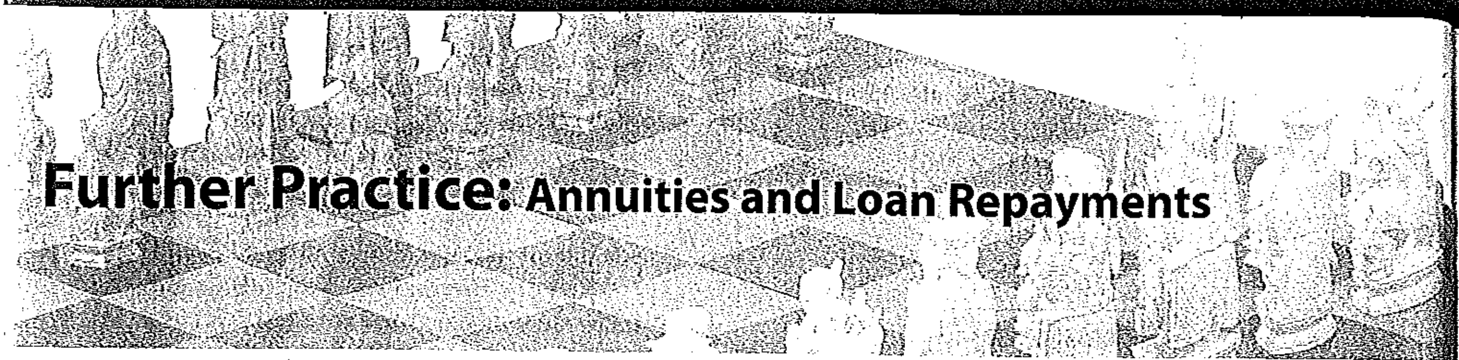

Further Practice: Annuities and Loan Repayments

Remember: all questions match the numbered examples on pages 19–29.

1 Find the amount to which $7000 accumulates if it is invested for five years at 8% p.a. interest, compounded annually.

2 Find the amount to which $95 000 accumulates when invested for eight years, if it is earning compound interest at the rate of 0.7% per month.

3 $46 000 is invested at 7.5% p.a., compounded monthly. What is the value of the investment, to the nearest dollar, after six years?

4 Find the amount of interest earned if $23 400 is invested for four years at 8% p.a. interest, compounded quarterly.

5 At the end of a year Jennifer opens an account by investing $2000. Interest of 7% p.a., compounded annually, is paid on the account.

a What will Jennifer's investment be worth at the end of two years, if she invests another $2000 at that time?

b What will Jennifer's investment be worth at the end of three years, if she invests another $2000 at that time?

6 Rosa invests $3000 every year into an account that pays 6% p.a. interest compounded annually. Find, to the nearest dollar, the value of Rosa's investment after 20 years.

7 Niamh wants to travel overseas in four years time and estimates that she needs $15 000 for the trip. She plans to invest $250 into an account every month until her departure. If the account pays 6% p.a. interest, compounded monthly, will she have saved enough money for the trip?

8 Every year Karl invests $4500 into an account that earns 6.5% p.a. interest, compounded annually.

a What is the investment worth after 35 years?

b How much interest will Karl have earned in that time?

9 Isaac intends to annually invest an amount of money into an account earning 5% p.a. interest, compounded annually. What amount, to the nearest $10, should Isaac invest in order to have $28 000 at the end of seven years?

10 Beth wants to buy a new car in three years time and estimates that she will need $25 000. How much, to the nearest dollar, will she need to invest each month into an account earning 0.48% per month compound interest?

11 Shane plans to retire in 15 years time and wants to have saved $300 000 by that time. He decides to save $1000 a month and places it in an account earning 7.2% p.a. interest, compounded monthly.

a How much more than his goal will Shane have saved?

b How much less could he save each month in order to achieve his goal?

12 What sum of money would I need to invest today to give the same result as investing $6000 every year for seven years at the interest rate of $7\frac{1}{2}$ % p.a., compounded annually?

13 Harrison intends to save $180 every month for six years. Interest of 6.6% p.a., compounded monthly, is paid on the investment. How much would Harrison need to invest now, at the same interest rate for the same length of time, to achieve the same result?

14 Which would give the best result if placed in an account at 0.8% per month compound interest for five years? An annuity of $250 per month, or a lump sum of $12 000?

15 The future value of an annuity is $47 000 when the investment is over twelve years and earning 4.5% p.a. interest, compounded monthly. Find the present value.

16 What amount of money needs to be invested now at 8% p.a. interest, compounded quarterly, to be worth $20 000 after six years?

17 What sum of money, to the nearest dollar, would need to be saved each month to be equivalent to a lump sum of $25 000 if both were invested at 0.7% per month compound interest over four years?

18 Sid borrows $100 000 and agrees to repay it in regular monthly instalments over 12 years. Compound interest of 6.6% p.a., compounded monthly, is charged on the loan. Find the amount of each monthly instalment.

19 Trent intends to borrow $175 000 and repay it over 25 years. Interest of 8.1% p.a., compounded monthly, is charged on the loan.

a Find the amount of each monthly instalment.

b Find the total amount repaid.

c Find the amount of interest that Trent will pay.

20 At the end of four years Antonia estimates that she will need $18 000 to replace her car. How much should she save each month at 0.7% per month compound interest?

21 What sum could I invest now to produce the same result as $750 invested every quarter for five years at 6% p.a. interest, compounded quarterly?

22 Which would produce the better result at the end of eight years?

Option 1: a sum of $5000 invested at 7% p.a. interest, compounded annually.

Option 2: an annuity of $70 per month, earning 0.55% per month interest.

23 Brock takes out a loan of $330 000 and agrees to repay it in equal monthly instalments over 30 years. Interest of 7.2% p.a. is compounded monthly. Find the total amount that Brock repays.

24 The table gives the future value of an annuity of $1 for different interest rates and periods of time.

Period	Future values of an annuity of $1				
	Interest rate				
	2%	2.5%	4%	5%	8%
2	2.0222	2.0250	2.0400	2.0500	2.0800
4	4.1216	4.1525	4.2465	4.3101	4.5061
6	6.3081	6.3877	6.6330	6.8019	7.3359
8	8.5830	8.7361	9.2142	9.5491	10.637
10	10.950	11.203	12.006	12.578	14.487
12	13.412	13.796	15.026	15.917	18.977

a What would be the future value of an investment of $3000 per period at 5% per period for 10 periods?

b $4500 is invested every year for eight years at 4% p.a. compound interest. What is the future value?

c Which option would give the better result at the end of three years?

Option X: an annuity of $1200 every six months at 5% p.a. compounded half-yearly.

Option Y: an annuity of $560 a quarter at 8% p.a. compounded quarterly.

d An amount of $$M$ per period is invested at 8% per period. After 12 periods the value of the investment is $170 793. Find M.

e What sum of money would need to be invested each quarter to produce $16 598.59 after two years at 10% p.a. interest, compounded quarterly?

25 The table shows the present value of an annuity of $1.

Period	Present values of an annuity of $1				
	Interest rate				
	1%	2%	4%	6%	8%
4	3.9020	3.8077	3.6299	3.4651	3.3121
8	7.6517	7.3255	6.7327	6.2098	5.7466
12	11.255	10.575	9.3851	8.3838	7.5361
16	14.718	13.578	11.652	10.106	8.8514
20	18.046	16.351	13.590	11.470	9.8181
24	21.243	18.914	15.247	12.550	10.529

a What is the present value of an annuity of $400 invested for 20 periods at 4% per period?

b What is the present value of an annuity of $210 per month for two years at 12% p.a. interest, compounded monthly?

c An amount of $8825.70 invested now would produce the same result as an annuity of $$M$ per period for 16 periods at 2% per period compound interest. Find M.

d What amount, to the nearest dollar, would need to be repaid every year for eight years to repay a loan of $88 000 at 6% p.a. compound interest?

26 Samuel borrows $200 000. The interest rate is 7.5% p.a., compounded monthly.

a For the first three years of the loan, Samuel pays interest only. How much will he pay every month?

b If the loan is to be repaid in total over 25 years, how much extra will Samuel need to pay every month for the final 22 years?

c What would be the repayments if Samuel paid equal instalments over 25 years?

d How much extra will Samuel pay in total by taking the interest-only option for the first three years?

e Give a reason why Samuel should take the interest-only option.

f Give a reason why Samuel should not have taken the interest-only option.

27 Johanna wants to borrow $13 000 on a reducing-balance loan and repay it in yearly instalments over 6 years. She has the option of choosing a fixed interest rate of 9% p.a. or a variable rate. The current variable rate is 8.4% p.a.

a How much will Johanna repay in total if she chooses the fixed-rate option?

b What would be the yearly instalments when $r = 0.084$?

c Complete the following table for two years, assuming the interest rate is 8.4%.

Year	P	I	$P + I$	$P + I - R$
1	$13 000			
2				

d What is the value of P at the beginning of the third year?

e If from the beginning of the third year the variable interest rate rose to 9.5%, find the new yearly instalments.

f Would Johanna have been better off financially if she chose the fixed rate or the variable rate (assuming that it is 8.4% for two years and 9.5% for the remaining four years)? Justify your answer with appropriate calculations.

28 Russell wants to take out a loan for $80 000.

a He must pay stamp duty on the loan. He pays $8 for the first $15 000 and $3 for every additional $1000. Find the amount of stamp duty.

b Russell's bank charges establishment fees totalling $750. Find the total amount of the loan, establishment fees and stamp duty.

c If Russell repays that total amount over 12 years and the interest rate is 6.3% p.a., compounded monthly, find the amount of interest that Russell will pay.

d Russell pays an additional $5 every month as a service fee. Find the total of all the costs involved with the loan.

29 Matilda takes out a loan of $260 000 and agrees to repay it over a period of 30 years. The fixed-interest rate is 7.2% p.a., compounded monthly.

a Find the amount of each monthly instalment.

b Find the difference between the total amount that Matilda would pay and the amount that she has paid after 14 years.

c After 14 years Matilda decides to pay out her loan. The lending institution charges a fee of 1.2% of the difference found in part **b** for early repayment. How much will this be?

Go to pp 283–4 for **Quick Answers**
or to pp 300–303 for **Worked Solutions**

Challenge: Annuities and Loan Repayments

1 $7000 is invested at 0.8% per month compound interest for five years. What rate of simple interest will give the same result? *Hint 1*

2 What rate of compound interest per month will mean that an amount of $4000 will accumulate to $4455 after 18 months? *Hint 2*

3 John pays $2185.93 per month on his reducible-interest loan over twenty years. The interest rate is 0.9% per month. How much did John borrow? *Hint 3*

4 Suzanne is considering two options for a prize that she has won.
Option A: a lump sum of $50 000
Option B: $1000 a month for five years.
The compound interest rate is 0.75% per month.
Which option would you recommend? *Hint 4*

5 Liz places $500 into an account every month. The account pays 0.84% per month compound interest. How long will it take her to save $20 000? *Hint 5*

6 Nancy is considering two reducing-balance loans of $70 000.
Loan I: interest rate is 0.72% per month over ten years.
Loan II: interest rate is 0.75% per month over twelve years.
Which loan will have the higher monthly repayments and by how much? *Hint 6*

7 Jemima wants to save some money for a holiday in three years time. She decides to invest an amount every quarter into an account that earns 10% p.a., compounded quarterly. Jemima looks up a table of future values of an annuity of $1 in order to determine how much money she should save.

a Explain why Jemima should look for an interest rate of 2.5% per period over 12 periods.
b Jemima found the value in the table was 13.796. How much extra will Jemima have if she saves $1700 every quarter rather than $1600? *Hint 7*

8 Justin takes out a loan of $90 000 and will repay it over 15 years. The reducible-interest rate charged is 0.7% per month.

a For the first three years Justin pays interest only on the loan. How much are Justin's monthly repayments during the first three years?
b How much more will Justin pay per month during the last twelve years of the loan? *Hint 8*

9 Rosie is buying a car on these terms: No deposit, $726 per month for five years. The reducible-interest rate is 9.6% p.a., charged monthly. How much is the car? *Hint 9*

10 Chad invested $70 000 into an account earning 6% p.a. interest, compounded monthly. He intends to withdraw $1000 every month.

a Explain why the amount in the account ($$A$) after the first month is given by:
$A = 70\,000(1.005) - 1000$ *Hint 10*
b Chad has calculated that the account will run out of money after n months when

$$70 = \frac{1.005^n - 1}{1.005^n(0.005)}$$

Use a calculator and the estimation and refinement technique to find for how long Chad will be able to withdraw $1000 each month. *Hint 11*

Go to p 284 for **Quick Answers**
or to pp 303–4 for **Worked Solutions**

Hint 1: Find the total amount of interest earned and then the amount per year. What percentage is this of the principal?
Hint 2: Solve an equation or use your calculator and trial and error.
Hint 3: Find the present value.
Hint 4: Find the present value of option B.
Hint 5: Use a calculator and trial and error. The future value is $20 000.
Hint 6: Use the present-value formula to find M for both loans.
Hint 7: There is no need to use the future-value formula. The future value will be the amount of the annuity multiplied by 13.796.
Hint 8: Justin will still owe the original amount after the first three years. The term of the loan, as far as calculations are concerned, is twelve years.
Hint 9: Find the present value.
Hint 10: At the end of the month, one month's interest is paid and then $1000 is withdrawn.
Hint 11: If no interest was earned, the money would last 70 months. n > 70

Ch 2: Annuities and Loan Repayments

Further Practice p30

1 $P = \$7000,\ r = 0.08,\ n = 5$

$A = P(1+r)^n$

$= \$7000(1.08)^5$

$= \$10\,285.296\,54\ldots$

$= \$10\,285$ (nearest dollar)

2 $P = \$95\,000,\ r = 0.007,\ n = 96$

$A = P(1+r)^n$

$= \$95\,000(1.007)^{96}$

$= \$185\,589.2336\ldots$

$= \$185\,589$ (nearest dollar)

3 $P = \$46\,000,\ r = 0.00625,\ n = 72$

$A = P(1+r)^n$

$= \$46\,000(1.00625)^{72}$

$= \$72\,041.401\,78\ldots$

$= \$72\,041$ (nearest dollar)

The investment is worth \$72 041, to the nearest dollar, after six years.

4 $P = \$23\,400,\ r = 0.02,\ n = 16$

$A = P(1+r)^n$

$= \$23\,400(1.02)^{16}$

$= \$32\,123.1855\ldots$

$= \$32\,123$ (nearest dollar)

$I = \$32\,123 - \$23\,400$

$= \$8723$

The compound interest earned is \$8723, to the nearest dollar.

5 a $P = \$2000,\ r = 0.07,\ n = 1$

$A = P(1+r)^n$

$= \$2000(1.07)^1$

$= \$2140$

Total value $= \$2140 + \$2000$

$= \$4140$

Jennifer's investment will be worth \$4140 at the end of two years.

b $P = \$4140,\ r = 0.07,\ n = 1$

$A = P(1+r)^n$

$= \$4140(1.07)^1$

$= \$4429.80$

Total value $= \$4429.80 + \$2000$

$= \$6429.80$

Jennifer's investment will be worth \$6429.80 at the end of three years.

6 $M = \$3000,\ r = 0.06,\ n = 20$

$$A = M\left\{\frac{(1+r)^n - 1}{r}\right\}$$

$$= \$3000\left\{\frac{(1.06)^{20} - 1}{0.06}\right\}$$

$= \$110\,356.7736\ldots$

$= \$110\,357$ (nearest dollar)

7 $M = \$250,\ n = 48,\ r = 0.005$

$$A = M\left\{\frac{(1+r)^n - 1}{r}\right\}$$

$$= \$250\left\{\frac{(1.005)^{48} - 1}{0.005}\right\}$$

$= \$13\,524.458\,05\ldots$

$= \$13\,524$ (nearest dollar)

No, Niamh will need an additional \$1476 to achieve her goal.

8 a $M = \$4500,\ r = 0.065,\ n = 35$

$$A = M\left\{\frac{(1+r)^n - 1}{r}\right\}$$

$$= \$4500\left\{\frac{(1.065)^{35} - 1}{0.065}\right\}$$

$= \$558\,156.1062\ldots$

$= \$558\,156$ (nearest dollar)

b Total contributions $= 35 \times \$4500$

$= \$157\,500$

Interest $= \$558\,156 - \$157\,500$

$= \$400\,656$

Karl will have earned \$400 656 interest to the nearest dollar.

9 $A = \$28\,000,\ r = 0.05,\ n = 7$

$$A = M\left\{\frac{(1+r)^n - 1}{r}\right\}$$

$$\$28\,000 = M\left\{\frac{(1.05)^7 - 1}{0.05}\right\}$$

$\$28\,000 = M \times 8.142\,008\,453\ldots$

$M = \$28\,000 \div 8.142\,008\,453\ldots$

$= \$3438.954\,917\ldots$

$= \$3440$ (nearest \$10)

Isaac should invest \$3440, to the nearest ten dollars.

10 $A = \$25\,000,\ r = 0.0048,\ n = 36$

$$A = M\left\{\frac{(1+r)^n - 1}{r}\right\}$$

$$\$25\,000 = M\left\{\frac{(1.0048)^{36} - 1}{0.0048}\right\}$$

$\$25\,000 = M \times 39.195\,225\,21\ldots$

$M = \$25\,000 \div 39.195\,225\,21\ldots$

$= \$637.832\,7938\ldots$

$= \$638$ (nearest dollar)

Beth should save \$638 each month.

11 a $M = \$1000, \quad n = 180, \quad r = 0.006$

$$A = M\left\{\frac{(1+r)^n - 1}{r}\right\}$$
$$= \$1000\left\{\frac{(1.006)^{180} - 1}{0.006}\right\}$$
$$= \$322\,532.0164\ldots$$
$$= \$322\,532 \quad \text{(nearest dollar)}$$

Extra = \$322 532 − \$300 000
= \$22 532

Shane will have an extra \$22 532, to the nearest dollar.

b $A = \$300\,000, \quad r = 0.006, \quad n = 180$

$$A = M\left\{\frac{(1+r)^n - 1}{r}\right\}$$
$$\$300\,000 = M\left\{\frac{(1.006)^{180} - 1}{0.006}\right\}$$
$$\$300\,000 = M \times 322.532\,0164\ldots$$
$$M = \$300\,000 \div 322.532\,0164\ldots$$
$$= \$930.140\,218\ldots$$
$$= \$930 \quad \text{(nearest dollar)}$$

Difference = \$1000 − \$930
= \$70

Shane could save \$70 less each month.

12 $M = \$6000, \quad r = 0.075, \quad n = 7$

$$N = M\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\}$$
$$= \$6000\left\{\frac{(1.075)^7 - 1}{0.075(1.075)^7}\right\}$$
$$= \$31\,779.607\,93\ldots$$
$$= \$31\,780 \quad \text{(nearest dollar)}$$

I would need to invest \$31 780 now to achieve the same result.

13 $M = \$180, \quad r = 0.0055, \quad n = 72$

$$N = M\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\}$$
$$= \$180\left\{\frac{(1.0055)^{72} - 1}{0.0055(1.0055)^{72}}\right\}$$
$$= \$10\,677.6877\ldots$$
$$= \$10\,678 \quad \text{(nearest dollar)}$$

Harrison would need to invest \$10 678.

14 $M = \$250, \quad r = 0.008, \quad n = 60$

$$N = M\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\}$$
$$= \$250\left\{\frac{(1.008)^{60} - 1}{0.008(1.008)^{60}}\right\}$$
$$= \$11\,876.053\,55\ldots$$
$$= \$11\,876 \quad \text{(nearest dollar)}$$

The lump sum of \$12 000 will give the better result.

15 $A = \$47\,000, \quad r = 0.045 \div 12 = 0.003\,75$

$n = 12 \times 12 = 144$

$$N = \frac{A}{(1+r)^n}$$
$$= \frac{\$47\,000}{(1.00375)^{144}}$$
$$= \$27\,416.844\,23\ldots$$
$$= \$27\,417 \quad \text{(nearest dollar)}$$

The present value is \$27 417, to the nearest dollar.

16 $A = \$20\,000, \quad r = 0.08 \div 4 = 0.02$

$n = 6 \times 4 = 24$

$$N = \frac{A}{(1+r)^n}$$
$$= \frac{\$20\,000}{(1.02)^{24}}$$
$$= \$12\,434.429\,76\ldots$$
$$= \$12\,434 \quad \text{(nearest dollar)}$$

An amount of \$12 434, to the nearest dollar, needs to be invested now.

17 $N = \$25\,000, \quad r = 0.007, \quad n = 48$

$$N = M\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\}$$
$$\$25\,000 = M\left\{\frac{(1.007)^{48} - 1}{0.007(1.007)^{48}}\right\}$$
$$\$25\,000 = M \times 40.648\,558\,43\ldots$$
$$M = \$25\,000 \div 40.648\,558\,43\ldots$$
$$= \$615.027\,9608\ldots$$
$$= \$615 \quad \text{(nearest dollar)}$$

18 $N = \$100\,000, \quad r = 0.0055, \quad n = 144$

$$N = M\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\}$$
$$\$100\,000 = M\left\{\frac{(1.0055)^{144} - 1}{0.0055(1.0055)^{144}}\right\}$$
$$\$100\,000 = M \times 99.286\,913\,33\ldots$$
$$M = \$100\,000 \div 99.286\,913\,33\ldots$$
$$= \$1007.182\,081\ldots$$
$$= \$1007.18 \quad \text{(nearest cent)}$$

The amount of each monthly instalment will be \$1007.18

19 a $N = \$175\,000, \quad r = 0.00675, \quad n = 300$

$$N = M\left[\frac{(1+r)^n - 1}{r(1+r)^n}\right]$$
$$\$175\,000 = M\left[\frac{(1.00675)^{300} - 1}{0.00675(1.00675)^{300}}\right]$$
$$\$175\,000 = M \times 128.460\,0057\ldots$$
$$M = \$175\,000 \div 128.460\,0057\ldots$$
$$= \$1362.291\,704\ldots$$
$$= \$1362.29 \quad \text{(nearest cent)}$$

The repayments would be \$1362.29 per month.

b Total repaid = \$1362.29 × 300
= \$408 687

Trent would repay a total of \$408 687.

c Interest = \$408 687 − \$175 000
= \$233 687

Trent pays a total of \$233 687 in interest.

20 $A = \$18\,000, \quad r = 0.007, \quad n = 48$

$$A = M\left\{\frac{(1+r)^n - 1}{r}\right\}$$
$$\$18\,000 = M\left\{\frac{(1+0.007)^{48} - 1}{0.007}\right\}$$
$$\$18\,000 = M \times 56.814\,571\,41\ldots$$
$$M = \$18\,000 \div 56.814\,571\,41\ldots$$
$$= \$316.820\,1318\ldots$$
$$= \$317 \quad \text{(nearest dollar)}$$

Antonia should save \$317 each month.

21 $M = \$750, \quad r = 0.015, \quad n = 20$

$$N = M\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\}$$
$$= \$750\left\{\frac{(1.015)^{20} - 1}{0.015(1.015)^{20}}\right\}$$
$$= \$12\,876.479\,09\ldots$$
$$= \$12\,876 \quad \text{(nearest dollar)}$$

I could invest \$12 876 now, to produce the same result.

22 Option 1: $P = \$5000, \; r = 0.07, \; n = 8$

$$A = P(1+r)^n$$
$$= \$5000(1.07)^8$$
$$= \$8590.930\,899\ldots$$
$$= \$8591 \quad \text{(nearest dollar)}$$

Option 2: $M = \$70, \; r = 0.0055, \; n = 8 \times 12 = 96$

$$A = M\left\{\frac{(1+r)^n - 1}{r}\right\}$$
$$= \$70\left\{\frac{(1.0055)^{96} - 1}{0.0055}\right\}$$
$$= \$8821.103\,227\ldots$$
$$= \$8821 \quad \text{(nearest dollar)}$$

Option 2 gives the better result.

23 $N = \$330\,000,\ r = 0.006,\ n = 360$

$$N = M\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\}$$

$$\$330\,000 = M\left\{\frac{(1.006)^{360} - 1}{0.006(1.006)^{360}}\right\}$$

$$\$330\,000 = M \times 147.321\,3568\ldots$$
$$M = \$330\,000 \div 147.321\,3568\ldots$$
$$= \$2240.001\,091\ldots$$
$$= \$2240.00 \quad \text{(nearest cent)}$$

Total repaid $= 360 \times \$2240$
$= \$806\,400$

Brock repays a total of $806 400.

24

Period	Future values of an annuity of $1				
	Interest Rate				
	2%	2.5%	4%	5%	8%
2	2.0222	2.0250	2.0400	2.0500	2.0800
4	4.1216	4.1525	4.2465	4.3101	4.5061
6	6.3081	6.3877	6.6330	6.8019	7.3359
8	8.5830	8.7361	9.2142	9.5491	10.637
10	10.950	11.203	12.006	12.578	14.487
12	13.412	13.796	15.026	15.917	18.977

a Interest rate: 5%, period: 10
Future value $= \$3000 \times 12.578$
$= \$37\,734$

b Interest rate: 4%, period: 8 years
Future value $= \$4500 \times 9.2142$
$= \$41\,463.90$

c Option X:
Interest rate $= (5 \div 2)\%$
$= 2.5\%$
Number of periods $= 3 \times 2$
$= 6$
Future value $= \$1200 \times 6.3877$
$= \$7665.24$
Option Y:
Interest rate $= (8 \div 4)\%$
$= 2\%$
Number of periods $= 3 \times 4$
$= 12$
Future value $= \$560 \times 13.412$
$= \$7510.72$
Option X gives the better result.

d Interest rate: 8%, period: 12
$\$170\,793 = \$M \times 18.977$
$\$M = \$170\,793 \div 18.977$
$= \$9000$
$M = 9000$

e Interest rate = 2.5%, period = 8
$\$16\,598.59 = \$M \times 8.7361$
$\$M = \$16\,598.59 \div 8.7361$
$= \$1900$
An amount of $1900 would need to be invested each quarter.

25

Period	Present values of an annuity of $1				
	Interest Rate				
	1%	2%	4%	6%	8%
4	3.9020	3.8077	3.6299	3.4651	3.3121
8	7.6517	7.3255	6.7327	6.2098	5.7466
12	11.255	10.575	9.3851	8.3838	7.5361
16	14.718	13.578	11.652	10.106	8.8514
20	18.046	16.351	13.590	11.470	9.8181
24	21.243	18.914	15.247	12.550	10.529

a Interest rate: 4%, period: 20
Present value $= \$400 \times 13.590$
$= \$5436$

b Interest rate $= (12 \div 12)\%$
$= 1\%$
Number of periods $= 2 \times 12$
$= 24$
Present value $= \$210 \times 21.243$
$= \$4461.03$

c Interest rate: 2%, period: 16
$\$8825.70 = \$M \times 13.578$
$\$M = \$8825.70 \div 13.578$
$= \$650$
$M = 650$

d Interest rate = 6%
Number of periods = 8
$\$88\,000 = \text{Repayment} \times 6.2098$
Repayment $= \$88\,000 \div 6.2098$
$= \$14\,171.148\,83$
$= \$14\,171$ (nearest dollar)

26 **a** Interest rate per month = 0.625%
Interest per month
$= 0.625\% \times \$200\,000$
$= \$1250$
Samuel will pay $1250 every month.

b $N = \$200\,000,\ r = 0.00625,\ n = 264$

$$N = M\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\}$$

$$\$200\,000 = M\left\{\frac{(1.00625)^{264} - 1}{0.00625(1.00625)^{264}}\right\}$$

$$\$200\,000 = M \times 129.113\,8247\ldots$$
$$M = \$200\,000 \div 129.113\,8247\ldots$$
$$= \$1549.020\,799\ldots$$
$$= \$1549.02 \quad \text{(nearest cent)}$$

Extra $= \$1549.02 - \$1250$
$= \$299.02$

c $N = \$200\,000,\ r = 0.00625,\ n = 300$

$$N = M\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\}$$

$$\$200\,000 = M\left\{\frac{(1.00625)^{300} - 1}{0.00625(1.00625)^{300}}\right\}$$

$$\$200\,000 = M \times 135.319\,6127\ldots$$
$$M = \$200\,000 \div 135.319\,6127\ldots$$
$$= \$1477.982\,356\ldots$$
$$= \$1477.98 \quad \text{(nearest cent)}$$

d Interest-only option:
Samuel pays
$36 \times \$1250 + 264 \times \$1549.02$
$= \$453\,941.28$
Other option:
Samuel pays $300 \times \$1477.98$
$= \$443\,394$
Extra $= \$453\,941.28 - \$443\,394$
$= \$10\,547.28$

e By taking the interest-only option Samuel has to find a smaller amount to make the repayments for the first 3 years.

f If he didn't take the interest-only option Samuel would save over $10 500 over the term of the loan. He also pays around $70 a month less for the final 22 years.

27 **a** $N = \$13\,000,\ r = 0.09,\ n = 6$

$$N = M\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\}$$

$$\$13\,000 = M\left\{\frac{(1.09)^6 - 1}{0.09(1.09)^6}\right\}$$

$$\$13\,000 = M \times 4.485\,918\,59\ldots$$
$$M = \$13\,000 \div 4.485\,918\,59\ldots$$
$$= \$2897.957\,183\ldots$$
$$= \$2897.96 \quad \text{(nearest cent)}$$

Total repaid $= 6 \times \$2897.96$
$= \$17\,387.76$

b $N = \$13\,000,\ r = 0.084,\ n = 6$

$$N = M\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\}$$

$$\$13\,000 = M\left\{\frac{(1.084)^6 - 1}{0.084(1.084)^6}\right\}$$

$$\$13\,000 = M \times 4.567\,314\,17\ldots$$
$$M = \$13\,000 \div 4.567\,314\,17\ldots$$
$$= \$2846.311\,752\ldots$$
$$= \$2846.31 \quad \text{(nearest cent)}$$

c $R = \$2846.31$

Year	Principal	Interest	$P+I$	$P+I-R$
1	$13 000.00	$1092.00	$14 092.00	$11 245.69
2	$11 245.69	$944.64	$12 190.33	$9 344.02

d At the beginning of the third year
$P = \$9344.02$

e $N = \$9344.02,\ r = 0.095,\ n = 4$

$$N = M\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\}$$

$$\$9344.02 = M\left\{\frac{(1.095)^4 - 1}{0.095(1.095)^4}\right\}$$

$$\$9344.02 = M \times 3.204\,481\,121\ldots$$
$$M = \$9344.02 \div 3.204\,481\,121\ldots$$
$$= \$2915.922\,936\ldots$$
$$= \$2915.92 \quad \text{(nearest cent)}$$

f Variable rate:

Total repaid
$= 2 \times \$2846.31 + 4 \times \$2915.92$
$= \$17\,356.30$

Difference $= \$17\,387.76 - \$17\,356.30$
$= \$31.46$

Johanna would pay $31.46 less in total if she chose the variable rate.

28 **a** Stamp duty $= \$8 + \$3 \times (80 - 15)$
$= \$203$

b Total $= \$80\,000 + \$203 + \$750$
$= \$80\,953$

c $N = \$80\,953, \quad r = 0.00525, \quad n = 144$

$$N = M\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\}$$

$$\$80\,953 = M\left\{\frac{(1.00525)^{144} - 1}{0.00525(1.00525)^{144}}\right\}$$

$\$80\,953 = M \times 100.862\,7975\ldots$
$M = \$80\,953 \div 100.862\,7975\ldots$
$= \$802.605\,1428\ldots$
$= \$802.61$ (nearest cent)

Total repaid $= 144 \times \$802.61$
$= \$115\,575.84$

Interest $= \$115\,575.84 - \$80\,953$
$= \$34\,622.84$

d Service fee $= \$5 \times 144$
$= \$720$

Total cost $= \$34\,622.84 + \$720 + \$953$
$= \$36\,295.84$

29 **a** $N = \$260\,000, \quad r = 0.006, \quad n = 360$

$$N = M\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\}$$

$$\$260\,000 = M\left\{\frac{(1.006)^{360} - 1}{0.006(1.006)^{360}}\right\}$$

$\$260\,000 = M \times 147.321\,3568\ldots$
$M = \$260\,000 \div 147.321\,3568\ldots$
$= \$1764.849\,345\ldots$
$= \$1764.85$ (nearest cent)

b Total to repay $= 360 \times \$1764.85$
$= \$635\,346$

Total paid after 14 years
$= 14 \times 12 \times \$1764.85$
$= \$296\,494.80$

Difference
$= \$635\,346.00 - \$296\,494.80$
$= \$338\,851.20$

c Early repayment fee
$= 1.2\% \times \$338\,851.20$
$= \$4066.2144$
$= \$4066.21$ (nearest cent)

Challenge p33

1 $P = \$7000, \quad r = 0.008, \quad n = 60$

$A = P(1+r)^n$
$= \$7000(1.008)^{60}$
$= \$11\,290.94$ (nearest cent)

$I = \$11\,290.94 - \$7000$
$= \$4290.94$

Annual interest $= \$4290.94 \div 5$
$= \$858.188$

$$\text{Interest rate} = \frac{858.188}{7000} \times 100\%$$
$= 12.3\%$ (1 d.p.)

2 $P = \$4000, \quad A = \$4455, \quad n = 18$

$A = P(1+r)^n$
$4455 = 4000(1+r)^{18}$
$1.11375 = (1+r)^{18}$
$1 + r = 1.006\,003\,097\ldots$
$r = 0.006\,003\,097\ldots$
$= 0.006$ (3 d.p.)

The rate of interest is 0.6% per month.

3 $M = \$2185.93, \quad r = 0.009, \quad n = 240$

$$N = M\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\}$$

$$= \$2185.93\left\{\frac{(1.009)^{240} - 1}{0.009(1.009)^{240}}\right\}$$

$= \$214\,598.8685\ldots$
$= \$214\,600$ (nearest $10)

John borrowed $214 600.

4 Option B:

$M = \$1000, \quad r = 0.0075, \quad n = 60$

$$N = M\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\}$$

$$= \$1000\left\{\frac{(1.0075)^{60} - 1}{0.0075(1.0075)^{60}}\right\}$$

$= \$48\,173.373\,52\ldots$
$= \$48\,173$ (nearest dollar)

The present value of option B is $48 173.
Option A ($50 000) is better.

5 $A = \$20\,000, \quad r = 0.0084, \quad M = \$500$

$$A = M\left\{\frac{(1+r)^n - 1}{r}\right\}$$

$$\$20\,000 = \$500\left\{\frac{(1.0084)^n - 1}{0.0084}\right\}$$

$$40 = \left\{\frac{(1.0084)^n - 1}{0.0084}\right\}$$

$0.336 = 1.0084^n - 1$
$1.336 = 1.0084^n$

Try $n = 30$
$1.0084^{30} = 1.2852\ldots$

Try $n = 40$
$1.0084^{40} = 1.3973\ldots$

Try $n = 35$,
$1.0084^{35} = 1.340\ldots$

Try $n = 34$,
$1.0084^{34} = 1.328\ldots$

It will take 35 months to save $20 000.

6 $N = \$70\,000$

Loan I: $r = 0.0072, \quad n = 120$

$$N = M\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\}$$

$$\$70\,000 = M\left\{\frac{(1.0072)^{120} - 1}{0.0072(1.0072)^{120}}\right\}$$

$\$70\,000 = M \times 80.169\,510\,21\ldots$
$M = \$70\,000 \div 80.169\,510\,21\ldots$
$= \$873.15$ (nearest cent)

Loan II: $r = 0.0075, \quad n = 144$

$$N = M\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\}$$

$$\$70\,000 = M\left\{\frac{(1.0075)^{144} - 1}{0.0075(1.0075)^{144}}\right\}$$

$\$70\,000 = M \times 87.87109195\ldots$
$M = \$70\,000 \div 87.871\,091\,95\ldots$
$= \$796.62$ (nearest cent)

Loan I will have the higher repayments.

Difference $= \$873.15 - \$796.62$
$= \$76.53$

7 **a** 10% p.a. $= (10 \div 4)\%$ per quarter
$= 2.5\%$ per quarter

3 years $= 3 \times 4$ quarters
$= 12$ quarters

b If Jemima saves an extra $100 she will have $13.796 \times \$100$ extra in total. Jemima will have $1379.60 extra.

8 **a** $P = \$90\,000, \quad r = 0.007$

Interest $= 0.007 \times \$90\,000$
$= \$630$

b $N = \$90\,000, \quad r = 0.007, \quad n = 144$

$$N = M\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\}$$

$$\$90\,000 = M\left\{\frac{(1.007)^{144} - 1}{0.007(1.007)^{144}}\right\}$$

$\$90\,000 = M \times 90.538\,291\,54\ldots$
$M = \$90\,000 \div 90.53829154\ldots$
$= \$994.05$ (nearest cent)

Difference $= \$994.05 - \$630$
$= \$364.05$

Justin will pay an extra $364.05 during the last twelve years.

9 $M = \$726,\ r = 0.008,\ n = 60$

$$N = M\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\}$$

$$= \$726\left\{\frac{(1.008)^{60} - 1}{0.008(1.008)^{60}}\right\}$$

$$= \$34\,488.0595\ldots$$

$$= \$34\,488 \quad \text{(nearest dollar)}$$

The car cost \$34 488.

10 **a** At the end of the month the account will earn interest. The amount will accumulate to \$70 000(1.005). Chad then draws out \$1000.

$A = 70\,000(1.005) - 1000$

b $70 = \dfrac{1.005^n - 1}{1.005^n(0.005)}$

Try $n = 80$,

$$\frac{1.005^{80} - 1}{1.005^{80}(0.005)} = 65.802\,305\,38\ldots$$

n is higher than 80

Try $n = 90$,

$$\frac{1.005^{90} - 1}{1.005^{90}(0.005)} = 72.331\,299\,58\ldots$$

n is lower than 90

Try $n = 86$,

$$\frac{1.005^{86} - 1}{1.005^{86}(0.005)} = 69.758\,711\,35\ldots$$

Try $n = 87$,

$$\frac{1.005^{87} - 1}{1.005^{87}(0.005)} = 70.406\,677\,96\ldots$$

The money will last 86 months, or seven years and two months.