4
C $\frac{1}{4}$ D $\frac{1}{2}$

15 $3(4m + 1) - 2(3m - 1) = ?$

- A $6m$ B $6m + 1$
C $6m + 2$ D $6m + 5$

16 $\frac{5a^2 \times 3a}{12a^3 \div 2a} = ?$

- A $\frac{5a}{2}$ B $\frac{5}{2a}$
C $\frac{5a}{8}$ D $\frac{5}{8a}$

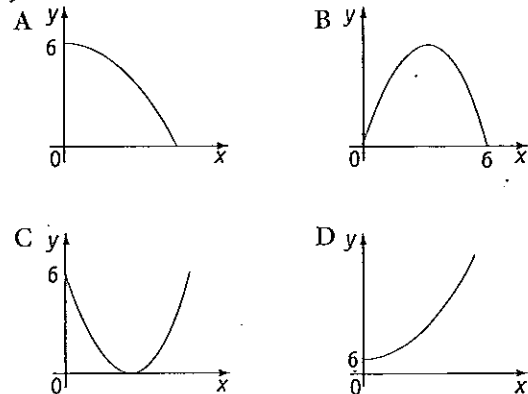
17 Which is a solution to the equation $2x^2 + 5 = 11$?

- A $x = \sqrt{3}$ B $x = \frac{\sqrt{6}}{2}$
C $x = \sqrt{6}$ D $x = \sqrt{8}$

18 The formula $S = V - 250n$ is used to find the value of a particular asset after n years. If $V = 3000$, what value of n does not make sense?

- A $n = 0$ B $n = 9\frac{1}{2}$
C $n = 12$ D $n = 15$

19 Which of the following could represent the equation $y = 6x - x^2$?



20 The sides of a cube are 0.004 m in length. What is the volume of the cube in cubic metres?

- A 6.4×10^{-6} B 6.4×10^{-7}
C 6.4×10^{-8} D 6.4×10^{-9}

Part B

(Suggested time: 40 minutes)

Show all working.

30 marks

- 21** Given that $s = ut + \frac{1}{2}at^2$, find s when $u = 32$,
 $t = 5$ and $a = 10$. 1 mark

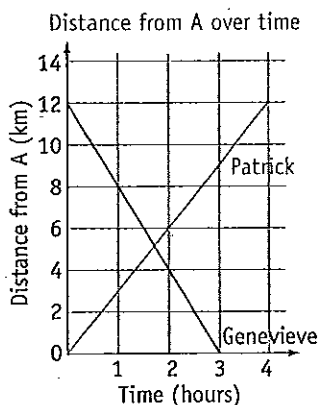
- 22** Simplify $5x^2 - (x^2 + 4x - 1)$. 2 marks

- 23** The population of a particular species on an island can be approximated by the expression $P = 50(0.84)^t$ where t is the number of years since 1991.
- What was the population in 1991? 1 mark
 - Use the estimation and refinement technique to find in what year the population will fall below 15. 2 marks
 - Explain why the expression will work as an approximation for only about twenty years. 2 marks

- 24** a Make I the subject of the formula $F = \frac{2LI^2}{d}$.
(You may assume $I > 0$.) 2 marks
- b If $F = 2.7 \times 10^4$, $d = 1.2$ and $L = 40$, find I .
Give the answer to one decimal place. 2 marks

- 25** Simplify $\frac{3x^2y^4 \times 2xy^3}{4x^3y^5}$ 2 marks

- 26** Patrick leaves point A and runs on a straight road to point B. At the same time, Genevieve leaves B and runs to A. The diagram shows their distance from A over time.



- How far is it from A to B? 1 mark
- How far from point B are they when they meet? 1 mark
- Who completes the journey in the shortest time? Justify your answer. 2 marks
- Write an expression for the distance (d km at t hours) of Genevieve from point A at time t . 2 marks

- 27** The number of stalls that can operate at a particular market varies inversely with the distance between the stalls. There will be 40 stalls if the distance between them is 7 metres.

- How many stalls are there if the distance between them is 5 metres? 2 marks
- What is the distance between the stalls if there are 35 stalls? 1 mark

- 28** a Complete the table of values for $y = 5 + 12x - 2x^2$.
- | | | | | | | | | |
|-----|---|---|---|---|---|---|---|---|
| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| y | | | | | | | | |
- 2 marks
- Sketch the graph of $y = 5 + 12x - 2x^2$. 2 marks
 - What is the maximum value of $y = 5 + 12x - 2x^2$? 1 mark
 - Use your calculator to find a value of x , to one decimal place, for which $y = 0$. 2 marks

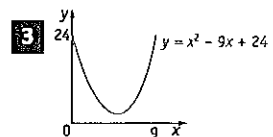
Go to p 294 for Quick Answers
or to pp 362–4 for Worked Solutions

Solutions

Unit 5 Test p270

1 $2x^2 + 3x \times 4x = 2x^2 + 12x^2$
 $= 14x^2$ C

2 If $a = 3$,
 $4a^2 - a = 4 \times 3^2 - 3$
 $= 33$ D



The minimum value occurs when
 $x = 4.5$.

When $x = 4.5$,
 $y = (4.5)^2 - 9 \times 4.5 + 24$
 $= 3.75$ B

4 $y = kt^3$
 When $t = 6$, $y = 27$
 $27 = k \times 6^3$
 $27 = 216k$
 $k = 0.125$
 $\therefore y = 0.125t^3$
 When $y = 64$
 $64 = 0.125t^3$
 $t^3 = 512$
 $t = 8$

B

5 Population of 2nd country is $\frac{4.8 \times 10^8}{3.2 \times 10^7}$ times larger than the first country.
 $\frac{4.8 \times 10^8}{3.2 \times 10^7} = 15$

A

6 The graph is an example of exponential decay.

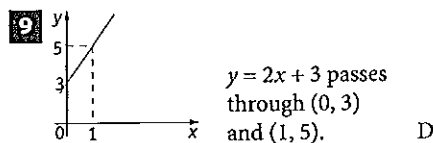
B

7 $y = ax^3$
 $x^3 = \frac{y}{a}$
 $x = \sqrt[3]{\frac{y}{a}}$

A

8 Number of people = $\$480 \div \$75 = 6.4$
 Seven people is the least number.

C



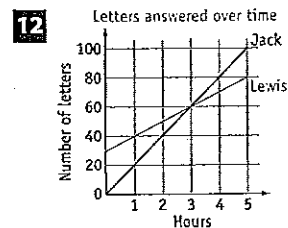
D

10 $V = \frac{1}{3}\pi r^2 h$
 If $V = 1020$ when $h = 4$,
 $1020 = \frac{1}{3}\pi r^2 \times 4$
 $r^2 = 1020 \div \left(\frac{1}{3}\pi \times 4\right)$
 $= 243.507 \dots$
 $r = \sqrt{243.507 \dots}$ ($r > 0$)
 $= 15.6$ (1 d.p.)

C

11 If $n = 13$,
 $27\,500(0.88)^{13} = 5219.241 \dots$
 If $n = 14$,
 $27\,500(0.88)^{14} = 4592.9329 \dots$
 Between 13 and 14.

C



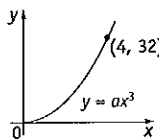
During the shift Jack answers 100 letters and Lewis answers 50 [80 - 30].
 Difference = 50.

C

13 If $p = 72$, $q = 18$ and $r = 12$,
 $\frac{p}{q+r} = \frac{72}{18+12} = 2.4$

D

14 $y = ax^3$
 When $x = 4$, $y = 32$
 $32 = a \times 4^3$
 $64a = 32$
 $a = \frac{32}{64} = \frac{1}{2}$



D

15 $3(4m + 1) - 2(3m - 1)$
 $= 12m + 3 - 6m + 2 = 6m + 5$

D

16 $\frac{5a^2 \times 3a}{12a^3 \div 2a} = \frac{15a^3}{6a^2} = \frac{5a}{2}$

A

17 $2x^2 + 5 = 11$
 $2x^2 = 6$
 $x^2 = 3$
 $x = \sqrt{3}$ ($x > 0$)

A

18 $S = V - 250n$
 $V = 3000$,
 $S = 3000 - 250n$
 If $n = 15$,
 $S = 3000 - 250 \times 15 = -750$
 But S is the value of an asset. It cannot be negative.

D

19 $y = 6x - x^2$
 When $x = 0$,
 $y = 6 \times 0 - 0^2 = 0$
 Only B passes through $(0, 0)$.

B

20 $V = (0.004)^3 = 6.4 \times 10^{-8}$

C

21 $s = ut + \frac{1}{2}at^2$
 When $u = 32$, $t = 5$ and $a = 10$
 $s = 32 \times 5 + \frac{1}{2} \times 10 \times 5^2 = 285$

✓

22 $5x^2 - (x^2 + 4x - 1) = 5x^2 - x^2 - 4x + 1 = 4x^2 - 4x + 1$

✓

23 $P = 50(0.84)^t$
 a In 1991, $t = 0$
 $P = 50(0.84)^0 = 50$
 In 1991 the population was 50.

✓

b When $t = 6$,
 $P = 50(0.84)^6 = 17.5649 \dots$
 When $t = 7$,
 $P = 50(0.84)^7 = 14.7545 \dots$
 The population will fall below 15 in the 7th year. In 1998.

✓

✓

c When $t = 20$,
 $P = 50(0.84)^{20} = 1.529 \dots$
 The population has fallen below 2. There cannot be any change except for the demise of the remaining animal.

✓

✓

✓

24 a $F = \frac{2LI^2}{d}$
 $Fd = 2LI^2$
 $I^2 = \frac{Fd}{2L}$
 $I = \sqrt{\frac{Fd}{2L}}$ ($I > 0$)

✓

✓

b $F = 2.7 \times 10^4$, $d = 1.2$, $L = 40$
 $I = \sqrt{\frac{Fd}{2L}} = \sqrt{\frac{2.7 \times 10^4 \times 1.2}{2 \times 40}} = \sqrt{405} = 20.124\,6118 \dots = 20.1$ (1 d.p.)

✓

✓

✓

✓

25 $\frac{3x^2y^4 \times 2xy^3}{4x^3y^5} = \frac{6x^3y^7}{4x^3y^5} = \frac{3y^2}{2}$

✓

✓

26 a It is 12 km from A to B.
 b When they meet they are approx. 5 km from A. They are 7 km from point B.

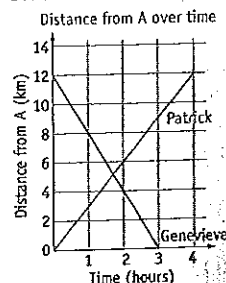
✓

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c Genevieve completes the journey in the shortest time. She begins from B whereas Patrick starts from A. Genevieve reaches A in 3 hours but Patrick takes 4 hours to reach B.

✓



$$d \quad m = \frac{\text{vertical change in position}}{\text{horizontal change in position}}$$

$$= \frac{-12}{3}$$

$$= -4 \quad \checkmark$$

Vertical intercept = 12

$$\therefore d = -4t + 12 \quad \checkmark$$

7 Let n be the number of stalls and d the distance between them.

a $n = \frac{k}{d}$

When $n = 40$, $d = 7$

$$40 = \frac{k}{7}$$

$$k = 40 \times 7$$

$$= 280 \quad \checkmark$$

$$\therefore n = \frac{280}{d}$$

If $d = 5$,

$$n = \frac{280}{5}$$

$$= 56$$

There will be 56 stalls if the distance between them is 5 metres. \checkmark

b $n = \frac{280}{d}$

If $n = 35$

$$35 = \frac{280}{d}$$

$$35d = 280$$

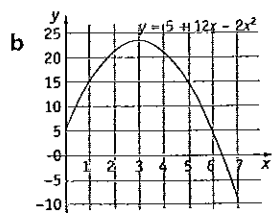
$$d = 8$$

If there are 35 stalls, the distance between them will be 8 metres. \checkmark

28 a $y = 5 + 12x - 2x^2$

x	0	1	2	3	4	5	6	7
y	5	15	21	23	21	15	5	-9

$\checkmark \checkmark$



$\checkmark \checkmark$

c The maximum value of $5 + 12x - 2x^2$ is 23 (when $x = 3$). \checkmark

d From the graph $y = 0$ when x is between 6 and 7.

When $x = 6.5$,

$$y = -1.5$$

When $x = 6.3$,

$$y = 1.22$$

When $x = 6.4$,

$$y = -0.12$$

The value of x for which $y = 0$ is $x = 6.4$ to one decimal place. $\checkmark \checkmark$