## Further Practice: Spherical Geometry

Remember: all questions match the numbered examples on pages 163-173.



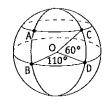
Find the arc length. Give the answer to the nearest metre.



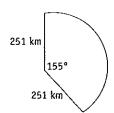
Points B and D lie on the equator. A has longitude 50°E. Give the position coordinates of:

a A В C c

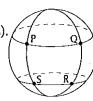
a P ь R



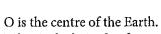
Find, to the nearest kilometre, the length of the arc of this sector.

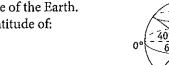


In the diagram below, Q has position (20°N, 130°W) and S is at (40°S, 80°E) Give the position of:



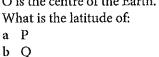
Find, to the nearest kilometre, the arc length of a circle with radius 83 km and angle subtended at the centre of 282°.

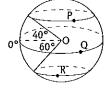




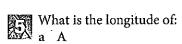
given that 1.852 km = 1 M.

Convert 80 nautical miles to kilometres,



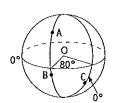


Given that 1.852 km = 1 M, how many kilometres have you travelled if you travel 500 nautical miles?



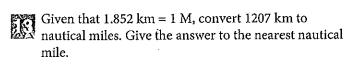
c R?

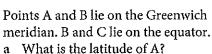
В Ь

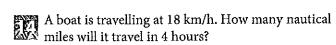


Find the number of nautical miles in 4630 km, given that 1.852 km = 1 M.

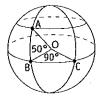
c C?



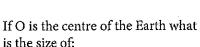




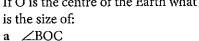
b What is the longitude of C?

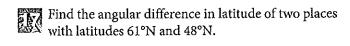


A plane is travelling at 150 km/h. Approximately how long will it take to travel 700 nautical miles?

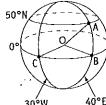


Find the angular difference in longitude of two places, one with longitude 27°E and the other 152°E.

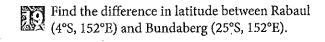








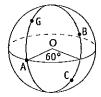
Find the angular difference in longitude between Beijing (40°N, 116°E) and Ankara (40°N, 33°E).



- Find the angular difference in latitude of two places, one with latitude 54°N and the other with latitude 19°S.
- Find the angular difference in longitude of two places, one with longitude 107°W and the other with longitude 38°E.
- Cape Town (34°S, 18°E) and Stockholm (59°N, 18°E) lie on the same meridian of longitude. What is their difference in latitude?
- Find the difference in longitude between Denver (40°N, 105°W) and Munich (48°N, 12°E).
- Find the smallest angular difference in longitude between Dili, East Timor (9°S, 126°E), and Salt Lake City, USA (41°N, 112°W).
- Two cities, lying on the same meridian of longitude, have a difference in latitude of 34°. Find the distance in kilometres between them, given that the radius of the Earth is approximately 6400 km.
- Nauru (longitude 167°E) and Nias (longitude 97°E) both lie on the equator. Given that the radius of the Earth is approximately 6400 km, find the distance between them in kilometres.
- Find the distance, in nautical miles, between Dublin (53°N, 6°W) and Seville (37°N, 6°W). (The radius of the Earth is approximately 6400 km. 1.852 km = 1 M)
- Find the distance, in nautical miles, between two points lying on the same meridian of longitude with angular difference in latitude of 70°.
- Find the distance, in nautical miles, between Tobruk (32°N, 24°E) and Athens (38°N, 24°E).
- Find the distance in nautical miles between Nagoya (35°N, 137°E) and Woomera (31°S, 137°E).
- Find the distance, in kilometres, between Djakarta (6°S, 107°E) and Ho Chi Minh City (11°N, 107°E). (1.852 km = 1 M)
- A ship is travelling at an average speed of 32 knots. How far will it travel in 6 hours?
- A boat travels 187 nautical miles in 17 hours. What is its average speed in knots?

- A yacht averaged 9 knots on a journey of 720 nautical miles. How long did the journey take?
- A helicopter flies 825 km in 3 hours. What is its average speed in knots? (1.852 km = 1 M)
- A plane is flying at an average speed of 260 knots. What is this speed in kilometres per hour? (1.852 km = 1 M)
- A plane flies directly from Buenos Aires (35°S, 58°W) to Georgetown (7°N, 58°W).
  - a Find the distance covered in kilometres. (Use the Earth's radius as 6400 km.)
  - b If the plane averaged 400 knots, how long did the journey take? (1.852 km = 1 M)
- A boat sails from Havana (23°N, 83°W) to Tampa, Florida (28°N, 83°W). Find the average speed in knots if the journey takes 2 days and 2 hours.
- If it is 10.30 am (WST) in Perth, what is the time in Brisbane (EST)?
- If it is 6 pm in Adelaide (CST) what is the standard time in Broome (WST)?
- If Melbourne is on EDST but Cairns is on EST, what time should a person in Cairns make a phone call to reach a person in Melbourne at 7.30 am?
- Perth is 8 hours ahead of GMT.
  - a What time is it in Perth if it is 6 am in London?
  - b What time will it be in London if it is 10.30 pm in Perth?
- When it is 12 noon in Greenwich it is 7 am in Santiago and 6 pm in Calcutta.
  - a If it is 3 am on April 13th in Greenwich, what is the time and date in Santiago?
  - b If it is 11 pm on 17th October in Santiago, what is the time and date in Calcutta?
- Fiji and Tahiti are both in the Pacific Ocean. Fiji lies to the west and Tahiti lies to the east of the International Date Line. If it is Friday afternoon in Tahiti, what day will it be in Fiji?
- It is 2.15 pm on 19th January on an island just to the west of the International Date Line. It is 3.45 pm on another island to the east of the 180° meridian. What is the date on the second island?

- If it is 3.45 pm in Greenwich, what is the time in Ottawa (45°N, 75°W). (Ignore time zones.)
- If it is 11.30 am on Thursday in Greenwich, what is the time and day in Mogadishu (2°N, 45°E)? (Ignore time zones.)
- If it is 6.30 am in Colombo (7°N, 80°E), what time is it in Greenwich? (Ignore time zones.)
- Given that G represents Greenwich and O is the centre of the Earth:



- a What time will it be at point A when it is 9 pm at Greenwich?
- b What time will it be at point B when it is 2.00 am at Greenwich?
- c What time will it be at Greenwich when it is 1 am at point C?
- At 9.00 pm on Tuesday, Mary in Copenhagen (56°N, 13°E), decides to phone her friend in Port Arthur (43°S, 148°E). What time and day is it at Port Arthur? (Ignore time zones.)
- If it is 5.15 am on 25th July in San Francisco (122°W), what is the time and date in Sydney (151°E)? (Ignore time zones.)
- While on business in New York (41°N, 74°W), Wasim decides to phone his partner in Karachi (22°N, 67°E). Wasim knows that the best times to reach his partner are around 9.00 am or 5.00 pm Karachi time. When should Wasim make the call? (Ignore time zones.)
- If it is 8.45 am daylight-saving time on Wednesday in Narrabri (30°S, 150°E), what is the standard time and day in St Petersburg (60°N, 30°E). (Ignore time zones.)
- At 6.00 am on Christmas Day, Bernadette in Great Falls, Montana (48°N, 111°W) decides to phone her friend on Lord Howe Island (31°S, 159°E). What time will it be on Lord Howe Island (daylight-saving time)? (Ignore time zones.)

- A plane leaves Singapore (1°N, 105°E) at 7.15 pm and flies to Brisbane (27°N, 153°E). The flight takes  $6\frac{3}{4}$  hours. What is the local time when the plane arrives? (Ignore time zones.)
- A plane is due to arrive at Auckland (longitude 175°E) at 9.30 am after an eleven-hour flight from Shanghai, which has longitude 121°E. What was the local time in Shanghai when the plane departed? (Ignore time zones.)
- A plane left Rome (42°N, 13°E) at 11.42 am Monday local time, and arrived in New York, located at (41°N, 74°W), at 7.48 pm Monday local time. How long was the flight? (Ignore time zones.)
- A plane flies from Sydney (151°E) to Los Angeles (118°W). If the flight departs at 9.45 am Saturday and takes 14 hours and 16 minutes, what time and day will it land in Los Angeles? (Ignore time zones.)

## Challenge: Spherical Geometry



A plane left Sydney (34°S, 151°E) at 11.40 am local time on January 28th and arrived at San Francisco (38°N, 122°W) at 6.43 am local time on January 28th. How long was the flight? *Hint* 1



The figure is made up of a semi-circle and a sector.



What is the perimeter of the figure? Hint 2



Charlene lives in Newcastle, which has latitude 33°S and longitude 153°E. 'The place diametrically opposite Newcastle on the Earth's surface is located at 33°N and 153°W', she said. Is Charlene correct? Justify your answer. *Hint 3* 



A boat is travelling at 20 knots. How long, in hours and minutes, will it take to travel 231.5 km? (1.852 km = 1 M) Hint 4



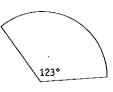
Quebec has position coordinates (47°N, 71°W) and Berne (47°N, 7°E).

- a What is the difference in longitude between Quebec and Berne?
- b Richard said: 'Using the radius of the Earth as 6400 km and the formula  $l=\frac{\theta}{360}2\pi r$ , I found that the distance between Quebec and Berne is about 8700 km.' Explain why Richard is wrong. Hint 5



The arc length of the sector is 88 m.

Find the radius. Hint 6





Frances is travelling around the world and knows that she shouldn't ring her parents in Canberra (35°S, 149°E) between 9 pm and 6 am unless there is an emergency. Between what local times might Frances ring her parents when she is in New York (41°N, 74°W)? *Hint 7* 



The diagram shows the relative position of two points A and B on the Earth's surface. O is the centre of the Earth.



If it is 8.40 pm on Saturday at point A, what is the time and day at point B? *Hint 8* 

Go to p 289 for **Quick Answers** or to pp 339–40 for **Worked Solutions** 

Hint 1: In January, Sydney is on daylight-saving time.

Hint 2: The radius of the sector is equal to the diameter of the semi-circle.

Hint 3: What is the angular difference in longitude?

Hint 4: Take care with the minutes.

Hint 5: On what circle do both cities lie?

Hint 6: Use the formula for finding the arc length and solve an equation to find r.

Hint 7: Find one of the times and the other will be a few hours later.

Hint 8: Use the angle at the centre of the Earth to find the difference in time.

# Solutions

### **Ch 9: Spherical Geometry** Further Practice . . . . p174

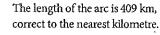
- = 7.330 382 858 ...
  - =7 (nearest unit) The length of the arc is 7 m, correct to the nearest metre.

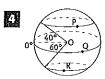
$$l = \frac{\theta}{360} 2\pi r$$

$$= \frac{155}{360} \times 2 \times \pi \times 251$$

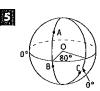
$$= 679.020 3455 ...$$
251 km

- = 679 (nearest unit) The length of the arc is 679 km, to the nearest kilometre.
- $l = \frac{\theta}{360} 2\pi r$  $= \frac{282}{360} \times 2 \times \pi \times 83$ = 408.511 7647 ... = 409 (nearest unit)

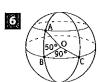




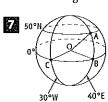
- [It lies on the equator.]
- c 60°S



- a 80°W
- c 0° [It lies on the prime meridian.]



- a The latitude of A is 50°N.
- **b** The longitude of C is 90°E.



- a  $\angle BOC = 70^{\circ}$  [30° + 40°]
- **b**  $\angle$ AOB = 50°



- a A is at (60°N, 50°E)
- **b** B is at (0°, 50°E).
- c C is at (60°N, 160°E)



- a P(20°N, 80°E)
- b R(40°S, 130°W)

80 M = 
$$80 \times 1.852$$
 km  
= 148.16 km

$$500 M = 500 \times 1.852 \text{ km}$$
$$= 926 \text{ km}$$

1207 km = 
$$(1207 \div 1.852)$$
 M  
=  $651.727 8618 \dots M$   
=  $652$  M (nearest M)

The boat will travel approximately 39 nautical miles in that time.

Angular difference = 
$$61^{\circ}$$
 -  $48^{\circ}$   
=  $13^{\circ}$   
The angular difference in latitude is  $13^{\circ}$ .

Angular difference = 
$$25^{\circ} - 4^{\circ}$$
  
=  $21^{\circ}$ 

Angular difference = 
$$54^{\circ} + 19^{\circ}$$
  
=  $73^{\circ}$   
The difference in latitude is  $73^{\circ}$ .

- Angular difference =  $107^{\circ} + 38^{\circ}$ =  $145^{\circ}$ The angular difference is  $145^{\circ}$ .
- Angular difference =  $34^{\circ} + 59^{\circ}$ =  $93^{\circ}$ The difference in latitude is  $93^{\circ}$ .
- Angular difference =  $105^{\circ} + 12^{\circ}$ =  $117^{\circ}$ The difference in longitude is  $117^{\circ}$ .
- Angular difference = 360° - (126° + 112°) = 122°
- 25  $l = \frac{\theta}{360} 2\pi r$ =  $\frac{34}{360} \times 2 \times \pi \times 6400$ = 3797.836 452 ...

The distance between the cities is approximately 3800 kilometres.

Angular difference =  $167^{\circ} - 97^{\circ}$ =  $70^{\circ}$ 

$$l = \frac{\theta}{360} 2\pi r$$

$$= \frac{70}{360} \times 2 \times \pi \times 6400$$

$$= 7819.075 049 ...$$

$$\approx 7820$$

The distance between the localities is approximately 7820 kilometres.

Angular difference = 53° - 37°

$$l = \frac{\theta}{360} 2\pi r$$

$$= \frac{16}{360} \times 2 \times \pi \times 6400$$

$$= 1787.217 154 ...$$

$$1787.217 ... \text{ km}$$

$$= (1787.217 ... \div 1.852) \text{ M}$$

$$= 965.020 0616 ... \text{ M}$$

$$\approx 965 \text{ M}$$

The distance between the localities is approximately 965 nautical miles.

- Distance =  $70 \times 60 \text{ M}$ = 4200 M. The distance between the points is 4200 nautical miles.
- Angular difference =  $38^{\circ} 32^{\circ}$ =  $6^{\circ}$ Distance =  $6 \times 60 \text{ M}$ = 360 MThe distance between the cities is 360 nautical miles.

Angular difference =  $35^{\circ} + 31^{\circ}$ =  $66^{\circ}$ Distance =  $66 \times 60 \text{ M}$ = 3960 MThe distance between the cities is

3960 nautical miles.

Angular difference =  $6^{\circ} + 11^{\circ}$ =  $17^{\circ}$ Distance =  $17 \times 60 \text{ M}$ = 1020 M=  $1020 \times 1.852 \text{ km}$ = 1889.04 km $\approx 1890 \text{ km}$ 

The distance between the two cities is approximately 1890 kilometres.

- Distance = (32 × 6) M = 192 M The ship will travel 192 nautical miles in that time.
- Speed = 187 M ÷ 17 h = 11 nautical miles per hour The average speed is 11 knots.
- Time = (720 ÷ 9) hours = 80 hours = 3 days and 8 hours
- Speed = (825 ÷ 3) km/h = 275 km/h = (275 ÷ 1.852) M per hour = 148.488 121 ... knots The average speed is approx. 148 knots.

36 Speed = 260 knots = 260 M per hour

 $= (260 \times 1.852) \text{ km per hour}$ = 481.52 km/hThe average speed is approx. 480 km/h.

a Angular difference = 35° + 7°

$$l = \frac{\theta}{360} 2\pi r$$

$$= \frac{42}{360} \times 2 \times \pi \times 6400$$

$$= 4691.445.029 ...$$

$$\approx 4690$$

The distance covered was approx. 4690 kilometres.

b 400 knots = (400 × 1.852) km/h = 740.8 km/h Time = distance ÷ speed = 4690 ÷ 740.8 = 6.330 993 521 ...

The journey took approximately 6.33 hours, or approximately 6 hours and 20 minutes.

- Angular difference = 28° 23° = 5° Distance = 5 × 60 M = 300 M Time = 2 days and 2 hours = 50 hours Speed = distance ÷ time = 300 M ÷ 50 hours = 6 knots The average speed of the boat was
- Perth is two hours behind EST.
  The time in Brisbane is 12.30 pm.

6 knots.

- CST is  $1\frac{1}{2}$  hours ahead of WST. Time in Broome is 4.30 pm.
- When it is 7.30 am in Melbourne it will be one hour earlier in Queensland. The call should be made at 6.30 am.
- a The time will be 8 hours ahead in Perth. It will be 2 pm.
  b It will be 8 hours before 10.30 pm.
  The time in London will be 2.30 pm.
- a Santiago is 5 hours behind
  Greenwich. It will be 10 pm on 12th April.
  b Calcutta is 11 hours ahead of Santiago. It will be 10 am on the
- It will be Saturday in Fiji.

18th of October.

- 18th January
- Angular difference = 75° 0° = 75° Time difference = (75 ÷ 15) hours = 5 hours Ottawa is 5 hours behind Greenwich. Time in Ottawa is 10.45 am.
- Angular difference = 45° 0° = 45° Time difference = (45 ÷ 15) hours = 3 hours Mogadishu is 3 hours ahead of Greenwich. Time in Mogadishu is 2.30 pm on Thursday.
- Angular difference = 80° 0° = 80° Time difference = 80 × 4 minutes = 320 minutes = 5 h 20 min Greenwich is 5 hours and 20 minutes

behind Colombo.
Time in Greenwich is 1.10 am.



a 9 pm

b Angular difference =  $60^{\circ}$ Time difference =  $(60 \div 15)$  hours = 4 hours

Point B is east of Greenwich so the time will be 4 hours ahead of Greenwich.

Time at B = 6 am

Time difference = 4 hours
 Greenwich is 4 hours behind point C.
 Time in Greenwich = 9 pm the
 previous day.

Angular difference = 148° - 13° = 135°

Time difference =  $(135 \div 15)$  hours = 9 hours

Port Arthur is 9 hours ahead of Copenhagen. Time in Port Arthur is 6 am on Wednesday.

Angular difference = 122° + 151°

= 273°

Time difference =  $(273 \div 15)$  hours

= 18.2 hours

= 18 h 12 min

Sydney is 18 hours and 12 minutes ahead of San Francisco.
Time in Sydney is 11.27 pm on 25th July.

Angular difference = 74° + 67° = 141°

Time difference =  $(141 \div 15)$  hours

= 9.4 hours

= 9 h 24 min

New York is 9 hours and 24 minutes behind Karachi.

When it is 9.00 am in Karachi, it is 11.36 pm in New York.

When it is 5.00 pm in Karachi, it is 7.36 am in New York.

Wasim should call at around 11.36 pm or 7.36 am.

Angular difference = 150° - 30° = 120°

Time difference =  $(120 \div 15)$  hours = 8 hours

On standard time, St Petersburg would be 8 hours behind Narrabri. On daylight-saving time, St Petersburg will be 9 hours behind.

Time in St Petersburg is 11.45 pm Tuesday. Angular difference = 111° + 159° = 270°

Time difference =  $(270 \div 15)$  hours = 18 hours

Lord Howe Island is normally 18 hours ahead of Great Falls. With daylightsaving, Lord Howe Island will be 19 hours ahead.

Time on Lord Howe Island is 1 am on Boxing Day.

Angular difference = 153° – 105° = 48°

Time difference =  $48 \times 4$  minutes

= 192 minutes

= 3 h 12 min

Brisbane is 3 hours and 12 minutes ahead of Singapore.

The plane leaves at 10.27 pm Brisbane time.

It arrives  $6\frac{3}{4}$  hours later. The plane arrives at 5.12 am the next day.

Angular difference = 175° - 121 ° = 54°

Time difference =  $(54 \div 15)$  hours

= 3.6 hours = 3 h 36 min

Shanghai is 3 hours and 36 minutes behind Auckland.

The time in Shanghai when the plane is due to arrive in Auckland is 5.54 am. The plane left 11 hours earlier. It left at 6.54 pm the previous day.

Angular difference = 13° + 74°

= 87°

Time difference =  $(87 \div 15)$  hours

= 5.8 hours

= 5 h 48 min

New York is 5 hours and 48 minutes behind Rome.

The plane left Rome at 5.54 am Monday, New York time.

The flight took 13 hours and 54 minutes.

Angular difference = 151° + 118°

=  $269^{\circ}$ Time difference =  $(269 \div 15)$  hours

= 17.9333 ... hours = 17 h 56 min

Los Angeles is 17 hours and 56 minutes behind Sydney.

Time in Los Angeles when the plane departs is 3.49 pm Friday.

The plane will land 14 h 16 min later. The plane lands at 6.05 am Saturday. Challenge ..... p177

Angular difference = 151° + 122°

= 273°

Time difference =  $(273 \div 15)$  hours = 18.2 hours

= 18 h 12 min

Sydney is 18 h 12 mins ahead of San Francisco on standard time. It will be 19 h 12 min ahead on daylight

saving time.
11.40 am in Sydney on 28th January
= 4.28 pm on 27th January in

 $\begin{array}{c} \text{San Francisco} \\ \text{Time of flight} = 14 \text{ h } 15 \text{ min} \end{array}$ 

2 Arc length of sector:

$$l = \frac{\theta}{360} 2\pi r$$
35

 $=\frac{35}{360}\times2\times\pi\times42$ 

= 25.656 3400 ...

Semi-circle:

diameter = 42 cm

radius = 21 cm

 $C = \pi r$  [half of  $2\pi r$ ]

 $=\pi\times21$ 

= 65.973 445 73 ...

P = 65.973 ... + 25.656 ... + 42

= 133.629 7857 ...

= 133.6 (1 d.p.)

The perimeter of the figure is 133.6 cm, to one decimal place.

No, Charlene is not correct.

The difference in longitude between

153°E and 153°W is 360° - (153 + 153)° = 54°

The place opposite Newcastle must have longitude differing by 180°. It is 27°W.

231.5 km =  $(231.5 \pm 1.852)$  M = 125 M

Time = distance + speed

= (125 ÷ 20) hours

= 6.25 hours

= 6 hours and 15 minutes

Quebec (47°N, 71°W), Berne (47°N, 7°E)

a Difference in longitude = 71° + 7°

= 78°

b Richard is wrong because Quebec and Berne lie on the same small circle not a great circle.

The distance between Quebec and Berne would be less than 8700 km, because the radius of the circle on which they both lie is less than 6400 km.



$$I = 88$$

$$l=\frac{\theta}{360}2\pi r$$

$$88 = \frac{123}{360} \times 2 \times \pi \times$$

$$88 = 2.14675498... \times r$$

$$r = 88 \div 2.14675498...$$

= 41 (nearest unit)

The radius of the sector is 41 m, to the nearest metre.

Angular difference = 149° + 74°

$$=223^{\circ}$$

Time difference =  $(223 \div 15)$  hours

New York is 14 hours and 52 minutes behind Canberra.

6 am Canberra time is 3.08 pm New York time.

Frances should ring in the 15 hours after 3.08 pm New York time. She should ring after 3.08 pm and before 6.08 am New York time.

Angular difference = 75°
Time difference

$$= (75 \div 15)$$
 hours

B is further east of A.

B will be five hours ahead of A.

It will be 1.40 am on Sunday.