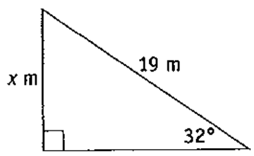
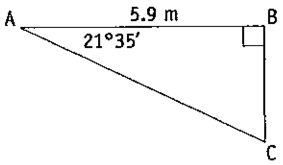
Further Practice: Applications of Trigonometry

Remember: all questions match the numbered examples on pages 137–153.

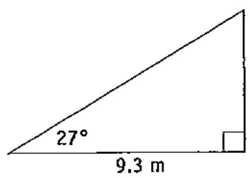
1 Find the value of x, correct to one decimal place.



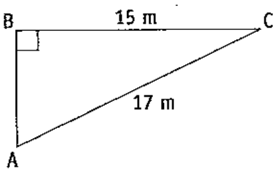
2 Find the length of side BC. (Give the answer correct to two decimal places.)



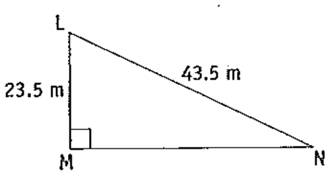
3 Find the length of the hypotenuse of the triangle. Give the answer to the nearest metre.



4 Find the size of angle BCA, to the nearest degree.



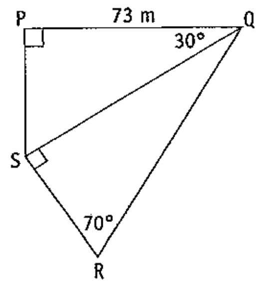
5 Find the size of ∠LNM, giving the answer correct to the nearest minute.



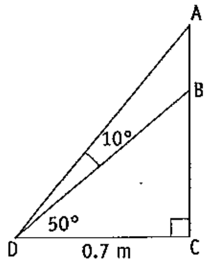
6 A 2.4 m long ladder leans against a wall. The foot of the ladder is 1.1 m from the base of the wall. Find the angle, to the nearest degree, between the ladder and the wall.

7 From a lookout at the top of a cliff, the angle of depression of a boat is 20°. If the boat is 650 m from the base of the cliff, find the height of the lookout.

8
a Find the length of QS.
b Find the length of QR.



9 AC is a post. Two pieces of timber are nailed to the post, one at A and one at B. The other ends of the timber are at D, which is 0.7 metres from the base of the post. ∠BDC = 50° and ∠ADB = 10°.
a Find the height of the post.
b Find the length of BD.
c How far from the bottom of the post is B?



10 A yacht sails 5.6 km due east from P, then turns and sails 10.5 km due south. It then sails directly back to P.
a Draw a diagram showing the path taken by the yacht.
b How far did the yacht sail altogether?

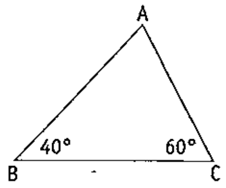
11 A helicopter leaves P and flies 32 km due east to Q. From Q it flies 15 km due north to R and 17 km due east to S. It then flies 24 km north-west to T, then straight back to P.
a Draw a diagram showing the path of the helicopter.
b In what general direction did the helicopter fly on the last leg?

12 Point K is 29 km due east of point L and 43 km due south of point J. Find the size of ∠KJL.

13 X is due west of Y and due north of Z. The bearing of Y from Z is 074°. Find the bearing of Z from Y.

14 The positions of three towns are shown in the diagram. C is due east of B. What is the bearing of:

a C from B
b B from C
c C from A
d A from B
e B from A
f A from C?



15 A plane left A and flew on a bearing of 220° to B. From B it flew on a bearing of 290° to C, which is due west of A.

a Draw a diagram showing the information.
b Find the size of ∠CAB.
c Find the size of ∠CBA.
d Find the bearing of B from C.

16 Town P is 72 km due east of town Q. Town R is due south of Q and on a bearing of 240° from P.

a Find the size of ∠QPR.
b Find the distance between towns Q and R, to the nearest kilometre.

17 A helicopter flies from its base 63 km due south and then it flies 84 km due east. On what bearing should it fly to return directly to base?

18 The bearing of X from P is 023°. X is 95 km due north of Y, which is on a bearing of 113° from P.

a Draw a diagram showing the above information.
b Find the size of ∠XPY.
c Find the distance between X and P, to the nearest kilometre.

19 Find, to three decimal places, cos 140°.

20 Find, correct to four decimal places, the value of sin 170°.

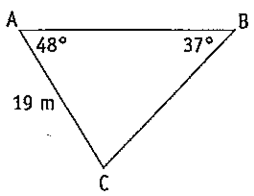
21 Find the value of tan 135°.

22 Find the value to one decimal place of $\dfrac{16 \sin 32^\circ}{\sin 107^\circ}$.

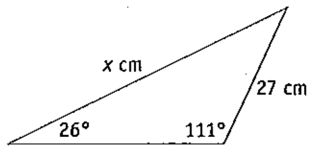
23 Find the value of θ to the nearest degree if

$\cos\theta = \dfrac{8^2 + 7^2 - 11^2}{2 \times 8 \times 7}$.

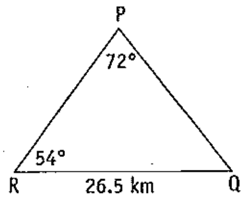
24 Find the length of side BC correct to one decimal place.



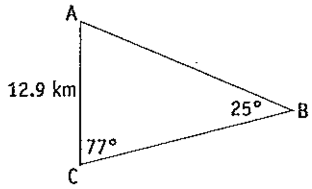
25 Find the value of x.



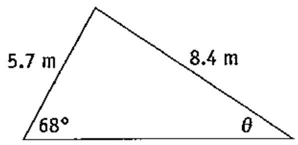
26 Find the length of side PQ of this triangle. Give the answer to one decimal place.



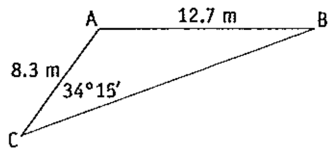
27 Find the length of side BC of this triangle. (Give the answer correct to two decimal places.)



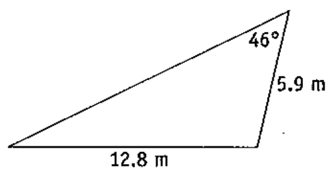
28 Find θ to the nearest degree.



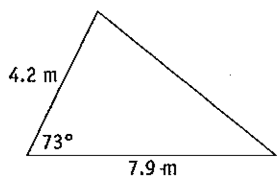
29 Find the size of ∠ABC, correct to the nearest minute.



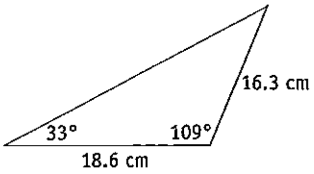
30 Find the size of the largest angle of this triangle. (Give the answer to the nearest degree.)



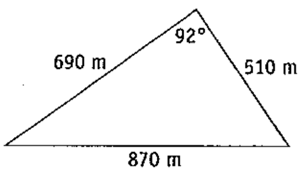
31 Find the area of the triangle. Give the answer correct to one decimal place.



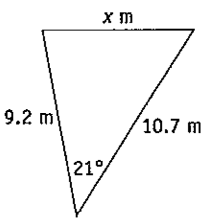
32 Find the area of the triangle to the nearest square centimetre.



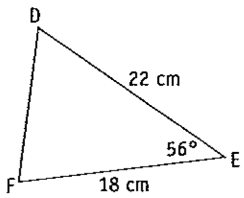
33 The diagram shows a triangular park, with measurements as shown. Find the area of the park in hectares, to two decimal places.



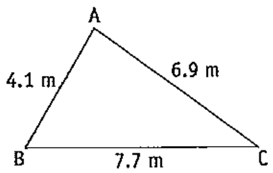
34 Use the cosine rule to find the length of the side marked x.



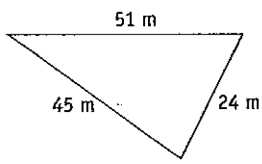
35 Find the perimeter of triangle DEF to the nearest centimetre.



36 Find the size of ∠ACB, to the nearest degree.

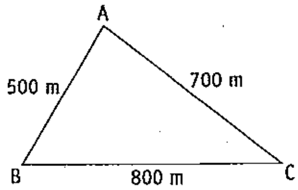


37 Find the size of the largest angle of this triangle.



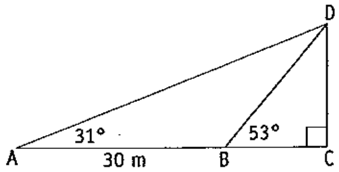
38 A boat leaves P and sails on a bearing of 167° to Q. From Q it sails 15 km on a bearing of 108° to R. The bearing of R from P is 130°. Use the sine rule to find the distance between P and R.

39 A triangular paddock has boundaries of length 500 m, 700 m and 800 m as shown in the diagram.



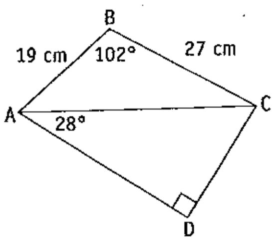
a Use the cosine rule to find the size of ∠ABC.
b Find the area of the paddock in hectares.

40 A and B are two points 30 metres apart on level ground. From A, the angle of elevation of the top of a building (D) is 31° and from B the angle of elevation of D is 53°.



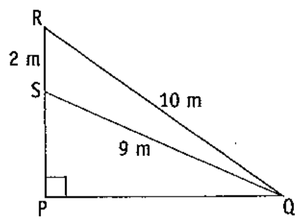
a Use the sine rule in ΔABD to find the length of BD.
b Find the height of the building.

41



a Use the cosine rule in triangle ABC to find the length of AC to the nearest millimetre.
b Find the length of CD to the nearest centimetre.

42 Two support wires, one 10 m long and one 9 m long, are attached to a pole from a common point, Q. The longer wire is attached to the top of the pole and the shorter wire is attached 2 m below the longer one.

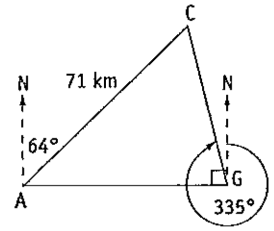


a Use the cosine rule in triangle RQS to show that $\cos R = \dfrac{23}{40}$.

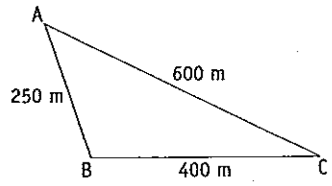
b Find the height of the pole.

43 A boat sails due east for a distance of 7 km and then on a bearing of 200° for 9 km. How far is it from its starting point, to the nearest kilometre?

44 Callistemon (C), Acacia (A) and Grevillea (G) are three towns. Callistemon bears 064° from Acacia and 335° from Grevillea. Grevillea is due east of Acacia. The distance from Acacia to Callistemon is 71 km. Find the distance, to the nearest kilometre, from Callistemon to Grevillea.

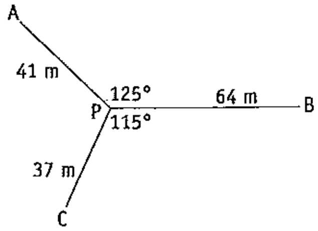


45 The diagram shows a plan of a paddock.



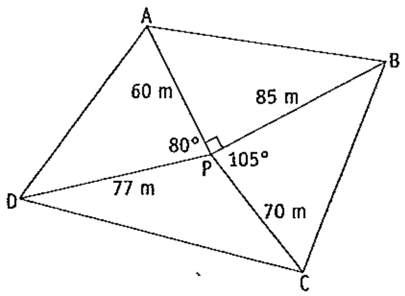
a Find the size of the angle at B.
b Find the area of the paddock in hectares, correct to two decimal places.

46 The diagram shows the result of a plane table survey of a triangular block of land.



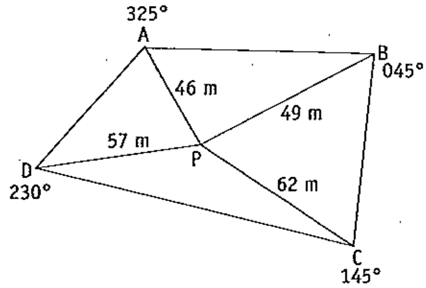
a Find the size of ∠APC.
b Find the length of AB.
c Find the area of the block of land, to the nearest square metre.

47 Adam has completed a radial survey of a paddock. A, B, C and D represent the corner posts of the paddock.



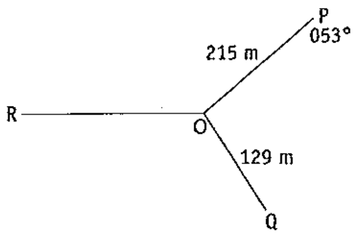
a Find the length of BC to the nearest metre.
b Find the size of ∠DPC.
c Adam has calculated that the area of ΔAPD is 2275 m² and ΔPCD is 2685 m². Find the area of the paddock in hectares.

48 A compass radial survey has been made of a block of land.



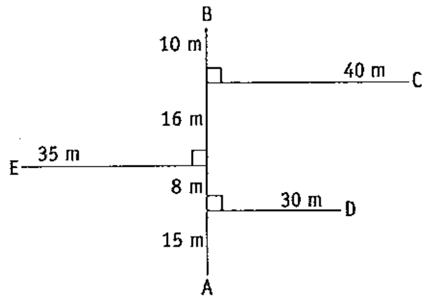
a Find the size of ∠APB.
b Find the length of boundary BC to the nearest metre.
c Find the area of ΔAPD.

49 The diagram shows an incomplete radial survey of a block of land. R is due west of O.



a Find the size of ∠POR.
b Use the cosine rule in ΔQOP to find the size of ∠POQ, if PQ is 301 metres.
c What is the bearing of Q from O?

50 An offset survey has been taken of a yard and the diagram shows the result.



a Find the length of the boundary from C to D.
b Find the area of the yard.

51 The following notebook entries have been given of a survey of a block of land. (All measurements are in metres.)

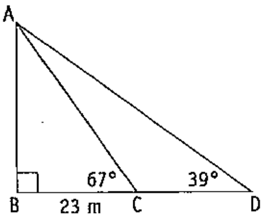
	B	
	90	
	60	40 C
D 50	20	
	0	
	A	

Complete the corresponding offset survey and sketch a plan of the block of land.

Go to p 288 for **Quick Answers**
or to pp 330–5 for **Worked Solutions**

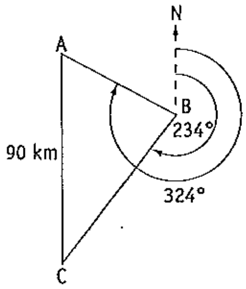
Challenge: Applications of Trigonometry

1



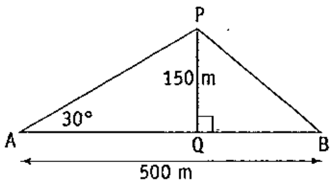
a Find the length of AB, correct to one decimal place.
b Find the length of CD. Give the answer to the nearest metre. *Hint 1*

2 C is 90 km due south of A. The bearing of A from B is 324° and of C from B is 234°.



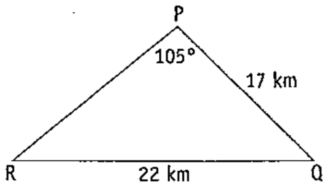
a Find the size of ∠ABC.
b Find the size of ∠BAC.
c Find the distance from B to C. *Hint 2*

3 A and B are two points 500 m apart on a straight road. P is 150 m from Q which is on the road.

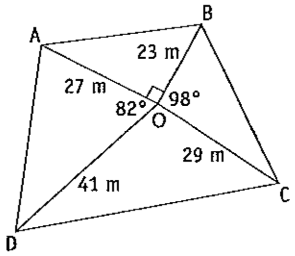


a Find the distance from A to P.
b Find the distance from P to B. *Hint 3*

4 Find the size of ∠Q, to the nearest degree. *Hint 4*



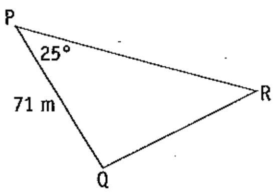
5 A plane table radial survey of a block of land has been made.



Find the area of the block of land. *Hint 5*

6 A helicopter flies 125 km due east and then 125 km due south. On what bearing should it fly to return straight to its starting point? *Hint 6*

7 The bearing of P from Q is 345° and the bearing of R from Q is 066°.



a Explain why ∠PQR is 81°.
b Find the distance from P to R. *Hint 7*

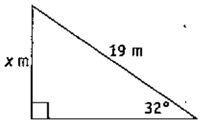
Go to p 289 for **Quick Answers**
or to pp 335–6 for **Worked Solutions**

Hint 1: Using triangle ABD, find the length of BD.
Hint 2: What was the answer to part a? There is no need to use the sine rule or the cosine rule.
Hint 3: Use the cosine rule in triangle APB.
Hint 4: First find angle R.
Hint 5: There is no need to use the formula Area = $\frac{1}{2}ab\sin C$ when the triangles are right-angled. Place the first area in the calculator's memory and add the others as they are found.
Hint 6: There is no need to use trigonometry to find the size of the angle. It is an isosceles triangle.
Hint 7: You will need to find the size of angle PRQ.

Ch 8: Applications of Trigonometry

Further Practice p154

1



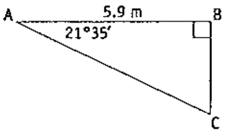
$\sin 32° = \frac{x}{19}$

$x = 19 \times \sin 32°$

$= 10.068\ 466\ 02\ ...$

$= 10.1 \quad (1\text{ d.p.})$

2



$\tan 21°35' = \frac{BC}{5.9}$

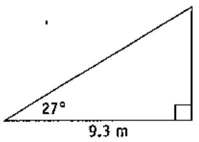
$BC = 5.9 \times \tan 21°35'$

$= 2.333\ 990\ 204\ ...$

$= 2.33 \quad (2\text{ d.p.})$

The length of BC is 2.33 m, correct to two decimal places.

3



Let the length of the hypotenuse be x m.

$\cos 27° = \frac{9.3}{x}$

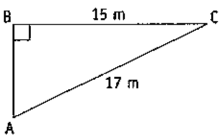
$x = \frac{9.3}{\cos 27°}$

$= 10.437\ 634\ 01\ ...$

$= 10 \quad (\text{nearest unit})$

The length of the hypotenuse is 10 m, to the nearest metre.

4

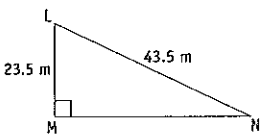


$\cos C = \frac{15}{17}$

$C = 28.072\ 486\ 94\ ...°$

$= 28° \quad (\text{nearest degree})$

5

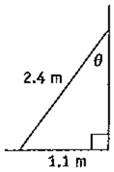


$\sin N = \frac{23.5}{43.5}$

$N = 32.699\ 289\ 49\ ...°$

$= 32°42' \quad (\text{nearest minute})$

6



Let θ be the angle between the ladder and the wall.

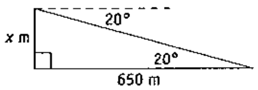
$\sin \theta = \frac{1.1}{2.4}$

$\theta = 27.279\ 612\ 74\ ...°$

$= 27° \quad (\text{nearest degree})$

The angle between the wall and the ladder is 27°, to the nearest degree.

7



Let x m be the height of the lookout.

$\tan 20° = \frac{x}{650}$

$x = 650 \times \tan 20°$

$= 236.580\ 6523\ ...$

$= 237 \quad (\text{nearest unit})$

The lookout is 237 metres high, to the nearest metre.

8 a In ΔPQS,

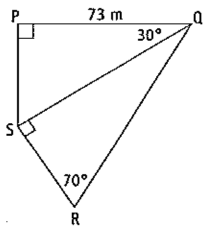
$\cos 30° = \frac{73}{QS}$

$QS = \frac{73}{\cos 30°}$

$= 84.293\ 1393\ ...$

$= 84.3 \quad (1\text{ d.p.})$

The length of QS is 84.3 m, to one decimal place.



b In ΔQSR,

$\sin 70° = \frac{QS}{QR}$

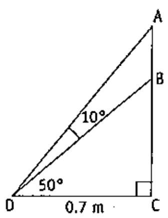
$QR = \frac{QS}{\sin 70°}$

$= 89.702\ 885\ 22\ ...$

$= 89.7 \quad (1\text{ d.p.})$

The length of QR is 89.7 m, to one decimal place.

9



a $\angle ADC = 10° + 50°$

$= 60°$

In ΔADC,

$\tan 60° = \frac{AC}{0.7}$

$AC = 0.7 \times \tan 60°$

$= 1.212\ 435\ 565\ ...$

$= 1.2 \quad (1\text{ d.p.})$

The height of the post is 1.2 m, correct to one decimal place.

b In ΔDBC,

$\cos 50° = \frac{0.7}{BD}$

$BD = \frac{0.7}{\cos 50°}$

$= 1.089\ 006\ 679\ ...$

$= 1.1 \quad (1\text{ d.p.})$

BD is 1.1 m long, correct to 1 decimal place.

c $\tan 50° = \frac{BC}{0.7}$

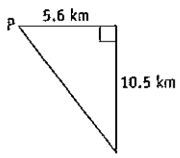
$BC = 0.7 \times \tan 50°$

$= 0.834\ 227\ 514\ ...$

$= 0.8 \quad (1\text{ d.p.})$

B is 0.8 metres from the ground, correct to one decimal place.

10 a



b Let x m be the distance back to P.

By Pythagoras' theorem:

$x^2 = 5.6^2 + 10.5^2$

$= 141.61$

$x = \sqrt{141.61} \quad (x > 0)$

$= 11.9$

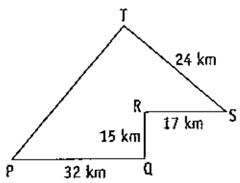
The distance back to P is 11.9 km.

Total distance $= (5.6 + 10.5 + 11.9)$ km

$= 28$ km

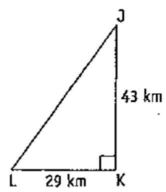
The yacht sailed 28 kilometres.

11 a



b The helicopter flew south-west on the last leg.

12



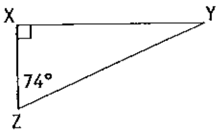
$\tan J = \frac{29}{43}$

$J = 33.996\ 459\ 15\ ...°$

$= 34° \quad (\text{nearest degree})$

$\angle KJL = 34°$ to the nearest degree.

13

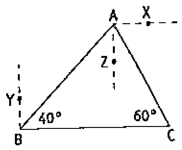


$\angle XYZ = 90° - 74°$
$= 16°$

The bearing of X from Y is 270°.

The bearing of Z from Y $= 270° - 16°$
$= 254°$

14 a C is due east of B. The bearing of C from B is 090°.



b B is due west of C. The bearing of B from C is 270°.

c Let X be a point due east of A.

$\angle XAC = \angle ACB$ (alternate angles, parallel lines)

$\angle XAC = 60°$

The bearing of C from A $= 90° + 60°$
$= 150°$

d Let Y be a point due north of B.

$\angle YBA + \angle ABC = 90°$ (C is east of B)

$\angle YBA = 90° - 40°$
$= 50°$

The bearing of A from B is 050°.

e Let Z be a point due south of A.

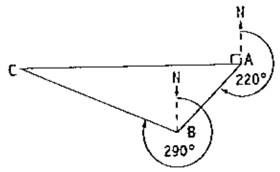
$\angle ZAB = \angle YBA$ (alternate angles, parallel lines)

$\angle ZAB = 50°$

The bearing of B from A $= 180° + 50°$
$= 230°$

f The bearing of A from C $= 270° + 60°$
$= 330°$

15 a

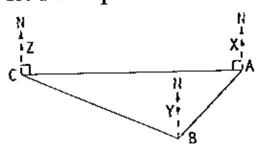


b C is due west of A.

The bearing of C from A is 270°.

$\angle CAB = 270° - 220°$
$= 50°$

c Let X be a point due north of A, and let Y be a point due north of B.



$\angle XAB = 90° + 50°$
$= 140°$

$\angle XAB + \angle YBA = 180°$

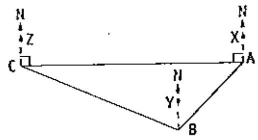
(co-interior $\angle$s, parallel lines)

$\angle YBA = 180° - 140°$
$= 40°$

$\angle CBY = 360° - 290°$
$= 70°$

$\angle CBA = 40° + 70°$
$= 110°$

d Let Z be a point due north of C.



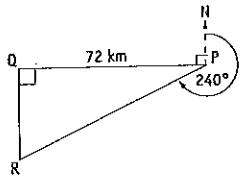
$\angle ZCB + \angle CBY = 180°$

(co-interior $\angle$s, parallel lines)

$\angle ZCB = 180° - 70°$
$= 110°$

The bearing of B from C is 110°.

16



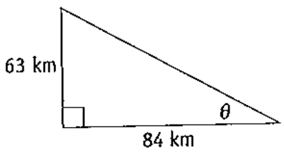
a $\angle QPR = 270° - 240°$
$= 30°$

b $\tan 30° = \frac{QR}{72}$

$QR = 72 \times \tan 30°$
$= 41.569\,219\,38\ldots$
$= 42$ (nearest unit)

The distance between Q and R is 42 km to the nearest kilometre.

17



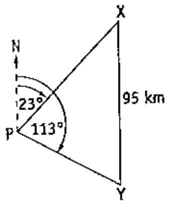
$\tan\theta = \frac{63}{84}$

$\theta = 36.869\,897\,65\ldots°$
$= 37°$ (nearest degree)

Required bearing $= 270° + 37°$
$= 307°$

The helicopter should fly on a bearing of 307°.

18 a



b $\angle XPY = 113° - 23°$
$= 90°$

c $\angle PXY = 23°$ (alternate angles, parallel lines)

$\cos 23° = \frac{PX}{95}$

$PX = 95 \times \cos 23°$
$= 87.447\,961\,08\ldots$
$= 87$ (nearest unit)

The distance between P and X is 87 km, to the nearest kilometre.

19 $\cos 140° = -0.766\,044\,443\ldots$
$= -0.766$ (3 d.p.)

20 $\sin 170° = 0.173\,648\,177\ldots$
$= 0.1736$ (4 d.p.)

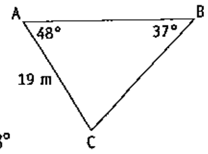
21 $\tan 135° = -1$

22 $\frac{16\sin 32°}{\sin 107°} = 8.866\,115\,299\ldots$
$= 8.9$ (1 d.p.)

23 $\cos\theta = \frac{8^2 + 7^2 - 11^2}{2 \times 8 \times 7}$
$= -\frac{1}{14}$

$\theta = 94.096\,043\,76\ldots°$
$= 94°$ (nearest degree)

24 $\frac{a}{\sin A} = \frac{b}{\sin B}$

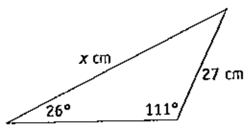


$\frac{a}{\sin 48°} = \frac{19}{\sin 37°}$

$a = \frac{19\sin 48°}{\sin 37°}$
$= 23.461\,946\,18\ldots$
$= 23.5$ (1 d.p.)

The length of BC is 23.5 m, correct to one decimal place.

25



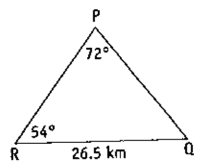
$\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{x}{\sin 111°} = \frac{27}{\sin 26°}$

$x = \frac{27\sin 111°}{\sin 26°}$
$= 57.500\,7541\ldots$
$= 57.5$ (1 d.p.)

The length of the side is 57.5 cm, to the nearest millimetre.

26



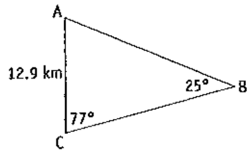
$\frac{r}{\sin R} = \frac{p}{\sin P}$

$\frac{r}{\sin 54°} = \frac{26.5}{\sin 72°}$

$r = \frac{26.5\sin 54°}{\sin 72°}$
$= 22.542\,246\,42\ldots$
$= 22.5$ (1 d.p.)

The length of side PQ is 22.5 kilometres, correct to one decimal place.

27

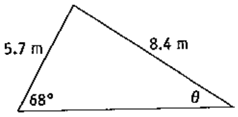


$$\angle A + 25° + 77° = 180° \quad \text{(angle sum } \Delta)$$
$$\angle A + 102° = 180°$$
$$\angle A = 78°$$

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{a}{\sin 78°} = \frac{12.9}{\sin 25°}$$
$$a = \frac{12.9 \sin 78°}{\sin 25°}$$
$$= 29.856\,977\,78\ldots$$
$$= 29.86 \quad (2\text{ d.p.})$$

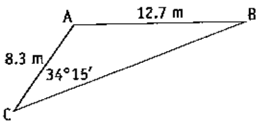
The length of the side BC is 29.86 km, correct to two decimal places.

28



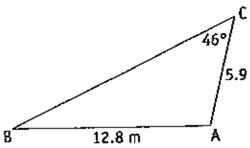
$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin \theta}{5.7} = \frac{\sin 68°}{8.4}$$
$$\sin \theta = \frac{5.7 \sin 68°}{8.4}$$
$$[= 0.629\,160\,472\ldots]$$
$$\theta = 38.988\,210\,86\ldots°$$
$$= 39° \quad \text{(nearest degree)}$$

29



$$\frac{\sin B}{b} = \frac{\sin C}{c}$$
$$\frac{\sin B}{8.3} = \frac{\sin 34°15'}{12.7}$$
$$\sin B = \frac{8.3 \sin 34°15'}{12.7} \quad [= 0.367\,817\,393\ldots]$$
$$B = 21.581\,073\,12\ldots°$$
$$= 21°35' \quad \text{(nearest minute)}$$

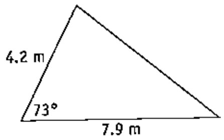
30



$$\frac{\sin B}{b} = \frac{\sin C}{c}$$
$$\frac{\sin B}{5.9} = \frac{\sin 46°}{12.8}$$
$$\sin B = \frac{5.9 \sin 46°}{12.8} \quad [= 0.331\,570\,689\ldots]$$
$$B = 19.364\,137\,69\ldots°$$
$$= 19° \quad \text{(nearest degree)}$$
$$\angle A + 19° + 46° = 180° \quad \text{(angle sum } \Delta)$$
$$\angle A + 65° = 180°$$
$$\angle A = 115°$$

The largest angle of the triangle is 115°.

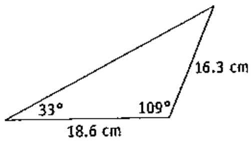
31



$$A = \frac{1}{2}ab\sin C$$
$$= \frac{1}{2} \times 4.2 \times 7.9 \times \sin 73°$$
$$= 15.865\,0959\ldots$$
$$= 15.9 \quad (1\text{ d.p.})$$

The area of the triangle is 15.9 m^2, correct to one decimal place.

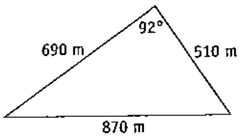
32



$$A = \frac{1}{2}ab\sin C$$
$$= \frac{1}{2} \times 16.3 \times 18.6 \times \sin 109°$$
$$= 143.331\,1609\ldots$$
$$= 143 \quad \text{(nearest unit)}$$

The area of the triangle is 143 cm^2, to the nearest square centimetre.

33

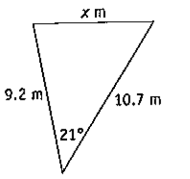


$$A = \frac{1}{2}ab\sin C$$
$$= \frac{1}{2} \times 690 \times 510 \times \sin 92°$$
$$= 175\,842.816\ldots$$

Area $= 175\,842.816\ldots\ m^2$
$= 17.584\,2816\ldots$ ha
$= 17.58$ ha (2 d.p.)

The area of the park is 17.58 ha, to two decimal places.

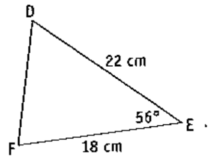
34



$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$x^2 = 9.2^2 + 10.7^2 - 2 \times 9.2 \times 10.7 \times \cos 21°$$
$$= 15.326\,685\,63\ldots$$
$$x = \sqrt{15.326\,685\,63\ldots} \quad (x > 0)$$
$$= 3.914\,931\,115\ldots$$
$$= 3.9 \quad (1\text{ d.p.})$$

The length of the side is 3.9 metres, correct to one decimal place.

35



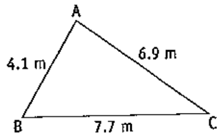
$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$[\text{or } e^2 = d^2 + f^2 - 2df\cos E]$$
$$e^2 = 18^2 + 22^2 - 2 \times 18 \times 22 \times \cos 56°$$
$$= 365.119\,2205\ldots$$
$$e = \sqrt{365.119\,2205\ldots} \quad (e > 0)$$
$$= 19.108\,093\,06\ldots$$
$$= 19 \quad \text{(nearest unit)}$$

The length of side DF is 19 cm to the nearest centimetre.

Perimeter ≈ 19 cm + 18 cm + 22 cm
= 59 cm

The perimeter of the triangle is 59 cm, to the nearest centimetre.

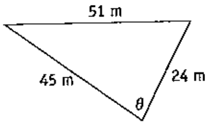
36



$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$
$$= \frac{7.7^2 + 6.9^2 - 4.1^2}{2 \times 7.7 \times 6.9}$$
$$[= 0.847\,826\,087\ldots]$$
$$C = 32.023\,995\,97\ldots°$$
$$= 32° \quad \text{(nearest degree)}$$

∠ACB is 32°, to the nearest degree.

37

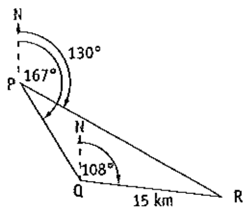


Let the required angle be θ.

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$
$$\cos \theta = \frac{24^2 + 45^2 - 51^2}{2 \times 24 \times 45}$$
$$= 0$$
$$\theta = 90°$$

The largest angle of the triangle is 90°.

38

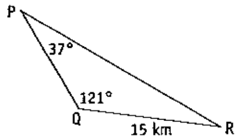


$$\angle RPQ = 167° - 130°$$
$$= 37°$$

Let X be a point due north of P and let Y be a point due north of Q.

$\angle XPQ + \angle PQY = 180°$ (co-interior ∠s, parallel lines)

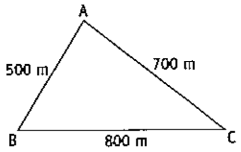
$$167° + \angle PQY = 180°$$
$$\angle PQY = 13°$$
$$\angle PQR = 13° + 108°$$
$$= 121°$$



$$\frac{q}{\sin Q}=\frac{p}{\sin P}$$
$$\frac{q}{\sin 121°}=\frac{15}{\sin 37°}$$
$$q=\frac{15\sin 121°}{\sin 37°}$$
$$=21.364\,553\,92\ldots$$
$$=21\quad\text{(nearest unit)}$$

The distance from P to R is 21 km, to the nearest kilometre.

39



a $\cos B=\frac{a^2+c^2-b^2}{2ac}$

$$=\frac{800^2+500^2-700^2}{2\times 800\times 500}$$
$$=0.5$$
$$B=60°$$
$$\angle ABC=60°$$

b $A=\frac{1}{2}ac\sin B$

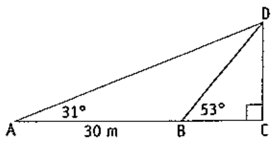
$$=\frac{1}{2}\times 800\times 500\times \sin 60°$$
$$=173\,205.0808\ldots$$
$$=173\,205\quad\text{(nearest unit)}$$

The area of the land is 173 205 m^2, to the nearest square metre.

$173\,205\ m^2=17.3205$ ha

The area of the land is 17.32 ha, correct to two decimal places.

40



a $\angle DBC=\angle BDA+\angle DAB$ (ext. angle of a triangle)

$$53°=\angle BDA+31°$$
$$\angle BDA=22°$$
$$\frac{a}{\sin A}=\frac{d}{\sin D}$$
$$\frac{a}{\sin 31°}=\frac{30}{\sin 22°}$$
$$a=\frac{30\sin 31°}{\sin 22°}$$
$$=41.246\,316\,85\ldots$$
$$=41.2\quad\text{(1 d.p.)}$$

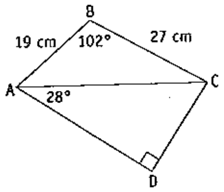
The length of BD is 41.2 metres, correct to one decimal place.

b In $\triangle DCB$,

$$\sin 53°=\frac{DC}{BD}$$
$$DC=BD\times\sin 53°$$
$$=32.940\,7733\ldots$$
$$=32.9\quad\text{(1 d.p.)}$$

The height of the building is 32.9 metres, correct to one decimal place.

41



a In $\triangle ABC$,

$$b^2=a^2+c^2-2ac\cos B$$
$$=27^2+19^2-2\times 27\times 19\times\cos 102°$$
$$=1303.317\,395\ldots$$
$$b=\sqrt{1303.317\,395\ldots}\quad(b>0)$$
$$=36.101\,487\,43\ldots$$
$$=36.1\quad\text{(1 d.p.)}$$

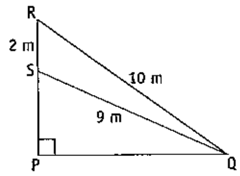
The length of AC is 36.1 cm, to the nearest millimetre.

b In $\triangle ACD$,

$$\sin 28°=\frac{CD}{AC}$$
$$CD=AC\times\sin 28°$$
$$=16.948\,621\,72\ldots$$
$$=17\quad\text{(nearest unit)}$$

The length of CD is 17 cm, to the nearest centimetre.

42



a $\cos R=\frac{q^2+s^2-r^2}{2qs}$

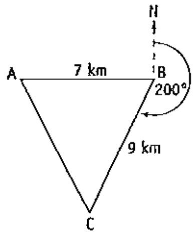
$$=\frac{2^2+10^2-9^2}{2\times 2\times 10}$$
$$=\frac{23}{40}$$

b In $\triangle RPQ$,

$$\cos R=\frac{RP}{10}$$
$$RP=10\times\cos R$$
$$=10\times\frac{23}{40}$$
$$=5.75$$

The height of the pole is 5.75 metres.

43



A is due west of B so the bearing of A from B is 270°.

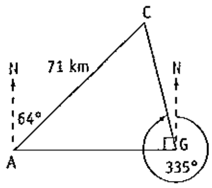
$$\angle ABC=270°-200°$$
$$=70°$$

By the cosine rule:

$$b^2=a^2+c^2-2ac\cos B$$
$$=9^2+7^2-2\times 9\times 7\times\cos 70°$$
$$=86.905\,461\,94\ldots$$
$$b=\sqrt{86.905\,461\,94\ldots}\quad(b>0)$$
$$=9.322\,309\,904\ldots$$
$$=9\quad\text{(nearest unit)}$$

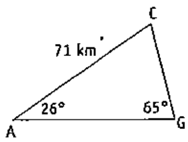
The boat is 9 km from its starting point, to the nearest kilometre.

44



$$\angle CAG=90°-64°$$
$$=26°$$
$$\angle CGA=335°-270°$$
$$=65°$$

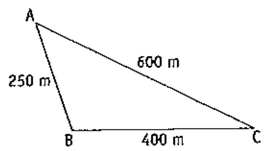
By the sine rule:



$$\frac{a}{\sin A}=\frac{g}{\sin G}$$
$$\frac{a}{\sin 26°}=\frac{71}{\sin 65°}$$
$$a=\frac{71\sin 26°}{\sin 65°}$$
$$=34.341\,9221\ldots$$
$$=34\quad\text{(nearest unit)}$$

The distance from Callistemon to Grevillea is 34 km, to the nearest kilometre.

45



a $\cos B=\frac{a^2+c^2-b^2}{2ac}$

$$=\frac{400^2+250^2-600^2}{2\times 400\times 250}$$
$$=-0.6875$$
$$B=133.432\,5366\ldots°$$
$$=133°\quad\text{(nearest degree)}$$

Angle B is 133°, to the nearest degree.

b $A = \frac{1}{2}ac\sin B$

$= \frac{1}{2} \times 400 \times 250 \times \sin B$

$= 36\,309.218\,87 \ldots$

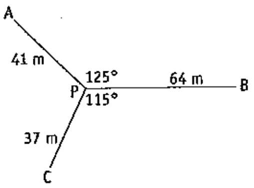
Area $= 36\,309.218\,87 \ldots \text{ m}^2$

$= 3.630\,921\,887 \ldots \text{ ha}$

$= 3.63 \text{ ha} \quad (2 \text{ d.p.})$

The area of the paddock is 3.63 hectares, correct to two decimal places.

46



a $\angle APC + 125° + 115° = 360°$ ($\angle$s at a point)

$\angle APC + 240° = 360°$

$\angle APC = 120°$

b $c^2 = a^2 + b^2 - 2ab\cos C$

Let x m be the length of AB.

$x^2 = 64^2 + 41^2 - 2 \times 64 \times 41 \times \cos 125°$

$= 8787.129\,138 \ldots$

$x = \sqrt{8787.129\,138 \ldots} \quad (x > 0)$

$= 93.739\,688\,17 \ldots$

$= 94 \quad$ (nearest unit)

The length of AB is 94 metres, to the nearest metre.

c $A = \frac{1}{2}ab\sin C$

ΔAPB: $A = \frac{1}{2} \times 64 \times 41 \times \sin 125°$

$= 1074.727\,482 \ldots$

ΔBPC: $A = \frac{1}{2} \times 37 \times 64 \times \sin 115°$

$= 1073.068\,42 \ldots$

ΔAPC: $A = \frac{1}{2} \times 41 \times 37 \times \sin 120°$

$= 656.880\,268 \ldots$

Total

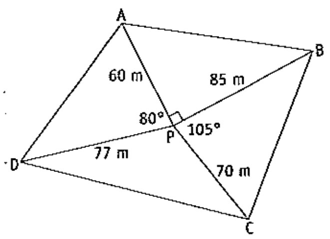
$= 1074.72 \ldots + 1073.06 \ldots + 656.88 \ldots$

$= 2804.676\,171 \ldots$

$= 2805 \quad$ (nearest unit)

The area of the block of land is 2805 m^2, to the nearest square metre.

47



a $c^2 = a^2 + b^2 - 2ab\cos C$

Let the length of BC be x m.

$x^2 = 85^2 + 70^2 - 2 \times 85 \times 70 \times \cos 105°$

$= 15\,204.946\,64 \ldots$

$x = \sqrt{15\,204.946\,64 \ldots} \quad (x > 0)$

$= 123.308\,3397 \ldots$

$= 123 \quad$ (nearest unit)

The length of BC is 123 m, to the nearest metre.

b $\angle DPC + 80° + 90° + 105° = 360°$ (angles at a point)

$\angle DPC + 275° = 360°$

$\angle DPC = 85°$

c ΔABP: $A = \frac{1}{2}bh$

$= \frac{1}{2} \times 60 \times 85$

$= 2550$

The area of ΔABP is 2550 m^2.

ΔBPC: $A = \frac{1}{2}ab\sin P$

$= \frac{1}{2} \times 85 \times 70 \times \sin 105°$

$= 2873.629\,333 \ldots$

$= 2874 \quad$ (nearest unit)

The area of ΔBPC is 2874 m^2, to the nearest square metre.

Total area

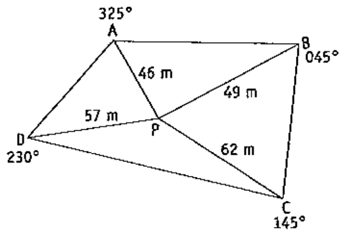
$= (2275 + 2685 + 2550 + 2874) \text{ m}^2$

$= 10\,384 \text{ m}^2$

$= 1.0384 \text{ ha}$

The area of the paddock is 1.04 hectares, to two decimal places.

48



a $\angle APB = (360 - 325)° + 45°$

$= 80°$

b $\angle BPC = 145° - 45°$

$= 100°$

$c^2 = a^2 + b^2 - 2ab\cos C$

$p^2 = 49^2 + 62^2 - 2 \times 49 \times 62 \times \cos 100°$

$= 7300.086\,327 \ldots$

$p = \sqrt{7300.086\,327 \ldots} \quad (p > 0)$

$= 85.440\,542\,65 \ldots$

$= 85 \quad$ (nearest unit)

The length of BC is 85 m, to the nearest metre.

c $\angle APD = 325° - 230°$

$= 95°$

$A = \frac{1}{2}ab\sin C$

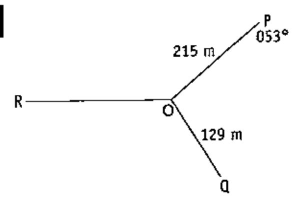
$= \frac{1}{2} \times 57 \times 46 \times \sin 95°$

$= 1306.011\,249 \ldots$

$= 1306 \quad$ (nearest unit)

The area of ΔAPD is 1306 m^2, to the nearest square metre.

49



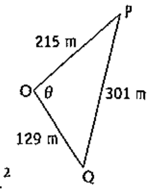
a R is due west of O.

$\therefore$ The bearing of R from O is 270°.

$\angle POR = (360 - 270)° + 53°$

$= 143°$

b $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$



$\cos \theta = \frac{215^2 + 129^2 - 301^2}{2 \times 215 \times 129}$

$= -0.5$

$\theta = 120°$

$\therefore \angle POQ = 120°$

c $\angle POQ = 120°$

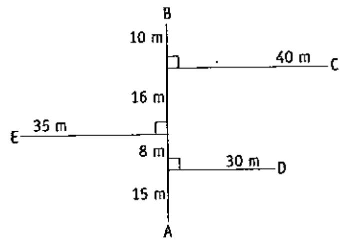
The bearing of P from O is 053°.

Bearing of Q from O $= (53 + 120)°$

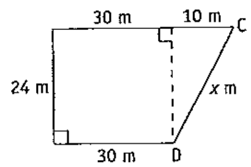
$= 173°$

$\therefore$ The bearing of Q from O is 173°.

50



a Let x m be the length of CD.



$x^2 = 24^2 + 10^2$

$= 676$

$x = \sqrt{676} \quad (x > 0)$

$= 26$

The length of CD is 26 m.

b $AB = (10 + 16 + 8 + 15)$ m
$= 49$ m

Left side: $A = \frac{1}{2}bh$
$= \frac{1}{2} \times 49 \times 35$
$= 857.5$

The area of the left side is 857.5 m^2.

Right side:

Top triangle: $A = \frac{1}{2}bh$
$= \frac{1}{2} \times 40 \times 10$
$= 200$

Trapezium: $A = \frac{1}{2}h(a+b)$
$= \frac{1}{2} \times 24 \times (40+30)$
$= 840$

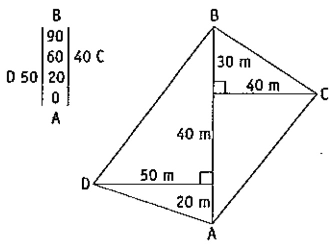
Bottom triangle: $A = \frac{1}{2}bh$
$= \frac{1}{2} \times 15 \times 30$
$= 225$

Total right side $= 200 + 840 + 225$
$= 1265$

The area of the right side is 1265 m^2.

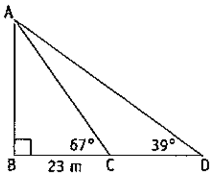
Total area of the yard
$= (857.5 + 1265)\ m^2$
$= 2122.5\ m^2$

51



Challenge p158

1



a In $\triangle ABC$,

$\tan 67° = \frac{AB}{23}$
$AB = 23 \times \tan 67°$
$= 54.184\,604\,41 \ldots$
$= 54.2$ (1 d.p.)

The length of AB is 54.2 m to one decimal place.

b In $\triangle ABD$,

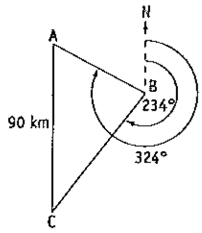
$\tan 39° = \frac{AB}{BD}$
$BD = \frac{AB}{\tan 39°}$
$= 66.912\,413\,92 \ldots$
$= 67$ (nearest unit)

The length of BD is 67 m, to the nearest metre.

$CD = 67 - 23$
$= 44$

The length of CD is 44 m, to the nearest metre.

2



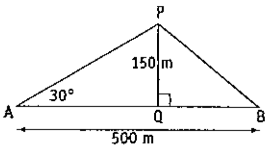
a $\angle ABC = 324° - 234°$
$= 90°$

b Let X be a point due north of B.
$\angle XBA = 360° - 324°$
$= 36°$
$\angle BAC = \angle XBA$ (alternate angles, parallel lines)
$\angle BAC = 36°$

c $\sin 36° = \frac{BC}{90}$
$BC = 90 \times \sin 36°$
$= 52.900\,672\,71 \ldots$
$= 52.9$ (1 d.p.)

The distance from B to C is 52.9 km, to one decimal place.

3



a In $\triangle PAQ$,

$\sin 30° = \frac{150}{AP}$
$AP = \frac{150}{\sin 30°}$
$= 300$

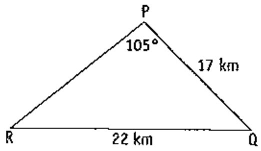
The distance from A to P is 300 metres.

b In $\triangle PAB$,

$a^2 = b^2 + p^2 - 2bp\cos A$
$= 300^2 + 500^2 - 2 \times 300 \times 500 \times \cos 30°$
$= 80\,192.378\,87 \ldots$
$a = \sqrt{80\,192.378\,87 \ldots}$ $(a > 0)$
$= 283.182\,5893 \ldots$
$= 283.2$ (1 d.p.)

The distance from P to B is 283.2 metres, to one decimal place.

4

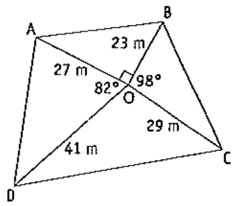


By the sine rule:

$\frac{\sin R}{r} = \frac{\sin P}{p}$
$\frac{\sin R}{17} = \frac{\sin 105°}{22}$
$\sin R = \frac{17 \sin 105°}{22}$
$= 0.746\,397\,229 \ldots$
$\angle R = 48.279\,251\,13 \ldots°$
$= 48°$ (nearest degree)

$\angle R + \angle P + \angle Q = 180°$ ($\angle$ sum $\triangle$)
$48° + 105° + \angle Q = 180°$
$153° + \angle Q = 180°$
$\angle Q = 27°$

5



$\triangle AOB$: $A = \frac{1}{2}bh$
$= \frac{1}{2} \times 23 \times 27$
$= 310.5$

$\triangle BOC$: $A = \frac{1}{2}bc\sin O$
$= \frac{1}{2} \times 29 \times 23 \times \sin 98°$
$= 330.254\,4009 \ldots$

$\triangle COD$:
$\angle COD + 98° + 90° + 82° = 360°$
$\angle COD + 270° = 360°$
$\angle COD = 90°$

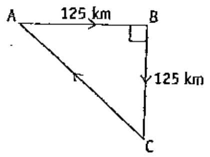
$A = \frac{1}{2}bh$
$= \frac{1}{2} \times 29 \times 41$
$= 594.5$

$\triangle AOD$:
$A = \frac{1}{2}ad\sin O$
$= \frac{1}{2} \times 41 \times 27 \times \sin 82°$
$= 548.113\,3761 \ldots$

Total $= 310.5 + 330.254 \ldots + 594.5 + 548.113 \ldots$
$= 1783.367\,777 \ldots$
$= 1783$ (nearest unit)

The area of the block of land is 1783 m^2, to the nearest square metre.

6



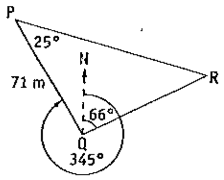
$\angle BCA = 45°$ ($\triangle ABC$ is an isosceles right-angled triangle)

The bearing of A from C $= 360° - 45°$

$= 315°$

The helicopter will need to fly on a bearing of 315°.

7



a Let X be a point due north of Q.

$\angle XQR = 66°$

$\angle PQX = 360° - 345°$

$= 15°$

$\angle PQR = 15° + 66°$

$= 81°$

b $\angle PRQ + 25° + 81° = 180°$ (angle sum $\triangle$)

$\angle PRQ = 74°$

$$\frac{q}{\sin Q} = \frac{r}{\sin R}$$

$$\frac{q}{\sin 81°} = \frac{71}{\sin 74°}$$

$$q = \frac{71 \sin 81°}{\sin 74°}$$

$= 72.951\,905\,27\ldots$

$= 73$ (nearest unit)

The distance from P to R is 73 metres, to the nearest metre.