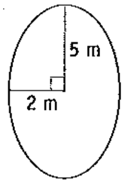
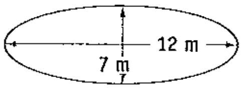
Further Practice: Further Applications of Area and Volume

Remember: all questions match the numbered examples on pages 118–128.

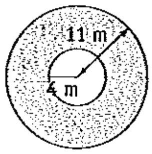
1 The diagram shows an ellipse. The semi-major axis is 5 m long and the semi-minor axis is 2 m in length. Find the area of the ellipse.



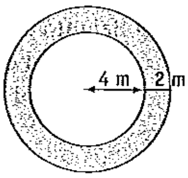
2 Find the area of the ellipse. Give the answer to the nearest square metre.



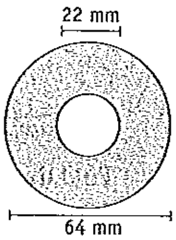
3 Find the area of the annulus shown in the diagram. Give the answer correct to one decimal place.



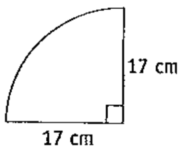
4 Find the area of the annulus shaded in the diagram. Give the answer to the nearest square metre.



5 The shaded area is between two circles with common centre. Find its area, to the nearest square millimetre.

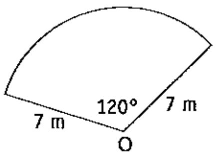


6 Find the area of the quadrant, correct to one decimal place.

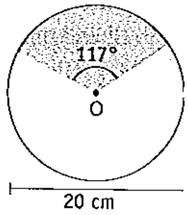


7 Find the area, to the nearest square metre, of a quadrant with radius 36 m.

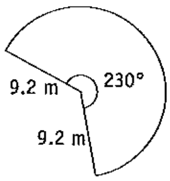
8 The diagram shows a sector of a circle, centre O. Find the area of the sector, giving the answer to the nearest square metre.



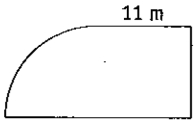
9 The diagram shows a circle of diameter 20 cm. Find the area of the shaded part, giving the answer correct to one decimal place.



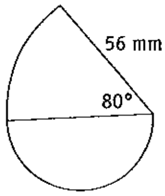
10 Find, to the nearest square metre, the area of the sector.



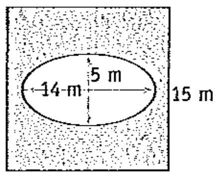
11 The shape in the diagram consists of a square and a quadrant. Find its area to the nearest square metre.



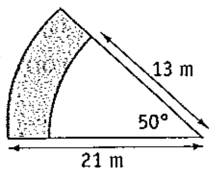
12 This figure consists of a semi-circle and a sector. Find its area to the nearest square millimetre.



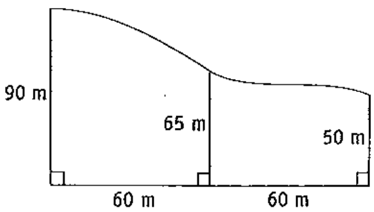
13 The diagram shows an ellipse and a square. Find the shaded area, correct to one decimal place.



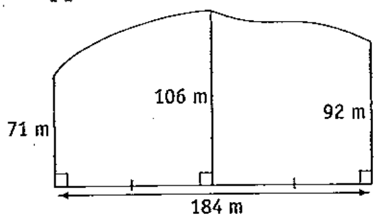
14 The diagram shows two sectors of circles with a common centre. Find the shaded area.



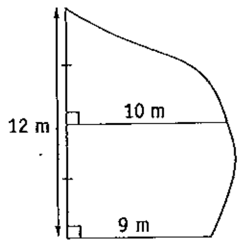
15 Use Simpson's rule to find the approximate area of the block of land shown in the diagram.



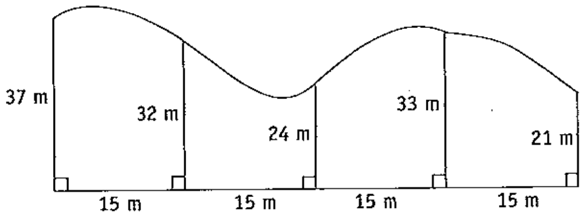
16 The diagram shows a sketch of a block of land. Find its approximate area.



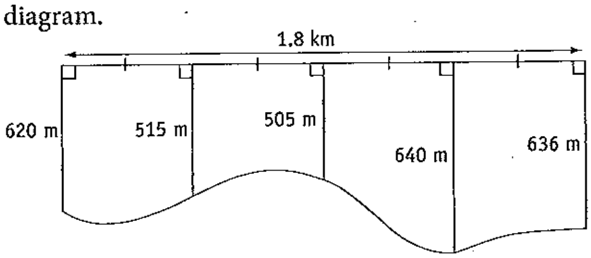
17 Find the approximate area of the region shown in the diagram.



18 The diagram shows a block of land. Find its approximate area.



19 Use two applications of Simpson's rule to find an approximation for the area of the land shown in the diagram.



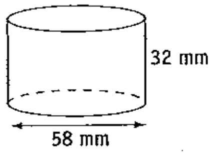
20 Find the external surface area of a completely open cylinder if its radius is 6 cm and height 9 cm. Give the answer correct to one decimal place.

21 An open cylindrical pipe is 8 m long and has a radius of 20 cm. Find its external surface area.

22 A closed drum is in the shape of a cylinder of radius 30 cm and height 50 cm. Find the surface area, to the nearest square centimetre.

23 A closed cylinder has diameter 34 cm and height 18 cm. Find its surface area to the nearest square centimetre.

24 The diagram shows a cylinder, open at the top. Find its surface area. Give the answer to the nearest square millimetre.



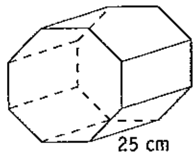
25 Find the surface area, to the nearest square metre, of a sphere of radius 5 m.

26 The planet Jupiter is roughly a sphere of radius 71 500 km. Find its approximate surface area.

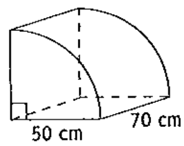
27 Find the external surface area of an open hemisphere with radius 4 m. (Give the answer to the nearest square metre.)

28 Find the surface area of a closed hemisphere with radius 36 mm.

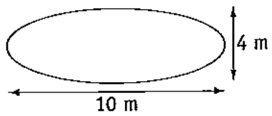
29 The diagram shows an octagonal prism. If the area of the octagonal face is 875 cm^2, find the volume.



30 The diagram shows a prism of height 70 cm. The cross-section is a quadrant of radius 50 cm. Find the volume of the prism.



31 A swimming pool has an elliptical cross-section, as shown in the diagram.



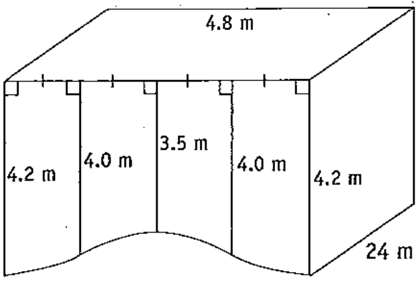
Find the volume of water required to fill the pool, if it can be filled to a height of 0.9 m. Give the answer to the nearest kilolitre. (1 m^3 = 1000 L)

32 A circular pond is surrounded by a garden bed one metre wide. If the radius of the pond is 1.8 m, find:

a the area of the garden bed

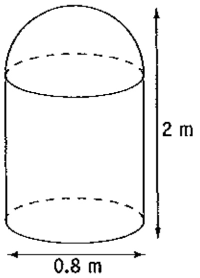
b the amount, in cubic metres, correct to one decimal place, of soil needed to add to the bed if the soil is to be 20 cm deep.

33 A man-made pool has constant vertical cross-sections as shown in the diagram.

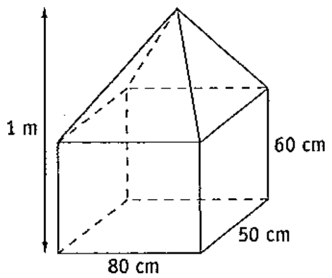


a Use Simpson's rule to find the approximate area of the cross-section.
b Find the volume of the pool.
c If the pool takes 3 hours and 57 minutes to empty, find the rate, in litres per second, at which the water is flowing from the pool. ($1\text{ m}^3 = 1000\text{ L}$)

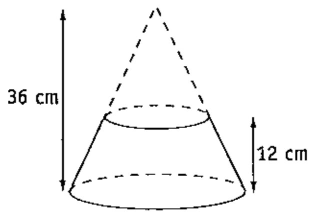
34 A post is made up of a cylinder with a hemi-spherical cap. Given the dimensions shown in the diagram, find the volume of the post.



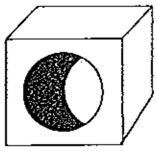
35 A storage bin consists of a rectangular prism and pyramid. Find its volume.



36 A container is a truncated cone. The diameter at the top is 28 cm and the diameter at the bottom is 42 cm. Find, to the nearest litre, the capacity of the container in litres. ($1000\text{ cm}^3 = 1\text{ L}$).



37 A concrete block is a cube of side length 40 cm. The block has a hole through it, as shown in the diagram. The diameter of the hole is 16 cm.



How many of these blocks could be made from 3 cubic metres of concrete?

38 Jackson measured the diameter of a sphere and found it to be 29 cm.
a Find the surface area of a sphere of diameter 29 cm.
b If the actual diameter was 30 cm, find the correct surface area.
c What is the error as a percentage of the correct surface area?

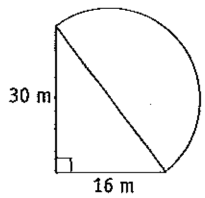
39 A rectangular prism has been measured, with the measurements correct to the nearest ten centimetres. The prism was found to be 70 cm long, 50 cm wide and 30 cm high.
a What is the bottom limit of the possible volume?
b What is the top limit of the possible volume?

40 The radius of a sphere has been given as 110 mm, to the nearest centimetre. Between what two limits must the volume lie?

Go to p 288 for **Quick Answers**
or to pp 325–8 for **Worked Solutions**

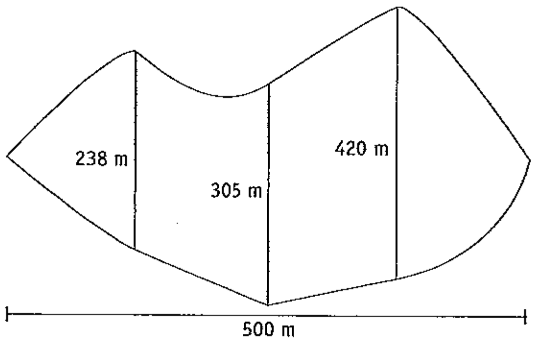
Challenge: Further Applications of Area and Volume

1 The figure consists of a right-angled triangle and a semi-circle.

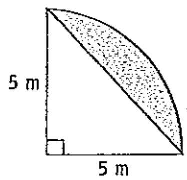


Find its area. *Hint 1*

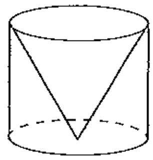
2 Use two applications of Simpson's rule to approximate the area of the region shown in the diagram. *Hint 2*



3 Find the area shaded in the diagram. *Hint 3*



4 The diagram shows a cylinder and cone, both with radius 8 cm and height 30 cm.

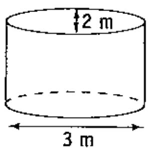


Find the volume between them. *Hint 4*

5 a Find the surface area of a cylinder, open at one end, with radius 3 cm and height 10 cm.

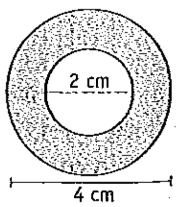
b Sophie wants to cut two pieces from a rectangular piece of cardboard, 20 cm long and 12 cm wide, to make the cylinder. Is this possible? Justify your answer. *Hint 5*

6 An above ground pool has an elliptical cross-section as shown in the diagram.



If the pool is filled with water to a depth of 1.2 metres, will it be too heavy to sit on a deck rated to hold 5 tonnes? *Hint 6*
($1\ \text{m}^3 = 1000$ L, 1 L of water = 1 kg)

7 Certain washers are made from an alloy material. Each washer is 5 mm thick and the other dimensions are shown on the diagram. How many of these washers could be made from a cubic metre of the alloy material? *Hint 7*



Go to p 288 for **Quick Answers**
or to p 328 for **Worked Solutions**

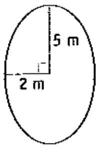
Hint 1: You will need to use Pythagoras' theorem to find the diameter of the circle.
Hint 2: It doesn't matter that neither side is straight. Just use the formula.
Hint 3: Subtract the area of the triangle from the area of the quadrant.
Hint 4: What fraction of the volume of the cylinder is the volume of the cone? What fraction remains when it is subtracted?
Hint 5: You will need to consider the size of the two pieces, not just the area.
Hint 6: Find the volume of the pool in m³. Convert first to litres, then to kilograms.
Hint 7: Two of the given measurements are in centimetres and one in millimetres. The amount of material is given in m³. Make sure you are working in the same units.

UNIT 3: MEASUREMENT

Ch 7: Further Applications of Area and Volume

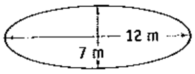
Further Practice p129

1 $A = \pi ab$
$= \pi \times 5 \times 2$
$= 31.415\,926\,54\ldots$
$= 31.4 \quad (1\text{ d.p.})$



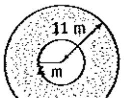
The area is 31.4 m^2, correct to one decimal place.

2 $a = 6 \quad [12 \div 2]$
$b = 3.5 \quad [7 \div 2]$



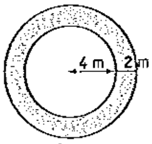
$A = \pi ab$
$= \pi \times 6 \times 3.5$
$= 65.973\,445\,73\ldots$
$= 66 \quad (\text{nearest unit})$
The area is 66 m^2, to the nearest square metre.

3 $A = \pi(R^2 - r^2)$
$= \pi(11^2 - 4^2)$
$= 329.867\,2286\ldots$
$= 329.9 \quad (1\text{ d.p.})$



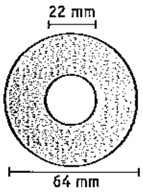
The area of the annulus is 329.9 m^2, correct to one decimal place.

4 $A = \pi(R^2 - r^2)$
$= \pi(6^2 - 4^2)$
$= 62.831\,853\,07\ldots$
$= 63 \quad (\text{nearest unit})$



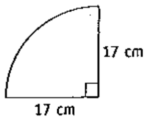
The area of the annulus is 63 m^2, correct to the nearest square metre.

5



Diameter of outer circle is 64 mm.
$\therefore R = 64 \div 2$
$= 32$
Diameter of inner circle is 22 mm.
$\therefore r = 22 \div 2$
$= 11$
$A = \pi(R^2 - r^2)$
$= \pi(32^2 - 11^2)$
$= 2836.858\,166\ldots$
$= 2837 \quad (\text{nearest unit})$
The area of the annulus is 2837 mm^2, to the nearest square millimetre.

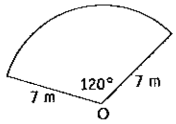
6 $A = \frac{1}{4}\pi r^2$
$= \frac{1}{4} \times \pi \times 17^2$
$= 226.980\,0692\ldots$
$= 227.0 \quad (1\text{ d.p.})$



The area of the quadrant is 227.0 cm^2, correct to one decimal place.

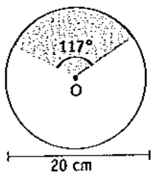
7 $A = \frac{1}{4}\pi r^2$
$= \frac{1}{4} \times \pi \times 36^2$
$= 1017.876\,02\ldots$
$= 1018 \quad (\text{nearest unit})$
The area of the quadrant is 1018 m^2, to the nearest square metre.

8 $A = \frac{\theta}{360}\pi r^2$
$= \frac{120}{360} \times \pi \times 7^2$
$= 51.312\,680\,01\ldots$
$= 51 \quad (\text{nearest unit})$



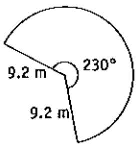
The area of the sector is 51 m^2, to the nearest square metre.

9 Diameter = 20 cm
Radius = 10 cm
$A = \frac{\theta}{360}\pi r^2$
$= \frac{117}{360} \times \pi \times 10^2$
$= 102.101\,7612\ldots$
$= 102.1 \quad (1\text{ d.p.})$



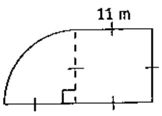
The area of the sector is 102.1 cm^2, correct to one decimal place.

10 $A = \frac{\theta}{360}\pi r^2$
$= \frac{230}{360} \times \pi \times 9.2^2$
$= 169.883\,3681\ldots$
$= 170 \quad (\text{nearest unit})$



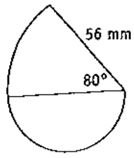
The area of the sector is 170 m^2, to the nearest square metre.

11 Radius = 11 m
$A = l^2 + \frac{1}{4}\pi r^2$
$= 11^2 + \frac{1}{4} \times \pi \times 11^2$
$= 216.033\,1778\ldots$
$= 216 \quad (\text{nearest unit})$



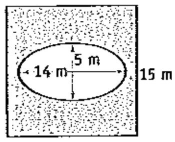
The area of the shape is 216 m^2, to the nearest square metre.

12 Radius of sector: $R = 56$
Area of sector:



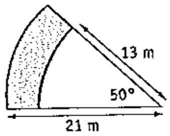
$A = \frac{\theta}{360}\pi R^2$
$= \frac{80}{360} \times \pi \times 56^2$
$= 2189.341\,014\ldots$
[The radius of the sector is the diameter of the semi-circle.]
Radius of semi-circle: $r = 28$
Area of semi-circle:
$A = \frac{1}{2}\pi r^2$
$= \frac{1}{2} \times \pi \times 28^2$
$= 1231.504\,32\ldots$
Total area
$= (2189.34\ldots + 1231.50\ldots)\ \text{mm}^2$
$= 3420.845\,334\ldots\ \text{mm}^2$
$= 3421\ \text{mm}^2 \quad (\text{nearest mm}^2)$

13 $a = 14 \div 2$
$= 7$
$b = 5 \div 2$
$= 2.5$



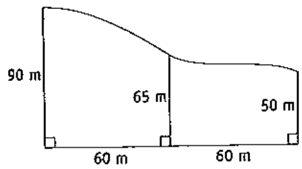
$A = s^2 - \pi ab$
$= 15^2 - \pi \times 7 \times 2.5$
$= 170.022\,1286\ldots$
$= 170.0 \quad (1\text{ d.p.})$
The shaded area is 170.0 m^2, correct to one decimal place.

14



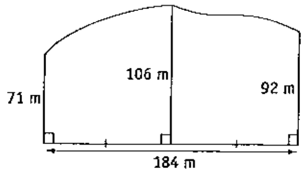
$A = \frac{\theta}{360}\pi(R^2 - r^2)$
$= \frac{50}{360} \times \pi \times (21^2 - 13^2)$
$= 118.682\,3891\ldots$
$= 118.7 \quad (1\text{ d.p.})$
The shaded area is 118.7 m^2, to one decimal place.

15



$A \approx \frac{h}{3}(d_f + 4d_m + d_l)$
$= \frac{60}{3}(90 + 4 \times 65 + 50)$
$= 8000$
The area is approximately 8000 m^2.

16

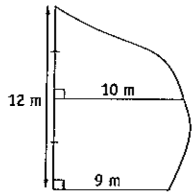


$h = 184 \div 2$
$= 92$

$A \approx \frac{h}{3}(d_f + 4d_m + d_l)$
$= \frac{92}{3}(71 + 4\times 106 + 92)$
$= 18\,001.333\,33\ldots$

The area is approximately 18 000 m^2, or 1.8 hectares.

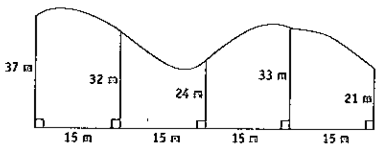
17



$A \approx \frac{h}{3}(d_f + 4d_m + d_l)$
$= \frac{6}{3}(0 + 4\times 10 + 9)$
$= 98$

The area is approximately 98 m^2.

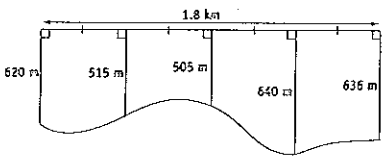
18



$A \approx \frac{h}{3}(d_f + 4d_m + d_l) + \frac{h}{3}(d_f + 4d_m + d_l)$
$= \frac{15}{3}(37 + 4\times 32 + 24)$
$\quad + \frac{15}{3}(24 + 4\times 33 + 21)$
$= 945 + 885$
$= 1830$

The area is approximately 1830 m^2.

19



Distance between successive measurements $= 1.8$ km $\div 4$
$= 450$ m

$\therefore h = 450$

$A \approx \frac{h}{3}(d_f + 4d_m + d_l) + \frac{h}{3}(d_f + 4d_m + d_l)$
$= \frac{450}{3}(620 + 4\times 515 + 505)$
$\quad + \frac{450}{3}(505 + 4\times 640 + 636)$
$= 477\,750 + 555\,150$
$= 1\,032\,900$

The area is approximately 1 032 900 m^2 or 103.29 ha.

20 $A = 2\pi rh$
$= 2\times\pi\times 6\times 9$
$= 339.292\,0066\ldots$
$= 339.3$ (1 d.p.)

The surface area is 339.3 cm^2, correct to one decimal place.

21 Radius = 20 cm
= 0.2 m

$A = 2\pi rh$
$= 2\times\pi\times 0.2\times 8$
$= 10.053\,096\,49\ldots$
$= 10.1$ (1 d.p.)

The surface area is 10.1 m^2, correct to one decimal place.

22 $A = 2\pi r^2 + 2\pi rh$
$= 2\times\pi\times 30^2 + 2\times\pi\times 30\times 50$
$= 15\,079.644\,74\ldots$
$= 15\,080$ (nearest unit)

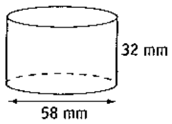
The surface area of the drum is 15 080 cm^2, to the nearest square centimetre.

23 Diameter = 34 cm
$\therefore$ radius = 17 cm

$A = 2\pi r^2 + 2\pi rh$
$= 2\times\pi\times 17^2 + 2\times\pi\times 17\times 18$
$= 3738.495\,258\ldots$
$= 3738$ (nearest unit)

The surface area of the cylinder is 3738 cm^2, to the nearest square centimetre.

24



Diameter = 58 mm
Radius = 29 mm

$A = \pi r^2 + 2\pi rh$
$= \pi\times 29^2 + 2\times\pi\times 29\times 32$
$= 8472.857\,387\ldots$
$= 8473$ (nearest unit)

The surface area is 8473 mm^2, to the nearest square millimetre.

25 $A = 4\pi r^2$
$= 4\times\pi\times 5^2$
$= 314.159\,2654\ldots$
$= 314$ (nearest unit)

The surface area is 314 m^2, to the nearest square metre.

26 $A = 4\pi r^2$
$= 4\times\pi\times 71\,500^2$
$= 6.424\,242\,817\ldots\times 10^{10}$
$= 64\,000\,000\,000$ (2 sig. figs)

The surface area is approximately 64 thousand million square kilometres.

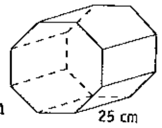
27 $A = 2\pi r^2$
$= 2\times\pi\times 4^2$
$= 100.530\,9649\ldots$
$= 101$ (nearest unit)

The surface area is 101 m^2, to the nearest square metre.

28 $A = 3\pi r^2$
$= 3\times\pi\times 36^2$
$= 12\,214.512\,24\ldots$
$= 12\,215$ (nearest unit)

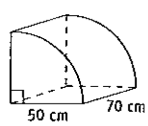
The surface area is 12 215 mm^2, to the nearest square millimetre.

29 $V = Ah$
$= 875\times 25$
$= 21\,875$



The volume of the prism is 21 875 cm^3.

30 $A = \frac{1}{4}\pi r^2$

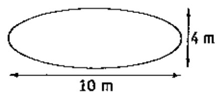


$= \frac{1}{4}\times\pi\times 50^2$
$= 1963.495\,409\ldots$

$V = Ah$
$= 1963.495\ldots\times 70$
$= 137\,444.6786\ldots$
$= 137\,445$ (nearest unit)

The volume of the prism is 137 445 cm^3, to the nearest cubic centimetre.

31 $a = 5,\quad b = 2$



$A = \pi ab$
$V = Ah$
$= \pi\times 5\times 2\times 0.9$
$= 28.274\,333\,88\ldots$
$= 28$ (nearest unit)

The volume of the pool is 28 m^3, to the nearest cubic metre.

28 m^3 = 28 000 L
= 28 kL

28 kilolitres of water are required to fill the pool.

32 a $A = \pi(R^2 - r^2)$
$= \pi(2.8^2 - 1.8^2)$
$= 14.451\,326\,21\ldots$
$= 14.5$ (1 d.p.)

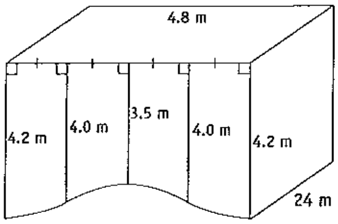
The area of the garden bed is 14.5 m^2, to one decimal place.

b Depth of soil = 20 cm
= 0.2 m

$V = Ah$
$= 14.451\,326\,21\ldots\times 0.2$
$= 2.890\,265\,241\ldots$
$= 2.9$ (1 d.p.)

The volume of soil required is 2.9 m^3, correct to one decimal place.

33



a The cross-section is symmetrical.

$h = 4.8 \div 4$
$= 1.2$

$A \approx 2\left[\frac{h}{3}(d_f + 4d_m + d_l)\right]$
$= 2\left[\frac{1.2}{3}(4.2 + 4 \times 4.0 + 3.5)\right]$
$= 18.96$

The area of the cross-section is approximately 18.96 m^2.

b $V = AH$
$= 18.96 \times 24$
$= 455.04$

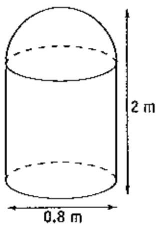
The volume of the pool is approximately 455.04 m^3.

c Capacity $= 455.04 \times 1000$ L
$= 455\,040$ L

3 hours and 57 minutes
$= (3 \times 60 + 57)$ minutes
$= 237$ minutes
$= 237 \times 60$ seconds
$= 14\,220$ seconds

Rate $= 455\,040$ L in $14\,220$ seconds
$= 32$ L/s

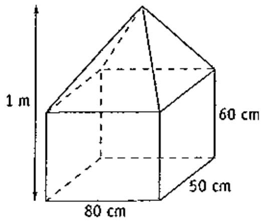
34 Diameter = 0.8 m
Radius = 0.4 m
Total height = 2 m
Radius of sphere = 0.4 m
∴ height of cylinder
$= (2 - 0.4)$ m
$= 1.6$ m



$V = \pi r^2 h + \frac{2}{3}\pi r^3$
$= \pi \times 0.4^2 \times 1.6 + \frac{2}{3} \times \pi \times 0.4^3$
$= 0.938\,289\,005\ldots$
$= 0.9$ (1 d.p.)

The volume of the post is 0.9 m^3, to one decimal place.

35



Rectangular prism:
$V = lbh$
$= 80 \times 50 \times 60$
$= 240\,000$

The volume of the rectangular prism is $240\,000 \text{ cm}^3$.

Rectangular pyramid:
Height $= (100 - 60)$ cm
$= 40$ cm

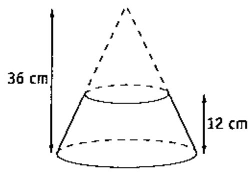
$V = \frac{1}{3}AH$
$= \frac{1}{3} \times 80 \times 50 \times 40$
$= 53\,333.3333\ldots$
$= 53\,333$ (nearest unit)

The volume of the rectangular pyramid is approximately $53\,333 \text{ cm}^3$.

Total volume
$= (240\,000 + 53\,333) \text{ cm}^3$
$= 293\,333 \text{ cm}^3$

The volume of the storage bin is approximately $293\,333 \text{ cm}^3$ [or 0.29 m^3].

36



$R = 21$ [$42 \div 2$]
$r = 14$ [$28 \div 2$]
$H = 36$
$h = 24$ [$36 - 12$]

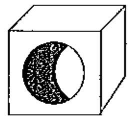
$V = \frac{1}{3}\pi R^2 H - \frac{1}{3}\pi r^2 h$
$= \frac{1}{3} \times \pi \times 21^2 \times 36 - \frac{1}{3} \times \pi \times 14^2 \times 24$
$= 11\,699.291\,04\ldots$
$= 11\,699$ (nearest unit)

The volume of the container is $11\,699 \text{ cm}^3$, to the nearest cubic centimetre.

Capacity
$= (11\,699.291\,04\ldots \div 1000)$ L
$= 11.699\,291\,04\ldots$ L
$= 12$ L (nearest litre)

The capacity of the container is 12 litres, to the nearest litre.

37 Side length of cube = 40 cm
$= 0.4$ m
Radius of hole = 8 cm
$= 0.08$ m



$A = l^2 - \pi r^2$
$= 0.4^2 - \pi \times (0.08)^2$
$= 0.139\,893\,807\ldots$

$V = Ah$
$= 0.139\,893\,807\ldots \times 0.4$
$= 0.055\,957\,522\ldots$

Each block requires approximately 0.056 m^3 of concrete.

Number of blocks
$= 3 \div 0.055\,957\,522\ldots$
$= 53.612\,094\,49\ldots$

∴ 53 blocks can be made from 3 cubic metres of concrete.

38 **a** Diameter = 29 cm
Radius = 14.5 cm
$A = 4\pi r^2$
$= 4 \times \pi \times 14.5^2$
$= 2642.079\,422\ldots$
$= 2642$ (nearest unit)

The surface area of the sphere is 2642 cm^2, to the nearest square centimetre.

b Diameter = 30 cm
Radius = 15 cm
$A = 4\pi r^2$
$= 4 \times \pi \times 15^2$
$= 2827.433\,388\ldots$
$= 2827$ (nearest unit)

The actual surface area of the sphere is 2827 cm^2, to the nearest square centimetre.

c Error $= 2827.433\ldots - 2642.079\ldots$
$= 185.353\,9666\ldots$

% error $= \frac{185.353\ldots}{2827.433\ldots} \times 100\%$
$= 6.5555\ldots\%$
$= 6.6\%$ (1 d.p.)

39 Each measurement is to the nearest ten centimetres. Error is 5 cm for each measurement.

a The smallest possible prism is 65 cm long, 45 cm wide and 25 cm high.
$V = lbh$
$= 65 \times 45 \times 25$
$= 73\,125$

The lower limit of the volume is $73\,125 \text{ cm}^3$.

b The largest possible prism is 75 cm long, 55 cm wide and 35 cm high.
$V = lbh$
$= 75 \times 55 \times 35$
$= 144\,375$

The top limit of the volume is $144\,375 \text{ cm}^3$.

40 Radius = (110 ± 5) mm

When $r = 105$,

$V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\times\pi\times 105^3$

$= 4\,849\,048.261\ldots$

$= 4\,849\,048$ (nearest unit)

Volume = 4 849 048 mm^3

When $r = 115$,

$V = \pi r^3$

$= \frac{4}{3}\times\pi\times 115^3$

$= 6\,370\,626.303\ldots$

$= 6\,370\,626$ (nearest unit)

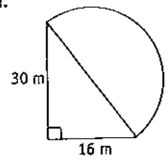
Volume = 6 370 626 mm^3

The volume is between 4 849 048 mm^3 and 6 370 626 mm^3.

Challenge p132

1 Let x m be the length of the hypotenuse.

By Pythagoras' theorem:



$x^2 = 16^2 + 30^2$

$= 256 + 900$

$= 1156$

$x = \sqrt{1156}$

$= 34$

The diameter of the semi-circle is 34 m.

The radius is 17 m.

Area of triangle:

$A = \frac{1}{2}bh$

$= \frac{1}{2}\times 16\times 30$

$= 240$

Area of semi-circle:

$A = \frac{1}{2}\pi r^2$

$= \frac{1}{2}\times\pi\times 17^2$

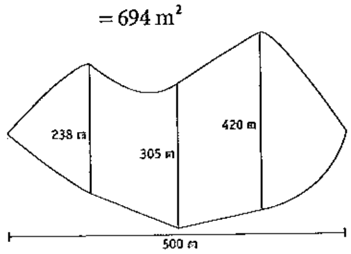
$= 453.960\,1384\ldots$

$= 454$ (nearest unit)

Total area ≈ 240 m^2 + 454 m^2

= 694 m^2

2



$h = 500 \div 4$

$= 125$

$A \approx \frac{h}{3}(d_f + 4d_m + d_l)$

$A \approx \frac{125}{3}(0 + 4\times 238 + 305)$

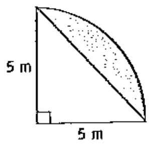
$+ \frac{125}{3}(305 + 4\times 420 + 0)$

$= 135\,083.333\ldots$

Area ≈ 135 083 m^2

The area is approximately 13.5 ha.

3 Area of quadrant:



$A = \frac{1}{4}\pi r^2$

$= \frac{1}{4}\times\pi\times 5^2$

$= 19.634\,954\,09\ldots$

$= 19.6$ (1 d.p.)

Area of triangle:

$A = \frac{1}{2}bh$

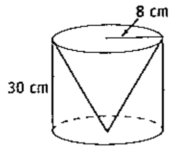
$= \frac{1}{2}\times 5\times 5$

$= 12.5$

Shaded area ≈ 19.6 m^2 − 12.5 m^2

= 7.1 m^2

4 $V = \frac{2}{3}\pi r^2 h$



$= \frac{2}{3}\times\pi\times 8^2\times 30$

$= 4021.238\,597\ldots$

$= 4021$ (nearest unit)

The volume between the cylinder and cone is 4021 cm^3 to the nearest cubic centimetre.

5 **a** $r = 3,\ h = 10$

$A = \pi r^2 + 2\pi rh$

$= \pi\times 3^2 + 2\times\pi\times 3\times 10$

$= 216.769\,8931\ldots$

$= 217$ (nearest unit)

The surface area is 217 cm^2, to the nearest square centimetre.

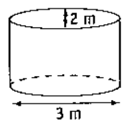
b Area of cardboard = 20 cm × 12 cm

= 240 cm^2

The curved surface area is made from a rectangle $2\pi r$ cm long and h cm wide. $2\times\pi\times 3 \approx 18.85$

The curved surface is a rectangle 18.85 cm long and 10 cm wide. There would be no room to also cut a circle of radius 3 cm. Although there is enough cardboard in area, there is not actually enough to cut the two pieces Sophie needs.

6 Ellipse: $a = 1.5,\ b = 1$



$A = \pi ab$

$= \pi\times 1.5\times 1$

$= 4.712\,388\,98\ldots$

$V = Ah$

$= 4.7123\ldots\times 1.2$

$= 5.654\,866\,776\ldots$

$= 5.65$ (2 d.p.)

The volume is 5.65 m^3, to two decimal places.

Capacity = 5.65 × 1000 litres

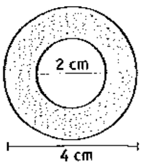
= 5650 litres

Weight = 5650 kg

= 5.65 t

The pool will be too heavy for the deck.

7 $A = \pi(R^2 - r^2)$



$= \pi(2^2 - 1^2)$

$= 9.424\,777\,961\ldots$

$V = Ah$

$= 9.424\,777\,961\ldots\times 0.5$

$= 4.712\,388\,98\ldots$

$= 4.7$ (1 d.p.)

The volume of each washer is 4.7 cm^3, to one decimal place.

One cubic metre

= (100 × 100 × 100) cm^3

= 1 000 000 cm^3

Number to be made

= 1 000 000 ÷ 4.712 388 98 ...

= 212 206.5908 ...

212 206 of the washers could be made.