



14

Mathematics and resources

The main mathematical ideas investigated are:

- ▶ interpreting information, making comparisons and performing calculations in relation to requirements and data relevant to water availability and usage
- ▶ performing calculations involving length, area and volume in relation to land and catchment areas and water storage
- ▶ estimating methods for area and volume
- ▶ calculating and comparing household electricity consumption and costs
- ▶ calculating and interpreting statistics related to electricity consumption and cost.

FOCUS STUDY

Syllabus references: FSRe1, FSRe2, FSRe3
Outcomes: MG2H-1, MG2H-2, MG2H-3, MG2H-4,
MG2H-5, MG2H-7, MG2H-9, MG2H-10

14A

Water availability and usage

EXERCISE 14A

- 1 a Use the information in the sample water bill on the opposite page to answer the following questions.
- What is the supply period for this bill?
 - How many days is this?
- b The fixed charges are those payable for having water and sewerage available to your property. What is the total of the fixed charges for this account?
- c
- How much water did this household use for the period shown?
 - Using the answer to part a ii, calculate the average daily water usage.
- d The usage charge is what you pay for the amount of water that you use.
- What is the cost per kilolitre of water?
 - What is the total cost (fixed and usage) for this period?
 - Using the answer to part a ii, calculate the average daily cost.
- e Assuming the same fixed and usage charges, was this bill smaller or larger than:
- the last bill?
 - the bill for the same period last year?
- f Assume the same fixed and usage charges.
- Calculate the water usage charge for the last bill.
 - Calculate the total charges for the last bill.
- g The water usage charge last year was \$2.13 per kL. What is the difference between the water usage charge for this bill and that for the bill for the same period last year?
- h How did the average daily water consumption for this bill compare with the local area average?
- i The table 'Targets for water-efficient households' shown on the bill allows you to compare a household's water usage with the supplier's targets for ideal water-efficient households. This bill is for a medium-sized property with five occupants.
- Is this household's usage smaller or larger than the ideal efficient usage shown in the table?
 - By how much would this household have to change its daily usage to reach the ideal?
 - If a bucket can hold 12 L of water, how many buckets is this equivalent to?
- 2 Use the water-efficient targets table to determine the ideal daily water consumption for these households.
- | | |
|---|--|
| a a large property with four people | b a small property with two people |
| c a 600 m ² property with three people | d an 18 m by 40 m property with six people |
- 3 In 2013, residential properties in the region managed by Hunter Water were charged a water availability fee of \$18.92 p.a. and a water usage rate of \$2.08/kL. Calculate the total annual cost for a property that uses 190 kL of water.
- 4 a In 2013, residential properties managed by Shoalhaven Water were charged \$1.55/kL for water usage up to 450 kL and \$1.75/kL for usage over 450 kL. The water availability charge was \$81. What is the annual cost of water for a household that uses:
- 270 kL?
 - 480 kL?

MONT BAY WATER

Mr S Ample
12 Water Street
NORTH WATERVILLE

Customer Number	22988701
Due Date	23 April 2013
Amount	\$348.99

ACCOUNT DETAILS

Account for residential property

Usage Charges (GST does not apply)
For period 01/01/2013 to 31/03/2013 (90 days)

Water Usage	
68 kL @ \$2.17 per kL =	\$147.56
Total Usage Charges	\$147.56

Service Charge Details (GST does not apply)

Water Service Charge	\$47.68
Sewerage Service Charge	\$153.75
Total Service Charges	\$201.43

Total Current Charges **\$348.99**

YOUR CHARGES EXPLAINED

Water Usage

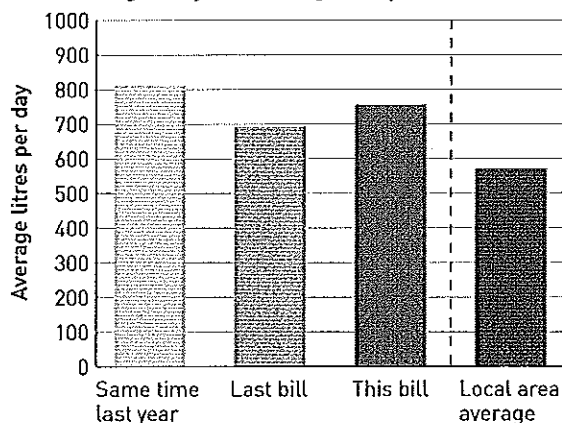
Recorded by your water meter, this charge covers the amount of water used at your property. This charge is billed per kilolitre (kL) of water you use.

Service Charges

Your water and sewerage service charges are fixed charges for access to our water supply and sewerage systems. They also help us maintain, renew and expand these systems so we can continue to provide you with high quality drinking water and safe sewerage removal now and into the future.

NOTICEBOARD

Your average daily water usage comparison



Targets for water-efficient households

People per household	Property size		
	Small	Medium	Large
1 person	247 L/day	249 L/day	267 L/day
2 people	367 L/day	375 L/day	396 L/day
3 people	463 L/day	477 L/day	498 L/day
4 people	546 L/day	565 L/day	587 L/day
5 people	621 L/day	645 L/day	666 L/day
6 people	689 L/day	718 L/day	739 L/day

Note:

A small property has an area of <math><500\text{ m}^2</math>.

A medium property has an area between

A large property has an area between

- b For non-residential properties, the water usage charge is \$1.55/kL. The water access charge is based on the size (diameter) of the water meter service connection(s), as shown. If you have more than one connection you must pay for each one. What would be the water availability charge for a non-residential property that is supplied by:
- one 32 mm connection?
 - one 25 mm and one 40 mm connection?
 - one 50 mm connection?
 - two 25 mm connections?

Meter connection	Charge (\$)
20 mm	81
25 mm	127
32 mm	207
40 mm	324
50 mm	506
100 mm	2025

- c Explain why the answers to parts b iii and iv are not the same. (Consider the cross-sectional areas of the pipes supplying the water.)
- d A non-residential property has one 20 mm, one 32 mm and one 40 mm connection. If the amount of water used is 560 kL, calculate the total annual cost.

- 5 The charges by Kempsey Shire Council for water access in 2012 are given in the table. The water supply usage charges are listed below.

Residential usage

For 0–250 kL: charge is \$1.67/kL

Each kL >250 kL: charge is \$2.35/kL

Non-residential usage

All consumption: charge is \$1.67/kL

- a Calculate the total annual cost of water for a residential property with a 20 mm connection that uses:
- 210 kL
 - 330 kL
- b Calculate the total annual cost of water for a non-residential property that uses 480 kL of water and has one 20 mm and one 50 mm connection.

Meter connection	Charge (\$)
20 mm	235
25 mm	367
32 mm	602
40 mm	940
50 mm	1469
100 mm	5875

- 6 The average rate of flow of a bathroom shower with a normal shower rose is 18 L/min.
- Jenny has two 8 minute showers each day. Calculate the annual cost of Jenny's showers if the water usage charge is \$2.17/kL. (Use 1 year = 365 days.)
 - Jenny's brother Sam has one 20 minute shower each day. What is the annual cost of Sam's showers?
 - How much could Jenny and Sam save each year by using a water-efficient shower rose that only uses 8.5 L/min?
- 7 A 7.0 kg top-loading washing machine, which costs \$556, uses 170 L of water per wash. A 7.0 kg front-loading machine by the same manufacturer costs \$660 and uses 80 L per wash. A large family who use their washing machine 4 times per week is considering whether to buy a top-loading or front-loading machine.
- Calculate the annual water usage charges for each machine if water costs \$2.25/kL. (Use 1 year = 52 weeks.)
 - What are the annual savings in water usage costs if they purchase the front loader?
 - What is the difference in the purchase prices of the machines?
 - If they purchase the front loader, how long will it take to break-even on total costs (purchase price and water usage)?
- 8
- There are 4 people in the Lee family. How many litres of water per year would they save by replacing their full-flush toilet with a 3-star rated dual-flush toilet, if the full-flush toilet uses 54 L per person per day and the 3-star rated dual-flush toilet uses 18 L per person per day?
 - What would be the cost savings if the water usage charge is \$2.75/kL?

INVESTIGATION 14.1

- 9 A rectangular backyard swimming pool is 10.3 m by 3.9 m and has an average depth of 1.4 m.
- How much water does it hold?
 - What is the cost of filling the pool if the water usage charge is \$1.98/kL?

10 The table below shows water usage data for the states and territories of Australia in 2010/11.

	NSW	VIC	QLD	SA	WA	TAS	NT	ACT	AUS
Total water consumption (GL)	5041	2359	2964	1023	1369	371	167	43	13 337
Average price of water for households (\$/kL)	2.53	2.34	2.95	3.09	1.75	1.75	1.51	2.52	2.31
Household consumption (GL)	527	311	311	115	310	69	31	25	1699
Household use per capita (kL)	72	55	68	70	132	135	136	69	92
Re-use water consumed (GL)	161	78	56	25	18	7	1	4	351

- For which state or territory is the total water consumption:
 - highest?
 - lowest?
 - What percentage of the total Australian consumption was the NSW consumption?
 - Which state or territory paid more than the Australian average price for water?
 - For which state or territory was the price of water:
 - highest?
 - lowest?
 - What percentage was Victoria's household consumption of its total water consumption?
 - Which state or territory was the highest consumer of re-use water?
 - What percentage was this state's consumption of the total Australian consumption, for re-use water?
 - For which state or territory was the household use per capita:
 - highest?
 - lowest?
 - Calculate the total revenue from the sale of water for household consumption in Queensland.
 - For which state or territory was household use per capita above the Australian average?
- 11 a Refer to the table, which shows water consumption by industry sector in Australia (GL) over different periods. For which industry sector was the water consumption:
- highest?
 - lowest?

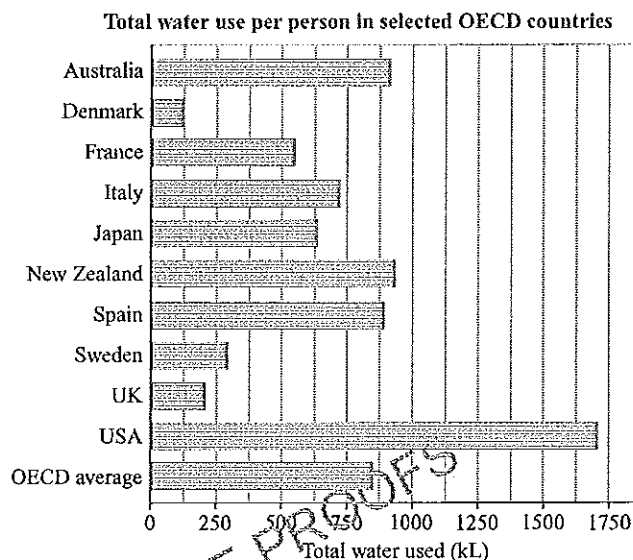
Industry sector	2000–01	2004–05	2008–09
Agriculture	14 989	12 191	6 996
Electricity and gas	255	271	328
Manufacturing	549	589	677
Forestry and fishing	40	47	101
Household	2 278	2 108	1 768
Mining	321	413	508
Water supply	2 165	2 083	2 396
Other industries	1 106	1 063	1 327
Total	21 703	18 767	14 101

- In which sectors was there a decrease in consumption from 2000–01 to 2008–09?
- What was the percentage increase in consumption for the sector 'Electricity and gas' from:
 - 2000–01 to 2004–05?
 - 2004–05 to 2008–09?
- What percentage of Australia's total water consumption in 2008–09 was in
 - agriculture?
 - mining?
 - manufacturing?
 - household?

- 12 Using the data for water prices in the table, calculate the percentage increase in the price from 2009/10 to 2010/11 for:
- household usage
 - agricultural usage
 - total usage.

	2009/10	2010/11
Household price (\$/kL)	2.10	2.44
Agricultural price (\$/kL)	0.11	0.14
Total price (\$/kL)	0.82	1.03

- 13 The graph shows the total water use per person per year across 10 OECD countries. (Total water use includes use for households, agriculture, industry, etc.)
- What is the OECD average?
 - For which country was the average
 - highest?
 - lowest?
 - Which countries had water use above the OECD average?



INVESTIGATION 14.2

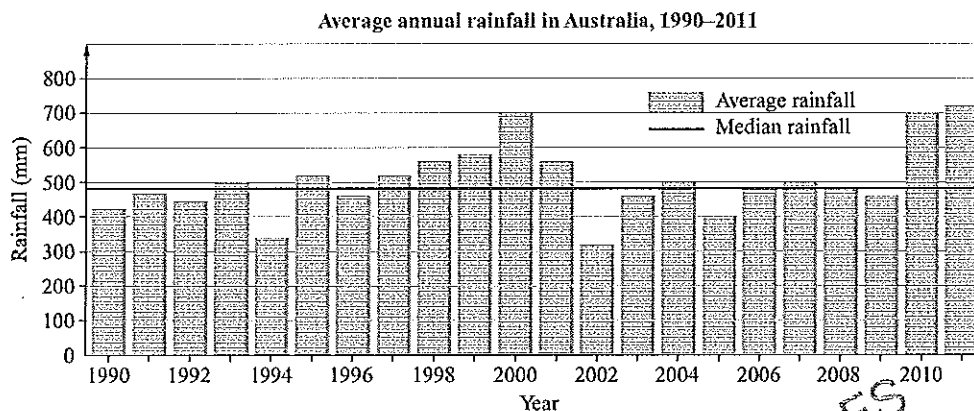
- 14 The table below is a global comparison of water resources and use in 2000/01.

Region	Available water per area (ML/ha)	Population density (People/km ²)	Available water per capita (ML/person/year)	Water consumed (1000 GL/year)
Australia	0.5	2.5	21.3	25
North America	2.8	20.7	13.4	603
Central America	11.2	115.7	9.6	23
Southern America	6.9	21.5	32.2	165
Western and Central Europe	4.3	107.1	4.0	265
Eastern Europe	2.5	11.5	21.4	110
Africa	1.3	32.7	4.0	215
Middle East	0.8	47.1	1.6	271
Central Asia	0.6	18.5	3.0	163
Southern and Eastern Asia	5.5	174.4	3.2	1991
Oceania and Pacific	1.1	3.3	33.0	26
World	3.2	50.4	6.4	3837

- Which region of the world has the most available water per area?
 - Does this region consume the most water per year?
- Which region of the world has the least available water per area?
 - Does this region consume the least water per year?

- c Consider the data for 'Available water per capita'.
- Draw a box-and-whisker plot for this data.
 - What is the range of the data?
 - What is the interquartile range?
 - Comment on the spread of the scores.
- d Consider the data for 'Population density'.
- Calculate the mean and standard deviation for this data.
 - Which region(s) have a population density that is more than one standard deviation above the mean?

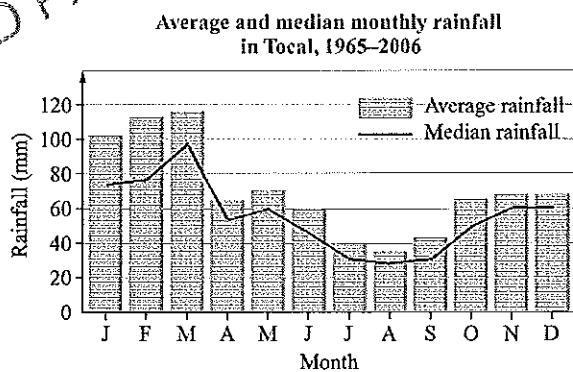
15 The graph shows the average annual rainfall for Australia from 1990 to 2011.



- In which years, in this period, did the highest and lowest rainfalls occur?
- In how many years in the decade 2000–2009 was the average rainfall below the median?
 - Comment on the weather for this decade.

16 The graph shows the mean and median monthly rainfall in Tocal from 1965 to 2006.

- For which season of the year was the rainfall:
 - greatest?
 - least?
- For each month the mean is greater than the median. Explain what weather conditions might have caused this to be so.



17 The table below shows the rainfall in mm over the summer months for Sydney for the decade 1990–1999. Summer in 1990 is December 1989 to February 1990.

	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999
Dec	161	44	190	202	36	81	102	40	29	55
Jan	84	120	66	61	52	95	150	186	99	178
Feb	631	63	409	100	99	51	55	137	31	143
Total										

- Complete the row for the total rainfall for each summer of this decade.
- Draw a column graph for the total rainfall data for part a.

c Calculate the mean and standard deviation of the total rainfall for each summer.

d Draw a box-and-whisker plot for the summer totals.

The table below shows the rainfall in mm over the summer months for Sydney for the decade 2000–2009.

	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Dec	82	46	40	75	60	76	25	74	123	54
Jan	56	186	98	14	51	68	121	45	57	25
Feb	19	109	348	59	129	125	51	108	258	128
Total										

e Complete the row for the total rainfall for each summer of this decade.

f Draw a column graph for the data of part e.

g Calculate the mean and standard deviation of the total rainfall for each summer.

h Draw a box-and-whisker plot for the summer totals.

i Using the information above, compare the summer rainfall for these two decades in Sydney. Write a short report and discuss your findings with the class.

18 The tables below show the summer rainfall in mm in Melbourne and Brisbane for the decade 2000–2009.

a Summer in Melbourne (2000–2009)

	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Dec	94	35	49	12	71	59	82	18	71	77
Jan	38	12	38	11	63	28	54	33	33	1
Feb	33	12	68	20	13	167	72	13	25	3
Total										

b Summer in Brisbane (2000–2009)

	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Dec	190	152	78	143	147	233	70	59	106	88
Jan	74	25	57	6	263	108	173	66	216	84
Feb	67	256	37	274	137	29	63	98	188	156
Total										

i Complete the row for total rainfall in each city.

ii Draw a column graph for the data of part i.

iii Calculate the mean and standard deviation of the total summer rainfall for each city.

iv Draw a box-and-whisker plot for the summer totals for each city.

c Compare the summer rainfall in each of these cities with the summer rainfall in Sydney for the decade 2000–2009.

19 The table shows the mean monthly rainfall in mm for Cairns and Alice Springs.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Cairns	419	422	460	264	111	73	39	42	44	50	98	203
Alice Springs	43	41	33	16	16	15	14	10	9	20	25	36

a Draw a box-and-whisker plot for each city.

b Compare the weather in these two cities. Write a short report and discuss it with the class.

- 20 a This table gives the mean number of days of rain for each month for all years of record for Sydney.

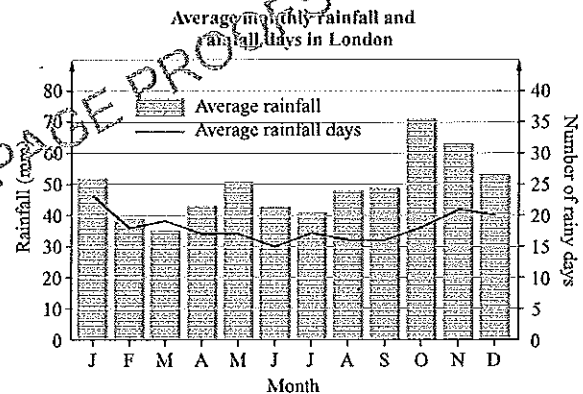
Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Number rainy days	12.2	12.5	13.6	12.8	13.1	12.5	11.2	10.4	10.6	11.7	11.7	11.5

- What is the mean number of days of rain for August?
 - Using this data, find the probability that it will rain on any particular day in August.
 - What is the probability that it will rain on two consecutive days?
- b This table gives the mean number of days of rain for each month for the years 1981–2010 for Sydney.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Number rainy days	12.3	12.9	13.3	11.1	12.2	10.5	10.2	8.4	8.8	11.1	12.7	11.2

- Use this data to find the mean number of days of rain for August.
 - What is the probability that it will rain on any particular day in August?
 - What is the probability that it will rain on two consecutive days?
- c Which set of data do you think would be the most accurate in predicting the probability that it will rain on any particular day in August next year? Discuss this with your class.
- 21 Go to the Bureau of Meteorology's website www.bom.gov.au/nsw and find the past weather data for your town. Use this data to calculate the probability that it will rain on your birthday next year.

- 22 The graph shows the average rainfall for each month in London, UK, and the average number of days of rain for each month.



- Does London have more rain in summer or winter?
- Which month is the:
 - wettest?
 - driest?
- Find the mean and standard deviation of the:
 - average monthly rainfall
 - average number of rainy days per month.
- Comment on the pattern of rainfall in London.
- Find the probability that it will rain next year on:
 - 5 August
 - 5 January.



23 The table shows the average rainfall and the average number of rainy days for each month in Tokyo, Japan.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Average rainfall (mm)	62	61	125	139	151	169	183	186	215	235	108	45
Average rainy days	11	10	16	17	18	19	18	16	18	17	12	9

- Draw a column graph for average rainfall each month.
- On the column graph draw a line graph for the number of rainy days.
- Does Tokyo have more rain in summer or winter?
- Which month is the:
 - wettest?
 - driest?
- Calculate the mean and standard deviation of the:
 - average monthly rainfall
 - average number of rainy days each month.
- Comment on the pattern of rainfall in Tokyo.
- Compare the rainfall in London with that in Tokyo.
- Find the probability that it will rain in Tokyo next year on:
 - 5 August
 - 5 January.

INVESTIGATION 14.3

14B Volume and collection of water

WORKED EXAMPLE 1

The surface area of a lake is $460\,000\text{ m}^2$. If an average of 6 mm of rain falls on the lake, what will be the increase in the volume of water in the lake? Ignore any run-off from the surrounding area.

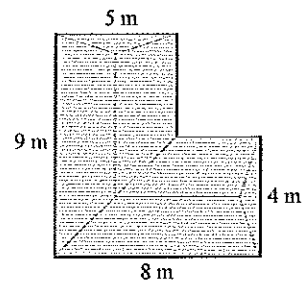
Solve	Think	Apply
$V = 460\,000 \times 0.006$ $= 2760\text{ m}^3$ $= 2\,760\,000\text{ L}$ or 2.76 ML	$6\text{ mm} = 0.006\text{ m}$ 1 m^3 holds 1000 L . $1\text{ ML} = 1\,000\,000\text{ L}$	Use $V = A \times h$.

EXERCISE 14B

- The surface area of a lake is $785\,000\text{ m}^2$. If an average of 4 mm of rain falls on the lake, what will be the increase in the volume of water in the lake? Ignore any runoff from the surrounding area.
- The rainfall over a rectangular paddock that measures 280 m by 196 m is 8 mm .
 - If all the rain flows into a dam on the property, what will be the increase in the volume of water in the dam?
 - If 20% of the rain soaks into the ground before flowing into the dam, what will be the increase in the volume of the dam?
- Lake Burragorang is formed by Warragamba dam, which supplies most of Sydney's population with water.
 - The area of Lake Burragorang is 75 km^2 . If an average of 3 mm of rain falls over the lake, what will be the increase in the volume of water in the lake?
 - The total catchment area for Lake Burragorang is 9051 km^2 . If an average of 1 mm of rain falls over the catchment and 85% of this runs into the lake, what will be the increase in the volume of water in the dam?

WORKED EXAMPLE 2

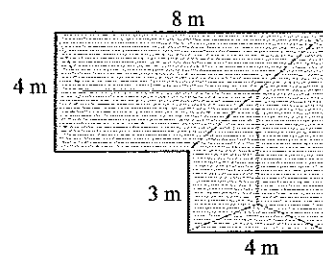
- a Calculate the plan-view area of the roof of the house shown.
 b How much water could be collected from this roof if 6 mm of rain fell on it, allowing 10% for wastage?



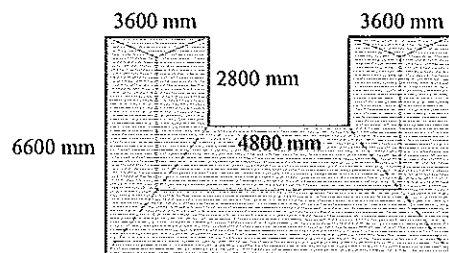
	Solve	Think	Apply
a	Area of roof = $5 \times 9 + 4 \times 3$ = 57 m^2	Divide the plan-view area into rectangles.	Find the plan-view area of the roof by dividing the shape of the roof into rectangles.
b	Volume of water collected = $0.9 \times 57 \times 0.006 \text{ m}^3$ = 0.3078 m^3 = 307.8 L = 308 L (to nearest L)	If all the rain is collected then $V = 57 \times 0.006 \text{ m}^3$. However it is usual to allow 10% for wastage. So $V = 90\%$ of 57×0.006 (90% is known as the water runoff coefficient).	Volume of rain collected $V (\text{m}^3) = C \times A \times R$ where C = the water runoff coefficient, as a decimal A = roof area (m^2) R = amount of rainfall (m)

Note: The plan-view area of the roof includes the width of the eaves and so, in general, is not the same as the floor area. The plan-view area is sometimes referred to as the 'footprint' of the 'drip line' of the house.

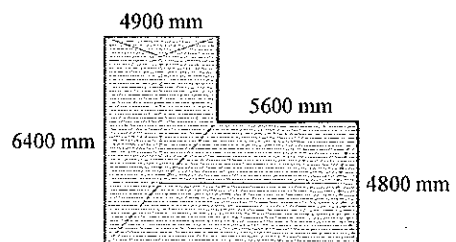
- 4 a i Calculate the plan-view area of the roof of the house shown.
 ii How much water could be collected from this roof if 4 mm of rain fell on it, allowing 10% for wastage?



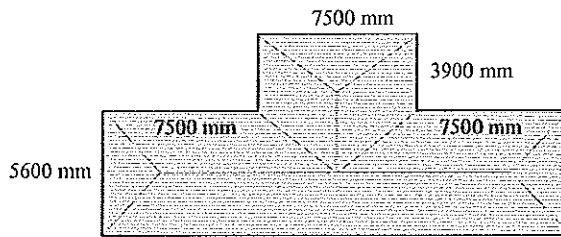
- b i Calculate the plan-view area of the roof of the house shown.
 ii This house is situated in Newcastle. The average rainfall for Newcastle in January is 91.4 mm. How much water could be expected to be collected from this roof in January next year, allowing 10% for wastage?



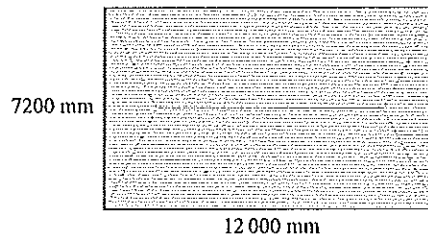
- c i Calculate the plan-view area of the roof of the house shown.
 ii This house is situated in Wollongong. The average rainfall for Wollongong in spring is 206.7 mm. How much water could be expected to be collected from this roof in spring next year, allowing 15% for wastage?



- d i Calculate the plan-view area of the roof of the house shown.
- ii This house is situated in Canberra. The average annual rainfall for Canberra is 633.1 mm. How much water could be expected to be collected from this roof next year, allowing 15% for wastage?



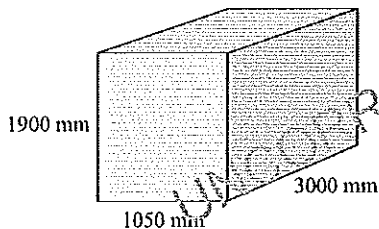
- e i Calculate the plan-view area of the roof of the house shown.
- ii This house is situated in Bourke. The average annual rainfall for Bourke is 354.7 mm. How much water could be expected to be collected from this roof next year?



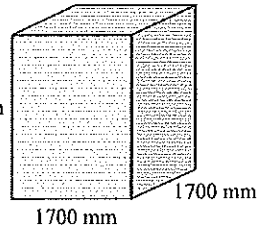
- iii Because of pollution it is necessary to discard the first 40 L each time there is a rain event (that is it rains). Water tanks often have a flushing device fitted to discard the first water collected. The mean annual number of rainy days in Bourke is 47. Treating each rainy day as a separate rain event, how much water will be discarded each year.
- v Calculate the amount of water that, on average, could actually be collected each year.
- vi The cost of town water in Bourke is \$1.86/kL. If all the water collected in the tank is used for household consumption, how much could this household save each year?

5 Calculate the capacity of the following water tanks to the nearest kL.

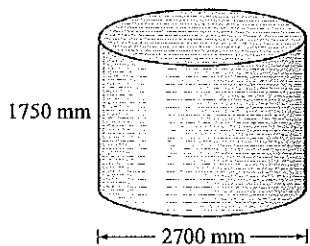
a



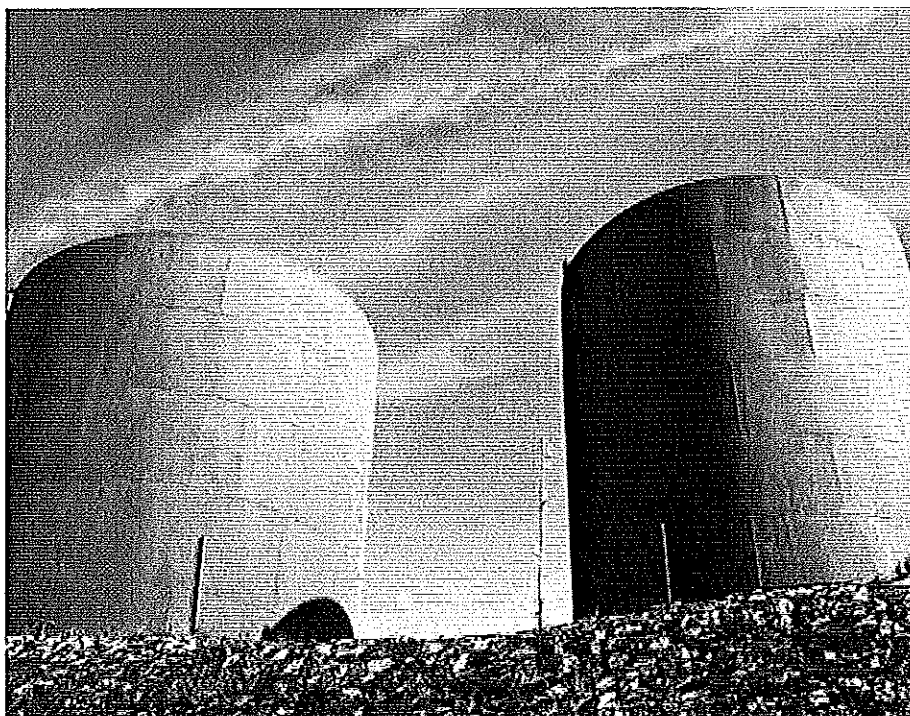
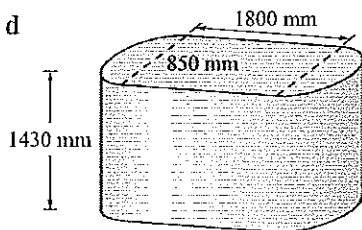
b



c



d



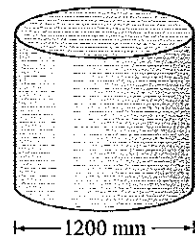
WORKED EXAMPLE 3

If 308 L of rain is collected in the tank shown, what will be the rise in the height of the water?

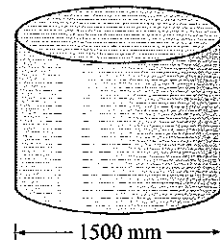


Solve	Think	Apply
Cross-sectional area of tank $= \pi \times 0.45^2 \text{ m}^2$ $= 0.636 \text{ m}^2$ Volume of water collected $= 0.308 \text{ m}^3$ $0.308 = 0.636 \times h$ $h = \frac{0.308}{0.636}$ $= 0.484 27\dots \text{ m}$ $= 484 \text{ mm (to nearest mm)}$ The water will rise by 484 mm.	Cross-sectional area of tank $= \pi \times r^2, r = 450 \text{ mm} = 0.45 \text{ m}$ Volume of water collected $= \frac{308}{1000} \text{ m}^3 (1 \text{ m}^3 = 1000 \text{ L})$	Calculate the horizontal cross-sectional area of the tank. Convert the volume of water to cubic metres. Use volume of water $= \text{cross-sectional area} \times \text{height}$

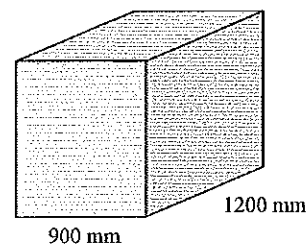
- 6 a If 295 L of rain is collected in the tank shown, what will be the rise in the height of the water?



- b If 370 L of rain is collected in the tank shown, what will be the rise in the height of the water?



- c If 300 L of rain is collected in the tank shown, what will be the rise in the height of the water?



UNCORRECTED PAGE PROOFS