Unit 3 Test: Measurement

Further Applications of Area and Volume and Applications of Trigonometry and Spherical Geometry

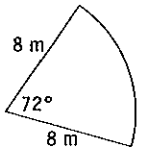
Remember: these are HSC-type questions.

Time allowed: 1 hour 10 minutes **Total marks:** 50

Part A (Suggested time: 30 minutes)

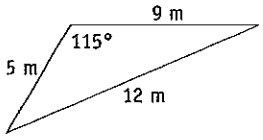
Choose the correct answer (A, B, C or D) for each question. One mark each

1 Find the area of the sector, correct to one decimal place.



A 10.1 m²
B 210.1 m²
C 40.2 m²
D 80.4 m²

2 Find the area of the triangle. Give the answer to the nearest square metre.



A 10 m²
B 20 m²
C 27 m²
D 48 m²

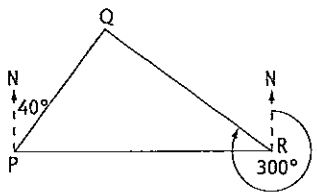
3 What is the difference in longitude between points P(25°N, 66°W) and Q(38°S, 113°E)?

A 13°
B 47°
C 63°
D 179°

4 The surface area of a sphere is 39 408 cm². What is the radius (to the nearest centimetre)?

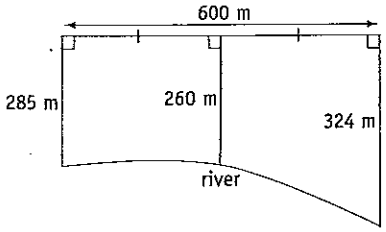
A 97 cm
B 21 cm
C 15 cm
D 56 cm

5 In the diagram, R is due east of P. The bearing of Q from P is 040° and of Q from R is 300°. What is the size of ∠PQR?



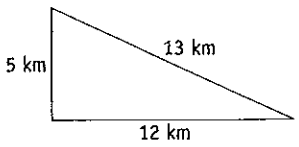
A 80°
B 90°
C 100°
D there is not enough information to find the size of ∠PQR

6 The diagram shows a paddock bounded by a river on one side. Using Simpson's rule, what is the approximate area of the paddock?



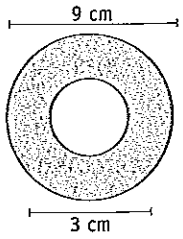
A 16.5 ha
B 3 ha
C 33 ha
D 10.5 ha

7 A boat averaged 12 knots when sailing the course shown. How long did the boat take to complete the course? (1.852 km = 1 M).



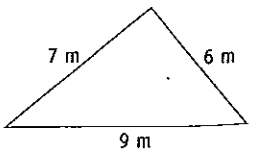
A 1 hour and 21 minutes
B 4 hours and 38 minutes
C 2 hours and 30 minutes
D 2 hours and 50 minutes

8 The diagram shows two circles with common centre. What is the shaded area (to one decimal place)?



A 28.3 cm²
B 113.1 cm²
C 56.5 cm²
D 226.2 cm²

9 To the nearest degree, what is the size of the largest angle of this triangle?

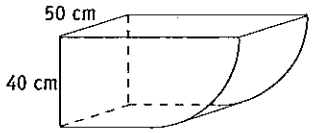


A 87°
B 51°
C 42°
D 99°

10 City P is located at (55°N, 73°E) and point Q is at (35°S, 148°E). If it is 6 pm local time at Q, what is the local time at P? (Ignore time zones.)

A 12 am B 1 pm
C 11 pm D 12 pm

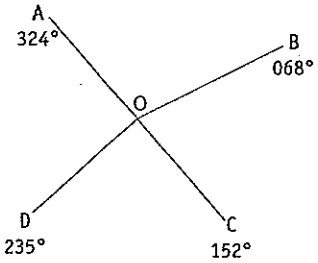
11 A bin has a constant cross-section made up of a square and a quadrant. What is the volume of the bin to the nearest cubic centimetre?



A 142 832 cm^3 B 205 664 cm^3
C 64 432 cm^3 D 81 257 cm^3

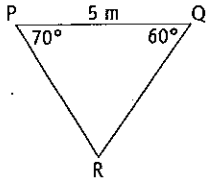
12 What is the size of $\angle AOB$?

A 68°
B 32°
C 84°
D 104°

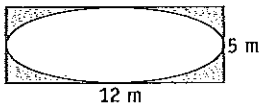


13 Find the length of side QR, correct to one decimal place.

A 5.4 m
B 6.1 m
C 4.1 m
D 5.7 m



14 Find the shaded area to the nearest square metre.



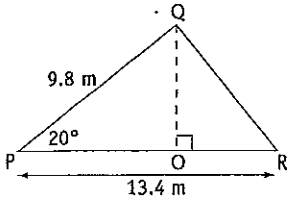
A 13 m^2 B 33 m^2
C 47 m^2 D 128 m^2

15 There are two particular points on the equator. One has longitude 27°E and the other has longitude 56°W. What is the approximate distance between them? (The radius of the Earth is approximately 6400 km; 1.852 km = 1 M)

A 3200 km B 1740 km
C 9250 km D 5000 km

16 In the diagram PR is 13.4 m. What is the length of OR (to one decimal place)?

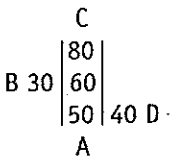
A 4.2 m
B 9.2 m
C 5.4 m
D 3.4 m

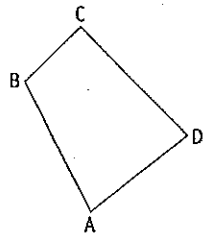


17 A closed cylinder has diameter 9 cm and height 8 cm. Find the surface area to the nearest square centimetre.

A 290 cm^2 B 353 cm^2
C 452 cm^2 D 961 cm^2

18 A sketch of a paddock has been drawn from the notebook entries shown. (All measurements are in metres.)



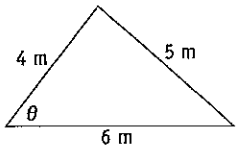


What is the area of the paddock?

A 1900 m^2 B 6650 m^2
C 2800 m^2 D 3800 m^2

19 What is the value of $\cos\theta$?

A $\frac{9}{16}$ B $\frac{3}{4}$



C $\frac{2}{3}$ D $\frac{1}{8}$

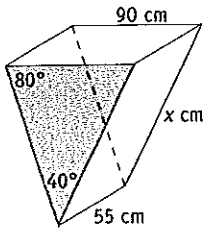
20 A plane flies directly from Hobart (43°S, 147°E) to Townsville (19°S, 147°E) in $3\frac{1}{2}$ hours. What is its approximate average speed? (The radius of the Earth is approximately 6400 km; 1.852 km = 1 M)

A 410 knots B 220 knots
C 440 knots D 770 knots

Part B (Suggested time: 25 minutes)

Show all working. **30 marks**

23 A part of a piece of machinery is a hopper in the shape of a triangular prism.



a Use the sine rule to find the length of the side marked x. 2 marks

b Find the area of the shaded triangle, to the nearest square centimetre. 2 marks

c Find the capacity of the hopper in litres, to the nearest litre. ($1\ cm^3 = 1\ mL$) 2 marks

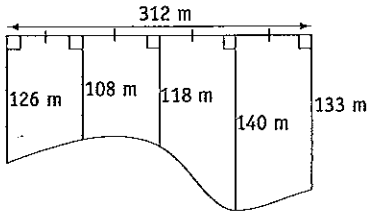
24 a Ignoring time zones, find the time difference between Entebbe (0°, 32°E) and Quito (0°, 79°W). 1 mark

b Show that the distance between Entebbe and Quito is approximately 12 400 km. (You may assume that the radius of the Earth is 6400 km; 1.852 km = 1 M) 1 mark

c A plane leaves Entebbe at 8.25 pm local time and arrives in Quito at 6.31 am the next day local time. How long was the flight? 2 marks

d Find the average speed of the flight in kilometres per hour. 1 mark

25 A paddock is bounded on three sides by straight fences, and on the fourth side by a creek.



a Use two applications of Simpson's rule to find the approximate area of the paddock in hectares, correct to two decimal places. 3 marks

b The paddock is to be sprayed to remove noxious weeds. The spray is to be used at the rate of one litre per 1.25 hectares. Approximately how many litres of spray will be needed? 1 mark

26 A helicopter left its base (B) and flew 97 km on a bearing of 206° to P. From P it flew 132 km on a bearing of 311° to Q, then straight back to its base.

a Draw a diagram showing the flight path of the helicopter. 1 mark

b How far did the helicopter fly? 3 marks

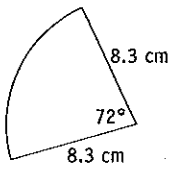
c On what bearing did it fly from Q back to its base? 3 marks

27 A large concrete pipe is going to be used when constructing a creek crossing.
The pipe is 3.8 m long. The inside diameter of the pipe is 1.1 m and the outside diameter is 1.4 m.

a Find the amount of concrete needed to construct the pipe. 2 marks

b The whole pipe (inside, outside and both ends) is to be painted with a particular type of sealant. Find the area to be painted. 3 marks

28



a Find the length of the arc of the sector. 1 mark

b The radius of the sector was incorrectly copied. The actual measurement should be 8.5 cm. What is the error in the calculation as a percentage of the actual arc length? 2 marks

Go to p 289 for **Quick Answers**
or to pp 340–3 for **Worked Solutions**

Solutions

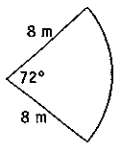
Unit 3 Test p182

1 $A = \frac{\theta}{360}\pi r^2$

$= \frac{72}{360} \times \pi \times 8^2$

$= 40.212\,385\,97\ldots$

$= 40.2$ (1 d.p.)

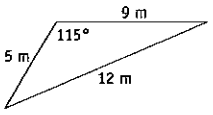


The area of the sector is 40.2 m^2, to one decimal place.

C

Solutions continued on next page

2



$A = \frac{1}{2}ab\sin C$

$= \frac{1}{2} \times 9 \times 5 \times \sin 115°$

$= 20.391\,925\,21 \ldots$

$= 20$ (nearest unit)

The area of the triangle is 20 m^2, to the nearest square metre. B

3 Difference in longitude = 113° + 66°

= 179° D

4

$A = 4\pi r^2$

$39\,408 = 4\pi r^2$

$r^2 = \frac{39\,408}{4\pi}$

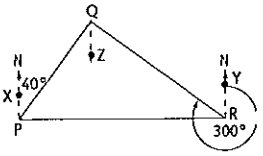
$r = \sqrt{\frac{39\,408}{4\pi}} \quad (r > 0)$

$= 55.999\,901\,77 \ldots$

$= 56$ (nearest unit)

The radius is 56 cm, to the nearest centimetre. D

5 Let X be a point due north of P. Let Y be a point due north of R and let Z be a point due south of Q.



$\angle PQZ = \angle XPQ$ (alternate angles, parallel lines)

$\angle PQZ = 40°$

$\angle YRQ = 360° - 300°$ (angles at a point)

$\angle YRQ = 60°$

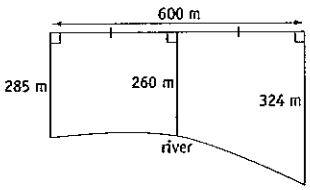
$\angle ZQR = \angle YRQ$ (alternate angles, parallel lines)

$\angle ZQR = 60°$

$\angle PQR = 40° + 60°$

$= 100°$ C

6



$h = 600 \div 2$

$= 300$

$A \approx \frac{h}{3}(d_f + 4d_m + d_l)$

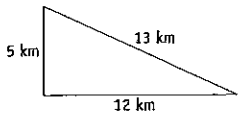
$= \frac{300}{3}(285 + 4 \times 260 + 324)$

$= 164\,900$

The approximate area is 164 900 m^2.

164 900 m^2 ≈ 16.5 ha A

7



Length of course = (5 + 12 + 13) km

= 30 km

30 km = (30 ÷ 1.852) M

= 16.198 7041 ... M

Time = (16.198 ... ÷ 12) hours

= 1.349 892 009 ... hours

= 1 hour and 21 minutes A

8 $R = 9 \div 2$

$= 4.5$

$r = 3 \div 2$

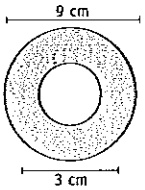
$= 1.5$

$A = \pi(R^2 - r^2)$

$= \pi \times (4.5^2 - 1.5^2)$

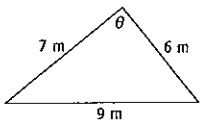
$= 56.548\,667\,76 \ldots$

$= 56.5$ (1 d.p.)



The shaded area is 56.5 cm^2, to one decimal place. C

9



The largest angle is opposite the largest side.

$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

$\cos\theta = \frac{7^2 + 6^2 - 9^2}{2 \times 7 \times 6}$

$\theta = 87.270\,597\,36 \ldots °$

$= 87°$ (nearest degree)

The largest angle measures 87°. A

10 Angular difference = 148° − 73°

= 75°

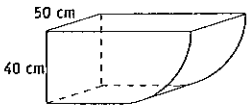
Time difference = (75 ÷ 15) hours

= 5 hours

P is west of Q so P is 5 hours behind Q.

Time at P is 1 pm. B

11 The cross-section is a square of side 40 cm plus a quadrant of radius 40 cm.



$V = Ah$

$= (40^2 + \frac{1}{4} \times \pi \times 40^2) \times 50$

$= 142\,831.8531 \ldots$

$\fallingdotseq 142\,832$ (nearest unit)

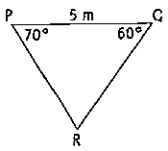
The volume of the bin is 142 832 cm^3, to the nearest cubic centimetre. A

12 $\angle AOB = (360 - 324)° + 68°$

= 104° D

13 $\angle R + 70° + 60° = 180°$ (angle sum of Δ)

$\angle R = 50°$



$\frac{p}{\sin P} = \frac{r}{\sin R}$

$\frac{p}{\sin 70°} = \frac{5}{\sin 50°}$

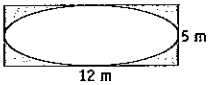
$p = \frac{5 \sin 70°}{\sin 50°}$

$= 6.133\,407\,985 \ldots$

$= 6.1$ (1 d.p.)

The length of QR is 6.1 m, to one decimal place. B

14 Shaded area is area of rectangle minus area of ellipse.



$a = 12 \div 2$

$= 6$

$b = 5 \div 2$

$= 2.5$

$A = 12 \times 5 - \pi \times 6 \times 2.5$

$= 12.876\,1102 \ldots$

$= 13$ (nearest unit)

The shaded area is 13 m^2, to the nearest square metre. A

15 Angular difference = 27° + 56°

= 83°

$l = \frac{\theta}{360} 2\pi r$

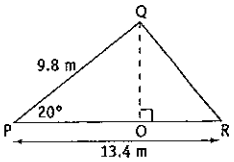
$= \frac{83}{360} \times 2 \times \pi \times 6400$

$= 9271.188\,987 \ldots$

$= 9250$ (nearest fifty)

The approx. distance is 9250 km. C

16



In ΔQPO,

$\cos 20° = \frac{OP}{9.8}$

$OP = 9.8 \times \cos 20°$

$= 9.208\,987\,684 \ldots$

$= 9.2$ (1 d.p.)

The length of OP is 9.2 m to 1 decimal place.

OR = 13.4 − 9.2

= 4.2

The length of OR is 4.2 m. A

17 $A = 2\pi r^2 + 2\pi rh$

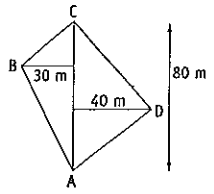
$= 2 \times \pi \times 4.5^2 + 2 \times \pi \times 4.5 \times 8$

$= 353.429\,1735 \ldots$

$= 353$ (nearest unit)

The surface area of the cylinder is 353 cm^2, to the nearest square centimetre. B

18

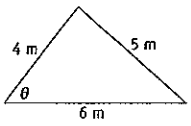


$A = \frac{1}{2} \times 80 \times 30 + \frac{1}{2} \times 80 \times 40$

$= 2800$

The area of the paddock is 2800 m². C

19



$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

$\cos\theta = \frac{4^2 + 6^2 - 5^2}{2 \times 4 \times 6}$

$= \frac{27}{48}$

$= \frac{9}{16}$ A

20 Angular difference = 43° − 19°

= 24°

Distance ≈ 24 × 60 M

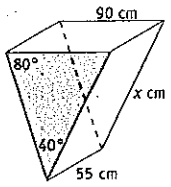
= 1440 M

Speed = 1440 M in 3.5 hours

= 411.428 5714 ... knots

The average speed is approximately 410 knots. A

21 a



$\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{x}{\sin 80°} = \frac{90}{\sin 40°}$

$x = \frac{90 \sin 80°}{\sin 40°}$ ✓

= 137.887 9998 ...

= 137.9 (1 d.p.)

The length of the side is 137.9 cm, to one decimal place. ✓

b Other angle of triangle

= 180° − (80 + 40)°

= 60°

$A = \frac{1}{2} ab \sin C$

$= \frac{1}{2} \times 137.887 \ldots \times 90 \times \sin 60°$ ✓

= 5373.652 98 ...

= 5374 (nearest unit)

The area of the triangle is 5374 cm², to the nearest square centimetre. ✓

c $V = Ah$

= 5373.652 ... × 55

= 295 550.9139 ...

= 295 550 (nearest ten)

The volume of the hopper is approximately 295 550 cm³. ✓

Capacity = 295 550 mL

= 295.55 L

The capacity of the hopper is 296 litres, to the nearest litre. ✓

22 a Angular difference = 32° + 79°

= 111°

Time difference

= (111 ÷ 15) hours

= 7.4 hours

= 7 hours and 24 minutes ✓

b $l = \frac{\theta}{360} 2\pi r$

$= \frac{111}{360} \times 2 \times \pi \times 6400$

= 12 398.819 01 ...

≈ 12 400

The distance between Entebbe and Quito is approximately 12 400 km. ✓

c Quito is 7 hours and 24 minutes behind Entebbe.

8.25 pm in Entebbe

= 1.01 pm in Quito ✓

The flight lasts from 1.01 pm until 6.31 am the next day.

Time of flight = 17.5 hours ✓

d Average speed

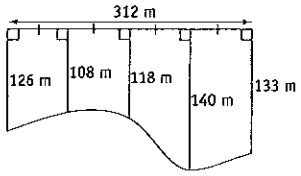
≈ (12 400 ÷ 17.5) km/h

= 708.571 4286 ... km/h

≈ 700 km/h

The average speed is about 700 km/h. ✓

23



a $h = 312 \div 4$

= 78

$A \approx \frac{h}{3}(d_f + 4d_m + d_l)$

$= \frac{78}{3}(126 + 4 \times 108 + 118)$

$+ \frac{78}{3}(118 + 4 \times 140 + 133)$ ✓

= 38 662

The approximate area of the paddock is 38 662 m². ✓

38 662 m² = 3.8662 ha

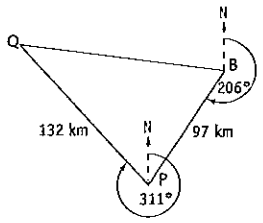
= 3.87 ha (2 d.p.) ✓

b Litres of spray = 3.87 ÷ 1.25

= 3.096 ...

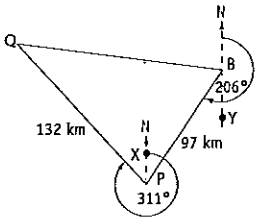
Approximately 3 litres of spray will be needed. ✓

24 a



✓

b Let X be a point due north of P and let Y be a point due south of B.



∠PBY = 206° − 180°

= 26°

∠XPB = ∠PBY (alternate angles, parallel lines)

∠XPB = 26°

∠XPQ = 360° − 311°

= 49°

∠QPB = 49° + 26°

= 75° ✓

By the cosine rule:

$p^2 = b^2 + q^2 - 2bq\cos P$

$= 132^2 + 97^2 - 2 \times 132 \times 97 \times \cos 75°$

= 20 205.161 89 ...

$p = \sqrt{20\,205.161\,89 \ldots}$ $(p > 0)$

= 142.144 8624 ...

= 142 (nearest unit)

The distance from B to Q is 142 km, to the nearest kilometre. ✓

Distance flown = (97 + 132 + 142) km

= 371 km

The helicopter flew 371 km, to the nearest kilometre. ✓

c By the sine rule:

$\frac{\sin B}{b} = \frac{\sin P}{p}$

$\frac{\sin B}{132} = \frac{\sin 75°}{142.144 \ldots}$

$\sin B = \frac{132 \sin 75°}{142.144 \ldots}$

B = 63.764 910 61 ... °

= 64° (nearest degree) ✓

∠QBP = 64°

∠QBY = 64° + 26°

= 90° ✓

Q is due west of B.

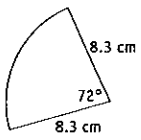
∴ B is due east of Q.

The bearing on which the helicopter must fly is 090°. ✓

25 **a** $R = 1.4 \div 2$
$= 0.7$
$r = 1.1 \div 2$
$= 0.55$
$A = \pi(R^2 - r^2)$
$= \pi \times (0.7^2 - 0.55^2)$
$= 0.589\,048\,622\ldots$ ✓
$V = Ah$
$= 0.589\ldots \times 3.8$
$= 2.238\,384\,766\ldots$
$= 2.2$ (1 d.p.)
The amount of concrete required is 2.2 m^3, to one decimal place. ✓

b Both ends have area of 0.589 ... m^2.
Inside:
$A = 2\pi rh$
$= 2 \times \pi \times 0.55 \times 3.8$
$= 13.131\,857\,29\ldots$ ✓
Outside:
$A = 2\pi Rh$
$= 2 \times \pi \times 0.7 \times 3.8$
$= 16.713\,272\,92\ldots$ ✓
Total
$= 2 \times 0.589\ldots + 13.13\ldots + 16.71\ldots$
$= 31.023\,227\,45\ldots$
$= 31$ (nearest unit)
The area to be painted is 31 m^2, to the nearest square metre. ✓

26 **a** $l = \frac{\theta}{360} 2\pi r$
$= \frac{72}{360} \times 2 \times \pi \times 8.3$
$= 10.430\,087\,61\ldots$
$= 10.43$ (2 d.p.)
The length of the arc is 10.43 cm to two decimal places. ✓



b $l = \frac{\theta}{360} 2\pi r$
$= \frac{72}{360} \times 2 \times \pi \times 8.5$
$= 10.681\,415\,02\ldots$
$= 10.68$ (2 d.p.)
Error = 10.681 415 02 ...
$- 10.430\,087\,61\ldots$
$= 0.251\,327\,412\ldots$ ✓
Percentage error
$= \frac{0.251\,327\,412\ldots}{10.681\,415\,02\ldots} \times 100\%$
$= 2.352\,941\,177\ldots\ \%$
$= 2.35\%$ (2 d.p.) ✓