Topic Test 12 — Similarity and right-angled triangles

Section I — Multiple choice
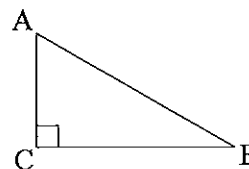
1 Two triangles are similar if they have the:

   A same shape.   B same area.   C same base.   D same height.

2 If $\triangle XYZ \;|||\; \triangle PQR$, which of the following is always true?

   A $\dfrac{XY}{PQ} = \dfrac{YZ}{QR}$ B $\dfrac{XZ}{PR} = \dfrac{XY}{QR}$ C $XY = PQ$ D $XZ = PQ$
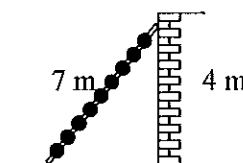
3 Which of the following statements is correct?

   A $\sin B = \dfrac{AC}{AB}$ B $\cos A = \dfrac{BC}{AB}$

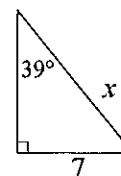   C $\tan C = \dfrac{BC}{AC}$ D $\tan A = \dfrac{AC}{BC}$

4 A ladder 7 metres long leans against a wall 4 metres high. Which of the following ratios could be used to calculate the angle the ladder makes with the wall?

   A $\sin\theta = \dfrac{4}{7}$ B $\cos\theta = \dfrac{4}{7}$

   C $\sin\theta = \dfrac{7}{4}$ D $\cos\theta = \dfrac{7}{4}$

5 What is the length of x? (Answer correct to 2 decimal places.)

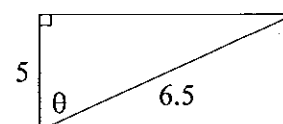   A 8.64   B 8.99

   C 11.12   D 11.57

6 What is the size of angle B, to the nearest minute, if the tangent of angle B is 0.5?

   A 26°33'   B 26°34'   C 63°26'   D 63°27'

7 What is the missing angle? (Answer correct to nearest degree.)

   A 38   B 39

   C 40   D 50

Section II — Short answer
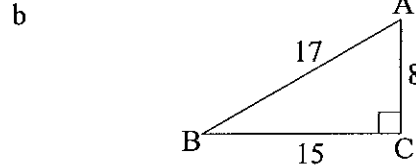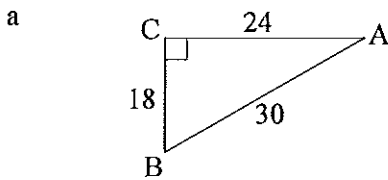
1
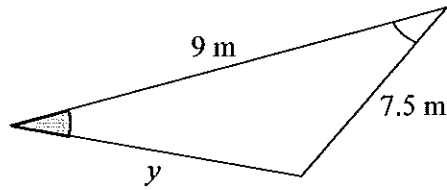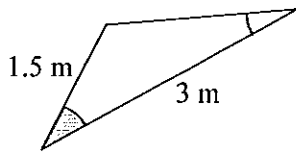
   a Why are these triangles similar?   b What is the scale factor?   c Find the value of y.

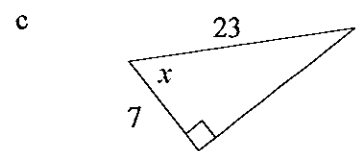2   Find the sine, cosine and tangent ratios in simplest form for angle A and B for each triangle.
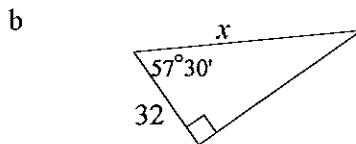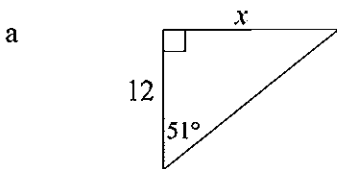
   a

   b

3   Find the value of x correct to the nearest whole number.
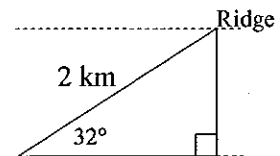
   a   b   c

4   A mountaineer climbed a ridge covering a distance of 2 km. The ridge was inclined at 32° to the horizontal. How far in the vertical did the mountaineer climb? (Answer correct to 1 decimal place.)
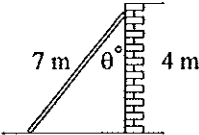
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Worked solutions

Section 1	Solution	Answer
1	Similar triangles are the same shape.	A
2	Corresponding sides are in the same proportion $$\frac{XY}{PQ} = \frac{YZ}{QR}$$	A
3	$\sin B = \dfrac{AC}{AB} \left(\dfrac{\text{Opposite}}{\text{Hypotenuse}} \right)$	A
4	$\cos\theta = \dfrac{4}{7}$	B
5	$\sin 39° = \dfrac{7}{x}$ $x = \dfrac{7}{\sin 39°}$ $= 11.1231101$ ≈ 11.12	C
6	$B = \tan^{-1} 0.5$ $= 26.56505118....$ $= 26°34'$	B
7	$\cos\theta = \dfrac{5}{6.5}$ $\theta = 39.71513723...$ $\approx 40°$	C

Section II	Solution
1a	Corresponding angles are equal.
b	Scale factor $= \dfrac{9}{3} = 3$ or 1:3
c	$\dfrac{y}{1.5} = \dfrac{9}{3}$ $3y = 9 \times 1.5$ $y = \dfrac{9 \times 1.5}{3}$ $= 4.5$
2a	$\sin A = \dfrac{18}{30} = \dfrac{3}{5}, \cos A = \dfrac{24}{30} = \dfrac{4}{5}, \tan A = \dfrac{18}{24} = \dfrac{3}{4}$ $\sin B = \dfrac{24}{30} = \dfrac{4}{5}, \cos B = \dfrac{18}{30} = \dfrac{3}{5}, \tan B = \dfrac{24}{18} = \dfrac{4}{3}$
b	$\sin A = \dfrac{15}{17}, \cos A = \dfrac{8}{17}, \tan A = \dfrac{15}{8}$ $\sin B = \dfrac{8}{15}, \cos B = \dfrac{15}{17}, \tan B = \dfrac{8}{15}$
3a	$\tan 51° = \dfrac{x}{12}$ $x = 12 \times \tan 51°$ $= 14.818765\ldots$ ≈ 15
b	$\cos 57°30' = \dfrac{32}{x}$ $x = \dfrac{32}{\cos 57°30'}$ $= 59.557087\ldots$ ≈ 60
c	$\cos x = \dfrac{7}{23}$ $x = \cos^{-1} \dfrac{7}{23}$ $= 72.281068\ldots$ $\approx 72°$
4	$\sin 32° = \dfrac{h}{2}$ $x = 2 \times \sin 32°$ $= 1.0598385\ldots$ $\approx 1.1 \text{ km}$ Mountaineer climbed 1.1 km in the vertical