Topic Test: Annuities and Loan Repayments

Remember: these are HSC-type questions.

Time allowed: 40 minutes **Total marks:** 25

Part A

(Suggested time: 15 minutes)

Choose the correct answer (A, B, C or D) for each question. One mark each

1 $2000 is placed in an account earning 8.4% p.a. interest, compounded monthly. Find the value (to the nearest dollar) of the investment after eight years.

A $3813 B $3907
C $148 496 D $252 806

2 What will be the monthly repayment on a loan of $90 000 over 15 years at 0.8% per month reducible interest?

A $945.24 B $536.72
C $1220 D $1476.94

3 What amount of money invested now would be equivalent to an annuity of $5000 per year for 25 years, compound interest rate of 7% p.a.?

A $58 268 B $80 709
C $316 245 D $171 640

4 If $1500 is invested every quarter into an account earning 6% p.a. interest, compounded quarterly, find the value of the investment at the end of twelve years.

A $51 064 B $97 305
C $81 147 D $104 348

5 The table shows the present value of an annuity of $1.

Period	Present values of an annuity of $1				
	Interest rate				
	1%	2%	6%	10%	12%
6	5.7955	5.6014	4.9173	4.3553	4.114
12	11.255	10.575	8.3838	6.8137	6.1944
18	16.398	14.992	10.828	8.2014	7.2497
20	18.046	16.351	11.470	8.5136	7.4694
24	21.243	18.914	12.550	8.9847	7.7843

Which has the greater present value?

I: an annuity of $500 a month for 2 years at 12% p.a., compounded monthly

II: an annuity of $2000 a year for six years at 10% p.a.

A I
B II
C the present value is the same
D there is not enough information

6 How much interest (to the nearest dollar) is earned if $1500 is invested into an account at the end of every year for three years, if the account pays 6.9% p.a. compound interest?

A $332 B $311
C $325 D $318

7 What amount of money would I need to invest at 7.8% p.a. interest, compounded monthly, to be worth $50 000 at the end of nine years?

A $24 836 B $25 433
C $29 854 D $34 871

8 How many years will it take for an annuity of $3000 per year to accumulate to $100 000 if interest is compounded annually at 7% p.a.?

A 16 B 17
C 18 D 19

9 Which would present the better option?

Option I: an annuity of $200 per month at 9% p.a., compounded monthly

Option II: an annuity of $500 per quarter at 10% p.a., compounded quarterly

A Option I
B Option II
C both options are the same
D there is not enough information

10 What amount, to the nearest $10, could be invested every month into an account earning 6% p.a. interest, compounded monthly, to accumulate to $70 000 at the end of ten years?

A $430 B $530
C $540 D $780

Part B (Suggested time: 25 minutes)

Show all working. 15 marks

11 Howard borrows \$60 000 over eight years. The interest rate is 8.4% p.a., charged monthly.

a Find the amount of each monthly repayment. 2 marks

b How much interest will Howard pay? 1 mark

12 a If Teagan invests \$400 every month into an account earning 0.8% per month compound interest, what will her investment be worth at the end of six years? 1 mark

b What sum of money would Teagan need to invest now to produce the same result? 1 mark

c After the end of six years Teagan keeps the money in the account but decides not to add any more amounts to it. What would her investment be worth at the end of a further four years? 1 mark

13 The table shows the future value of an annuity of \$1.

Period	Future values of an annuity of \$1				
	Interest rate				
	2%	3%	4%	5%	6%
2	2.0200	2.0300	2.0400	2.0500	2.0600
4	4.1216	4.1836	4.2465	4.3101	4.3746
6	6.3081	6.4684	6.6330	6.8019	6.9753
8	8.5830	8.8923	9.2142	9.5491	9.8975
10	10.950	11.464	12.006	12.578	13.181

Use the table to answer the following questions.

a What would be the future value of an investment of \$1200 every six months for five years if it earns interest at the rate of 6% p.a., compounded half-yearly? 1 mark

b Amber wants to have \$12 000 for an overseas holiday in two years time. If she invests \$1350 every quarter into an account earning 12% p.a., compounded quarterly, will she have enough? Justify your answer. 1 mark

c What amount of money needs to be invested every year into an account earning 5% p.a. interest to be worth \$30 000 after six years? 1 mark

14 Bailey wants to borrow \$85 000 and repay it over 15 years. One option he has is a fixed (reducible) interest loan with monthly repayments of \$773.54.

a How much will Bailey repay altogether if he takes this option? 1 mark

b Use your calculator and the estimation and refinement technique to find the monthly interest rate charged on the fixed rate loan. 2 marks

c Another option Bailey is considering is a loan with a variable interest rate. Bailey has calculated that with current interest rates he will save \$1717 with this loan. Give a reason why Bailey should not take this loan. 1 mark

15 Which is the better option when invested for four years at 0.5% per month compound interest? Justify your answer.

Option A: a lump sum of \$9000

Option B: an annuity of \$200 per month.

2 marks

Go to p 284 for **Quick Answers**
or to pp 304–5 for **Worked Solutions**

Solutions

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1 $P = \$2000,\ r = 0.007,\ n = 96$

$A = P(1+r)^n$

$= \$2000(1.007)^{96}$

$= \$3907$ (nearest dollar) **B**

2 $N = \$90\,000,\ r = 0.008,\ n = 180$

$$N = M\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\}$$

$$\$90\,000 = M\left\{\frac{(1.008)^{180} - 1}{0.008(1.008)^{180}}\right\}$$

$= M \times 95.213\,859\,53$

$M = \$90\,000 \div 95.213\,8593\ldots$

$= \$945.24$ (nearest cent) **A**

3 $M = \$5000,\ r = 0.07,\ n = 25$

$$N = M\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\}$$

$$= \$5000\left\{\frac{(1.07)^{25} - 1}{0.07(1.07)^{25}}\right\}$$

$= \$58\,268$ (nearest dollar) **A**

4 $M = \$1500,\ r = 0.015,\ n = 48$

$$A = M\left\{\frac{(1+r)^n - 1}{r}\right\}$$

$$= \$1500\left\{\frac{(1.015)^{48} - 1}{0.015}\right\}$$

$= \$104\,348$ (nearest dollar) **D**

5 I: $M = \$500,\ r = 1\%,\ n = 24$

Present value $= \$500 \times 21.243$

$= \$10\,621.50$

II: $M = \$2000,\ r = 10\%,\ n = 6$

Present value $= \$2000 \times 4.3553$

$= \$8710.60$

I has the greater present value. **A**

6 $M = \$1500,\ r = 0.069,\ n = 3$

$$A = M\left\{\frac{(1+r)^n - 1}{r}\right\}$$

$$= \$1500\left\{\frac{(1.069)^3 - 1}{0.069}\right\}$$

$= \$4818$ (nearest dollar)

Total contribution $= \$1500 \times 3$

$= \$4500$

Interest $= \$4818 - \$4500$

$= \$318$ **D**

7 $A = \$50\,000,\ r = 0.0065,\ n = 108$

$$N = \frac{A}{(1+r)^n}$$

$$= \frac{\$50\,000}{(1.0065)^{108}}$$

$= \$24\,836$ (nearest dollar) **A**

8 $A = \$100\,000,\ M = \$3000,\ r = 0.07$

$$A = M\left\{\frac{(1+r)^n - 1}{r}\right\}$$

$$\$100\,000 = \$3000\left\{\frac{(1.07)^n - 1}{0.07}\right\}$$

$n = 18$ **C**

[Using calculator, by trial and error.]

9 It is necessary to know the length of time of the investments to compare them. **D**

10 $A = \$70\,000,\ r = 0.005,\ n = 120$

$$A = M\left\{\frac{(1+r)^n - 1}{r}\right\}$$

$$\$70\,000 = M\left\{\frac{(1.005)^{120} - 1}{0.005}\right\}$$

$= M \times 163.879\,3468\ldots$

$M = \$70\,000 \div 163.879\,3468\ldots$

$= \$430$ (nearest ten dollars) **A**

11 a $N = \$60\,000,\ r = 0.007,\ n = 96$

$$N = M\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\}$$

$$\$60\,000 = M\left\{\frac{(1.007)^{96} - 1}{0.007(1.007)^{96}}\right\}$$ ✓

$= M \times 69.730\,979\,75\ldots$

$M = \$60\,000 \div 69.730\,979\,75\ldots$

$= \$860.45$ (nearest cent) ✓

b Total repaid $= \$860.45 \times 96$

$= \$82\,603.20$

Interest $= \$82\,603.20 - \$60\,000$

$= \$22\,603.20$ ✓

12 a $M = \$400,\ r = 0.008,\ n = 72$

$$A = M\left\{\frac{(1+r)^n - 1}{r}\right\}$$

$$= \$400\left\{\frac{(1.008)^{72} - 1}{0.008}\right\}$$

$= \$38\,741.8169$

$= \$38\,742$ (nearest dollar) ✓

b $$N = \frac{A}{(1+r)^n}$$

$$= \frac{\$38\,742}{(1.008)^{72}}$$

$= \$21\,828.491\,55$

$= \$21\,828$ (nearest dollar) ✓

c $A = P(1+r)^n$

$= \$38\,742(1.008)^{48}$

$= \$56\,792.054\,25$

$= \$56\,792$ (nearest dollar) ✓

13 a Interest rate per period = 3%

Number of periods = 10

Future value $= \$1200 \times 11.464$

$= \$13\,756.80$ ✓

b Interest rate per period = 3%

Number of periods = 8

Future value

$= \$1350 \times 8.8923$

$= \$12\,004.605$

$= \$12\,005$ (nearest dollar)

Amber will just have enough money. ✓

c Interest rate = 5% p.a., 6 years

$\$30\,000 = M \times 6.8019$

$M = \$30\,000 \div 6.8019$

$= \$4410.532\,351\ldots$

$= \$4411$ (nearest dollar) ✓

14 a Total repaid $= \$773.54 \times 12 \times 15$

$= \$139\,237.20$ ✓

b $N = \$85\,000$, $M = 773.54$, $n = 180$

$$N = M\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\}$$

$$\$85\,000 = \$773.54\left\{\frac{(1+r)^{180} - 1}{r(1+r)^{180}}\right\} \checkmark$$

Try $r = 0.006$

$N = \$85\,000$ (nearest dollar)

The monthly interest rate is 0.6%. ✓

c If interest rates rise Bailey could pay a lot more. ✓

15 Option B:

$M = \$200$, $r = 0.005$, $n = 48$

$$N = M\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\}$$

$$= \$200\left\{\frac{(1.005)^{48} - 1}{0.005(1.005)^{48}}\right\}$$

$= \$8516$ (nearest dollar) ✓

Option A ($9000) is the better option. ✓