Unit 1 Test: Financial Mathematics

Credit and Borrowing and Annuities and Loan Repayments and Depreciation

Remember: these are HSC-type questions.

Time allowed: 1 hour 10 minutes **Total marks:** 50

Part A

(Suggested time: 30 minutes)

Choose the correct answer (A, B, C or D) for each question. **One mark each**

1 Mackenzie borrows $16 000 at the flat rate of interest of 7.2% p.a. Find the total amount Mackenzie repays if the loan is over three years.

A $19 456 B $19 711
C $18 357 D $20 967

2 A van is depreciating using the declining-balance method at 12% p.a. If it is currently worth $12 800 and is five years old, what was the purchase price?

A $22 560 B $24 255
C $21 330 D $32 000

3 Rashid takes out a loan over eight years. The reducible interest is 9% p.a., charged monthly. His monthly repayments are $1062.14. How much did Rashid borrow?

A $68 100 B $58 500
C $72 500 D $85 100

4 What amount of money, to the nearest hundred dollars, needs to be invested now to be equivalent to an annuity of $300 a month for four years at 0.8% per month interest?

A $17 500 B $11 900
C $14 400 D $12 700

5 The monthly repayments on loans over twelve years at a particular rate of interest are $10.91 per $1000 borrowed. How much interest in total (to the nearest dollar), would be paid on a loan of $90 000?

A $13 092 B $40 920
C $51 394 D $98 190

6 A machine bought for $5325 is being depreciated by the straight-line method. The amount of depreciation apportioned per period is $350. After how many years will the value of the machine first fall below $1000?

A 11 B 12
C 13 D 14

7 Emma's credit card charges interest at 0.043% per day. There is no interest-free period. How much interest will Emma pay if she has an amount of $5200 outstanding for 94 days?

A $56.46 B $57.58
C $210.18 D $214.44

8 A certain amount is to be invested at the end of each month into an account earning 9% p.a. interest compounded monthly. At the end of five years the future value will be $60 339. What is the present value?

A $38 539 B $39 216
C $27 153 D $33 186

9 Patrick wants to save some money for a special project. What is the minimum amount he could invest each quarter at 7.6% p.a. interest, compounded quarterly, to have $8000 at the end of three years?

A $500 B $550
C $600 D $650

10 An asset is being depreciated using the declining-balance method. If the original value was $43 200 and the value after six years is $15 200, at what rate per annum is the asset being depreciated?

A 6% B 16%
C 18% D 35%

11 Ethan pays $2000 deposit when buying a car for $14 500. Simple interest of 7% p.a. is paid on the loan, which is over five years. Find the amount of each monthly repayment.

A $326.25 B $281.25
C $337.92 D $292.20

12 Regan places $250 into a special account every month. Interest of 0.5% per month is paid. What will be the value of the account at the end of thirty years (to the nearest dollar)?

A $251 129 B $542 032
C $199 317 D $388 975

13 A tractor is being depreciated by the declining-balance method at 18% p.a. The original price was $53 000. How much, to the nearest dollar, did the tractor depreciate in the third year?

A $9540 B $7822
C $6415 D $5260

14 Montana pays $500 a month on her loan of $20 000 over four years. What annual rate of simple interest is she paying?

A 5% B 7.5%
C 20% D 30%

15 Which will be worth the most at the end of three years?

A $9000 cash kept under the mattress
B $7500 earning 7% p.a., simple interest
C $7000 earning 0.7% per month compound interest
D an annuity of $2750 per year, earning 8% p.a. interest compounded yearly

16 A loan of $10 000 has repayments of $437.82 per month. Reducible interest of 4.8% p.a. is charged monthly. What is the term of the loan?

A 18 months B 2 years
C 3 years D 4 years

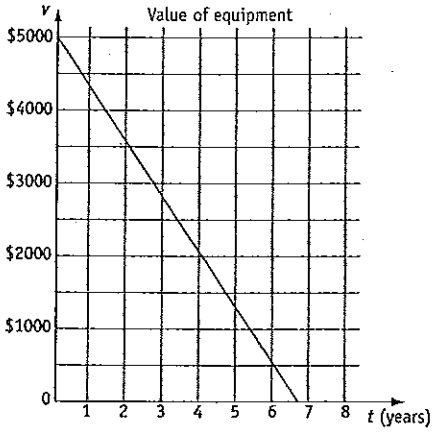
17 A boat was bought for $6000 on March 1st and is depreciated using the straight-line method. If the amount of depreciation claimed at the end of the financial year is $260, approximately how much depreciation is allocated per year?

A $260 B $640
C $780 D $1040

18 A loan has service fees of $12 per month and an establishment fee of $650. The loan was for $35 000 and will be repaid over five years with monthly repayments of $641.37. To the nearest per cent, what percentage of the amount borrowed are the total fees and interest?

A 12% B 10%
C 18% D 14%

19 The graph shows the value of some equipment over time. Which statement correctly describes the method of depreciation?



A straight-line method at $750 per year
B straight-line method at $830 per year
C declining-balance method at 15% per year
D declining-balance method at 32% per year

20 How much interest (to the nearest dollar) is earned if $9000 is placed in an account earning 15% p.a. interest, compounded quarterly for seven years?

A $9450 B $11 646
C $14 940 D $16 230

Part B (Suggested time: 40 minutes)

Show all working. 30 marks

21 Marcos wants to borrow $20 000 to buy a hay baler. He gets a reducible-interest loan and agrees to repay the money plus interest with three equal annual instalments. A table of loan repayments has been drawn up and partly filled in.

Year	Principal	Interest	$P+I$	$P+I-R$
1	$20 000.00	$1600	$21 600.00	
2				
3				

a What rate of interest is charged on the loan? 1 mark

b Use the present value formula to find the amount of each monthly repayment to the nearest cent. 3 marks

c By completing the table, show that the baler will be paid off at the end of the three years. 3 marks

d If the baler depreciates using the declining-balance method at 10% p.a., find its value when the loan is finally repaid. 1 mark

22 Josie wants to have $15 000 in three years time for a holiday.

a If she places $375 every month for three years into an account that earns 7.2% p.a., interest, compounded monthly, will she have enough money? Justify your answer. 2 marks

b If instead Josie placed an amount every quarter into an account earning 7.2% p.a. compounded quarterly, how much should she invest to have $15 000 at the end of three years? 2 marks

c How much extra would Josie need to invest altogether if she invested quarterly rather than monthly investments of $375? 1 mark

23 The table shows the monthly repayments per $1000 on loans at various interest rates over various periods of time.

Monthly repayment per $1000			
Period (years)	Interest rate 5%	5.5%	6%
5	$18.87	$19.10	$19.33
10	$10.61	$10.85	$11.10
15	$7.91	$8.17	$8.44
20	$6.60	$6.88	$7.16
25	$5.85	$6.14	$6.44
30	$5.37	$5.68	$6.00

a If Judy borrows $180 000 over 25 years at 5.5% p.a., how much interest will she pay? 2 marks

b Kieran intends to borrow $75 000 over 15 years. How much extra would he need to pay each month if the interest rate was 6% rather than 5%? 2 marks

c Lucy wants to borrow $135 000 to buy a home unit. The loan has a variable rate of interest, currently 5% p.a. Lucy can afford to pay at most $900 per month. Over how many years would you recommend she take the loan? Justify your answer. 3 marks

24 Marcia buys a new car and pays no deposit. She takes a loan for the full price of the car and pays it back (with interest) over five years with repayments of $709.67 each month. The reducible interest rate is 8% p.a., charged monthly.

a What was the price of the car? 2 marks

b The car was valued at $18 500 when the loan was repaid. If it was depreciated using the straight-line method, how much depreciation is allocated every year? 2 marks

25 Karla has a credit card that has up to 55 days interest free. There is an annual fee of $26 and the interest rate is 0.048% per day.

a One particular month Karla pays her credit card bill by the due date. Seven days later she buys some furniture and puts the amount ($4000) on her credit card. If Karla pays her next bill by the due date, how much interest will she pay? 1 mark

b Find the amount of interest Karla will pay if she pays the bill five days late. 2 marks

c Karla wanted to know whether it was worth paying the annual fee or whether she would be better off with a credit card that has no interest free days and no annual fee. She decided to work out how many days she could have an amount of $4000 outstanding to cost $26 in interest. She determined that she would find the answer by solving the equation $1.00048^n = 1.0065$. Briefly explain why this is so. 1 mark

d Use a calculator and the estimation and refinement technique to find the value of n. Justify the accuracy of your answer. 2 marks

Go to p 285 for **Quick Answers**
or to pp 309–10 for **Worked Solutions**

Solutions

Unit 1 Test p57

1 $I = Prn$
$= \$16\,000 \times 0.072 \times 3$
$= \$3456$
Total repaid $= \$16\,000 + \$3456$
$= \$19\,456$ A

2 $S = \$12\,800,\ r = 0.12,\ n = 5$
$S = V_0(1-r)^n$
$\$12\,800 = V_0(1-0.12)^5$
$= V_0(0.88)^5$
$V_0 = \$12\,800 \div (0.88)^5$
$= \$24\,254.739\,18\ldots$
$= \$24\,255$ (nearest dollar) B

3 $M = \$1062.14,\ r = 0.0075,\ n = 96$
$$N = M\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\}$$
$$= \$1062.14\left\{\frac{(1.0075)^{96} - 1}{0.0075(1.0075)^{96}}\right\}$$
$= \$72\,500$ (nearest dollar) C

4 $M = \$300,\ r = 0.008,\ n = 48$
$$N = M\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\}$$
$$= \$300\left\{\frac{(1.008)^{48} - 1}{0.008(1.008)^{48}}\right\}$$
$= \$11\,900$ (nearest \$100) B

5 Repayments $= \$10.91 \times 90$
$= \$981.90$
Total repaid $= \$981.90 \times 12 \times 12$
$= \$141\,393.60$
Interest $= \$141\,393.60 - \$90\,000$
$= \$51\,393.60$
$= \$51\,394$ (nearest dollar) C

6 $S = \$1000,\ V_0 = \$5325,\ D = \$350$
$S = V_0 - Dn$
$\$1000 = \$5325 - \$350 \times n$
$\$350n = \$5325 - \$1000$
$= \$4325$
$n = 12.357\,142\,86\ldots$
The value will first fall below \$1000 after 13 years. C

7 $P = \$5200,\ r = 0.00043,\ n = 94$
$A = P(1+r)^n$
$= \$5200(1.00043)^{94}$
$= \$5414.44$ (nearest cent)
Interest $= \$5414.44 - \$5200$
$= \$214.44$ D

8 $A = \$60\,339,\ r = 0.0075,\ n = 60$
$$N = \frac{A}{(1+r)^n}$$
$$= \frac{\$60\,339}{(1.0075)^{60}}$$
$= \$38\,539$ (nearest dollar) A

9 $A = \$8000,\ r = 0.019,\ n = 12$
$$A = M\left\{\frac{(1+r)^n - 1}{r}\right\}$$
$$\$8000 = M\left\{\frac{(1.019)^{12} - 1}{0.019}\right\}$$
$\$8000 = M \times 13.336\,920\,74\ldots$
$M = \$8000 \div 13.336\,920\,74\ldots$
$= \$600$ (nearest dollar) C

10 $S = \$15\,200,\ V_0 = \$43\,200,\ n = 6$
$S = V_0(1-r)^n$
$\$15\,200 = \$43\,200(1-r)^6$
$(1-r)^6 = 0.351\,851\,851\ldots$
$1 - r = 0.840\,220\,564\ldots$
$r = 0.159\,779\ldots$
$= 0.16$ (2 d.p.)
The rate is 16%. B

11 Amount borrowed $= \$14\,500 - \$2000$
$= \$12\,500$
Interest $= \$12\,500 \times 0.07 \times 5$
$= \$4375$
Total to repay $= \$12\,500 + \$4375$
$= \$16\,875$
Each repayment $= \$16\,875 \div 60$
$= \$281.25$ B

12 $M = \$250,\ r = 0.005,\ n = 360$
$$A = M\left\{\frac{(1+r)^n - 1}{r}\right\}$$
$$= \$250\left\{\frac{(1.005)^{360} - 1}{0.005}\right\}$$
$= \$251\,129$ (nearest dollar) A

13 $V_0 = \$53\,000,\ r = 0.18$
$S = V_0(1-r)^n$
After two years:
$S = \$53\,000(1-0.18)^2$
$= \$35\,637.20$
After three years:
$S = \$53\,000(1-0.18)^3$
$= \$29\,222.50$ (nearest cent)
Difference
$= \$35\,637.20 - \$29\,222.50$
$= \$6414.70$
$= \$6415$ (nearest dollar) C

14 Total repaid $= \$500 \times 48$
$= \$24\,000$
Total interest $= \$24\,000 - \$20\,000$
$= \$4000$
Annual interest $= \$4000 \div 4$
$= \$1000$
$$\text{Interest rate} = \frac{\$1000}{\$20\,000} \times 100\%$$
$= 5\%$ A

15 A: \$9000

B: Interest $= \$7500 \times 0.07 \times 3$
$= \$1575$
Total value $= \$7500 + \$1575$
$= \$9075$

C: Value $= \$7000(1.007)^{35}$
$= \$8998$ (nearest dollar)

D: Future value $= \$2750\left\{\dfrac{(1.08)^3 - 1}{0.08}\right\}$
$= \$8927.60$

B is the highest. B

16 $N = \$10\,000$, $M = \$437.82$, $r = 0.004$

$$N = M\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\}$$

$$\$10\,000 = \$437.82\left\{\frac{(1.004)^n - 1}{0.004(1.004)^n}\right\}$$

Try $n = 24$,

$$\$437.82\left\{\frac{(1.004)^{24} - 1}{0.004(1.004)^{24}}\right\} = 10\,000.028\,36$$

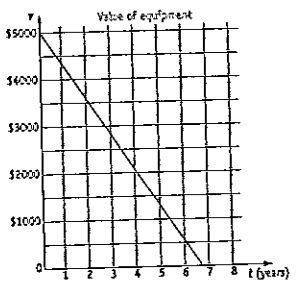
24 months = 2 years B

17 From 1st March until 30th June is 4 months.

$$\$260 \approx \frac{4}{12} \times D$$
$$D \approx \$780$$ C

18 Total repayments $= \$641.37 \times 60$
$= \$38\,482.20$
Total interest $= \$38\,482.20 - \$35\,000$
$= \$3482.20$
Total fees $= \$650 + \$12 \times 60$
$= \$1370$
Total fees and interest $= \$4852.20$
Percentage of total
$$= \frac{\$4852.20}{\$35\,000} \times 100\%$$
$= 14\%$ (nearest %) D

19 The graph is a straight-line, so the straight-line method is used.
The equipment reduces from \$5000 to \$0 in approximately $6\frac{2}{3}$ years.
$D = \$5000 \div 6\frac{2}{3}$
$= \$750$ A



20 $P = \$9000$, $r = 0.0375$, $n = 28$
$A = P(1+r)^n$
$= \$9000(1.0375)^{28}$
$= \$25\,230$ (nearest dollar)
Interest $= \$25\,230 - \$9000$
$= \$16\,230$ D

21 a Rate of interest $= \dfrac{\$1600}{\$20\,000} \times 100\%$
$= 8\%$ ✓

b $N = \$20\,000$, $r = 0.08$, $n = 3$

$$N = M\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\}$$

$$\$20\,000 = M\left\{\frac{(1.08)^3 - 1}{0.08(1.08)^3}\right\}$$ ✓

$\$20\,000 = M \times 2.577\,096\,987\ldots$ ✓
$M = \$20\,000 \div 2.577\,096\,987\ldots$
$= \$7760.67$ (nearest cent) ✓

c

Year	Principal	Interest	$P+I$	$P+I-R$
1	\$20 000.00	\$1600.00	\$21 600.00	\$13 839.33
2	\$13 839.33	\$1107.15	\$14 946.48	\$7 185.81
3	\$7 185.81	\$574.86	\$7 760.67	\$0.00

✓✓✓

d $V_0 = \$20\,000$, $r = 0.1$, $n = 3$
$S = V_0(1-r)^n$
$= \$20\,000(1 - 0.1)^3$
$= \$14\,580$ ✓

22 a $M = \$375$, $r = 0.006$, $n = 36$

$$A = M\left\{\frac{(1+r)^n - 1}{r}\right\}$$

$$= \$375\left\{\frac{(1.006)^{36} - 1}{0.006}\right\}$$ ✓

$= \$15\,019$ (nearest dollar)
Yes, Josie will have just enough money. ✓

b $A = \$15\,000$, $r = 0.018$, $n = 12$

$$A = M\left\{\frac{(1+r)^n - 1}{r}\right\}$$

$$\$15\,000 = M\left\{\frac{(1.018)^{12} - 1}{0.018}\right\}$$ ✓

$\$15\,000 = M \times 13.262\,251\,75\ldots$
$M = \$15\,000 \div 13.262\,251\,75\ldots$
$= \$1131$ (nearest dollar)
Josie should invest \$1131 each quarter. ✓

c Monthly investments:
Total invested $= \$375 \times 36$
$= \$13\,500$
Quarterly investments:
Total invested $= \$1131 \times 12$
$= \$13\,572$
Josie would need to invest an additional \$72 if she made quarterly investments. ✓

23 a 25 years, 5.5% p.a.
Repayment $= 180 \times \$6.14$
$= \$1105.20$ ✓
Total repaid $= \$1105.20 \times 25 \times 12$
$= \$331\,560$
Interest $= \$331\,560 - \$180\,000$
$= \$151\,560$ ✓

b Over 15 years at 6% p.a.
Repayments $= 75 \times \$8.44$
$= \$633.00$ ✓
Over 15 years at 5% p.a.
Repayments $= 75 \times \$7.91$
$= \$593.25$
Extra amount $= \$633.00 - \$593.25$
$= \$39.75$ ✓

c Repayment
$= \$900$
$= (\$900 \div 135)$ per \$1000
$= \$6.67$ per \$1000 ✓
Lucy can afford to repay the loan over 20 years, but this means repaying close to the maximum amount she can afford. If interest rates rise, Lucy may not be able to meet the repayments. ✓
Lucy would be better off to take the loan over 25 years, because she could then still afford the repayments if interest rates rose by 1% p.a. ✓

24 $M = \$709.67$, $r = 0.00666\ldots$, $n = 60$

a $$N = M\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\}$$

$$= \$709.67\left\{\frac{(1.0066\ldots)^{60} - 1}{0.0066\ldots(1.0066\ldots)^{60}}\right\}$$ ✓

$= \$35\,000$ (nearest dollar) ✓

b $S = \$18\,500$, $V_0 = \$35\,000$, $n = 5$
$S = V_0 - Dn$
$\$18\,500 = \$35\,000 - D \times 5$ ✓
$5D = \$35\,000 - \$18\,500$
$= \$16\,500$
$D = \$3300$ ✓

25 a \$0 ✓

b $P = \$4000$, $r = 0.00048$, $n = 5$
$A = P(1+r)^n$
$= \$4000(1.00048)^5$
$= \$4009.61$ (nearest cent) ✓
Interest $= \$4009.61 - \$4000$
$= \$9.61$ ✓

c Interest $= \$26$
$A = \$4026$
$A = P(1+r)^n$
$\$4026 = \$4000(1.00048)^n$
$1.0065 = (1.00048)^n$ ✓

d Try $n = 13$,
$(1.00048)^{13} = 1.006\,258\ldots$
Try $n = 14$,
$(1.00048)^{14} = 1.006741\ldots$
Try $n = 13.5$,
$(1.00048)^{13.5} = 1.006499\ldots$ ✓
The value is very close to 13.5. However, n must be a whole number of days. If Karla has an amount of \$4000 outstanding for 14 days, she will pay more than her \$26 fee. ✓