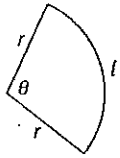


# UNIT: SPHERICAL GEOMETRY

## Arc length of a circle

The circumference of a circle is given by the formula  $C = 2\pi r$  where  $r$  is the radius.

A **sector** is part of a circle.

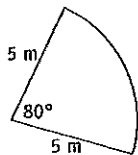


It is bounded by two radii and an arc. If  $\theta$  is the angle at the centre of the circle, then the Length of the arc of the sector is given by

$$l = \frac{\theta}{360} \times 2\pi r$$

### Example 1:

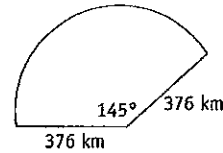
Find the arc length of the diagram below. Give the answer to the nearest metre.



ANS: 7m

### Example 2:

Find, to the nearest kilometre, the length of the arc of this sector.



ANS: 952km

### Example 3:

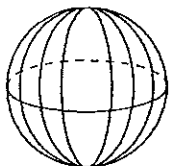
Find, to the nearest kilometre, the arc length of a circle with radius  $92 \text{ km}$  and angle subtended at the centre of  $234^\circ$

ANS: 376km

## Great and small circles

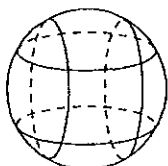
A circle drawn on the surface of a sphere can be either a great or a small circle.

A **great circle** is a circle whose **radius is the radius of the sphere**. The centre of the circle is the centre of the sphere.



great circles

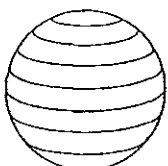
A **small circle** has a radius that is **smaller than the radius of the sphere**. The centre of a small circle is not the centre of the sphere.



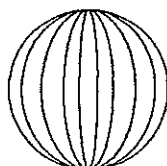
small circles

To identify any point on the Earth's surface, we imagine lines are drawn. These lines are **parallels of latitude** and **meridians of Longitude**.

The **parallels of Latitude** are **parallel to the equator**. The equator is a great circle. All other parallels of latitude are small circles.



parallels of latitude

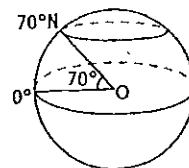


meridians of longitude

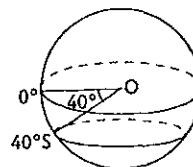
The **meridians of Longitude** all pass through the **north and south poles**. All meridians of Longitude are great circles.

**Latitude** is measured in degrees **north** and **south** of the **equator**, which is at  $0^\circ$ . The degree of latitude depends on the angle at the centre of the Earth.

For example, in the diagram below, every point on the upper circle is  $70^\circ$  north of the equator so the parallel of latitude is  $70^\circ$  N.

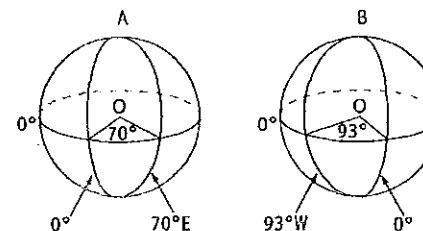


In the diagram below, every point on the lower circle is  $40^\circ$  south of the equator, so that parallel of latitude is  $40^\circ$  S.



**Longitude** is measured in degrees **east** and **west** of the **prime meridian**, which is at  $0^\circ$ . This meridian passes through **Greenwich** in London and is also known as the **Greenwich meridian**.

For example, in diagram A below, every point on the right meridian is  $70^\circ$  east of the prime meridian so the meridian is  $70^\circ$  E. In diagram B, every point on the left meridian is  $93^\circ$  west of the prime meridian so the meridian is  $93^\circ$  W.



## Giving Locations on the Earth's surface

A point on the globe is always given in brackets with the **LATITUDE (N or S)** first, followed by the **LONGITUDE (E or W)**.

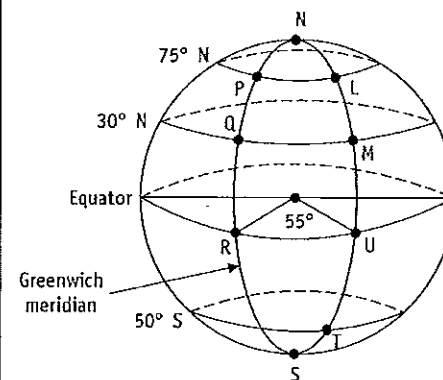
### Example 4:

Plot on a globe the position of Tokyo and Sydney if Tokyo is located  $(35^{\circ}N, 140^{\circ}E)$  and Sydney is  $(34^{\circ}S, 150^{\circ}E)$

### Example 5:

On the Earth's sphere given below, give the location of the points using latitude and longitude.

- (a) P
- (b) Q
- (c) T
- (d) L
- (e) Which point is at  $(30^{\circ}N, 55^{\circ}E)$  ?
- (f) Which point is at  $(0^{\circ}, 55^{\circ}E)$  ?



## Nautical miles

A nautical mile is a unit of length. It is generally used for measuring distances in air and sea navigation.

The symbol for a nautical mile is M.

A nautical mile is defined to be 1852 metres. Nautical miles can be converted to kilometres and vice versa using the conversion rate:

$$1 \text{ M} = 1.852 \text{ km}$$

- \* To change nautical miles into kilometres, MULTIPLY by 1.852
- \* To change kilometres into nautical miles, DIVIDE by 1.852

### Example 6:

Convert 70 nautical miles to kilometres.

ANS: 129.64 km

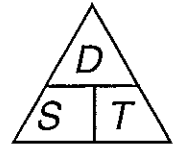
### Example 7:

Given that  $1.852 \text{ km} = 1 \text{ M}$ , convert 215 km to nautical miles. Give the answer to the nearest nautical mile.

ANS: 116M

### Example 8:

A boat is travelling at 15km/h. How many nautical miles will it travel in 3 hours?



ANS: 24 M

### Example 9:

A ship is travelling at 50 km/h. How long will it take to travel 350 nautical miles?

ANS: 13 hours

## Angular difference

The **angular difference** in either latitude or longitude can easily be found. It is the overall angle difference between the two points.

### Example 10:

Find the angular difference in longitude of two places, one with longitude  $19^{\circ}W$  and the other  $171^{\circ}W$

ANS:  $152^{\circ}$

### Example 11:

Find the angular difference in longitude between Sydney ( $34^{\circ}S, 151^{\circ}E$ ) and Cape Town ( $34^{\circ}S, 18^{\circ}E$ )

ANS:  $133^{\circ}$

### Example 12:

Lima ( $12^{\circ}S, 77^{\circ}W$ ) and Washington ( $38^{\circ}N, 77^{\circ}W$ ) lie on the same meridian of longitude. What is their difference in latitude?

ANS:  $50^{\circ}$

### Example 13:

Find the smallest angular difference in longitude between Wellington, New Zealand ( $41^{\circ}S, 175^{\circ}E$ ) and Edmonton Canada ( $54^{\circ}N, 113^{\circ}W$ ).

ANS:  $72^{\circ}$

## Distance between two points

We can easily find the distance between two points on the same great circle. One way to do this is to use the formula for the arc length of a circle,  $l = \frac{\theta}{360} \times 2\pi r$ , and the fact that the radius of the Earth is approximately 6400 km.

### Example 14:

Two cities, lying on the same meridian of longitude, have a difference in latitude of  $56^\circ$ . Find the distance in kilometres between them, given that the radius of the Earth is approximately 6400 km.

ANS: 6255 km

### Example 15:

Find the distance, in nautical miles, between Honolulu ( $22^\circ$  N,  $158^\circ$  W) and Rarotonga ( $20^\circ$  S,  $158^\circ$  W). (The radius of the Earth is approximately 6400 km;  $1.852 \text{ km} = 1 \text{ M}$ )

ANS: 2530 M

## $1^\circ = 60 \text{ M}$

There is a second method that can be used to find the distance between two points on the same great circle.

One degree of difference in latitude is equivalent to 60 nautical miles.

### Example 16:

Find the distance, in nautical miles, between Cairo ( $30^\circ$  N,  $31^\circ$  E) and Durban ( $30^\circ$  S,  $31^\circ$  E).

ANS: 3600 M

### Example 17:

Find the distance, in kilometres, between Boston ( $42^\circ$  N  $71^\circ$  W) and Santiago ( $34^\circ$  S,  $71^\circ$  W).

ANS: 8445 km

## Knots

A knot is a measure of speed.

One knot is one nautical mile per hour.

### Example 18:

A ship is travelling at an average speed of 14 knots. How far will it travel in 6 hours?

ANS: 84 M

### Example 19:

A boat travels 91 nautical miles in 13 hours. What is its average speed in knots?

ANS: 7 knots

### Example 20:

A yacht sails 1568 nautical miles at an average speed of 8 knots. How long did the journey take?

ANS: 8 days 4 hours

### Example 21:

A helicopter flies 360 km in 2 hours. What is its average speed in knots?

ANS: 97 knots

### Example 22:

A plane flies directly from Prague ( $50^\circ \text{ N}$ ,  $14^\circ \text{ E}$ ) to Malta ( $36^\circ \text{ N}$ ,  $14^\circ \text{ E}$ ).

(a) Find the distance covered in kilometres. (The radius of the Earth is approx 6400 km)

(b) If the plane averaged 180 knots, how long did the journey take? (1.852 km = 1 M)

ANS: (a) 1560 km (b) 4.7 hours

### Example 23:

A boat sailed from the Maldives ( $7^\circ \text{ N}$ ,  $73^\circ \text{ E}$ ) to Mumbai ( $19^\circ \text{ N}$ ,  $73^\circ \text{ E}$ ). Find the average speed in knots if the journey took 3 days and 18 hours.

ANS: 8 knots

## Time zones

The world is divided into time zones, so that a full revolution of the Earth comprises 24 hours. [This roughly matches the path of the sun as the Earth moves about it.]

Governments have control of the time zones for their nations, so that, for example, a whole country might fall into the same time zone.

Australia is divided into **three** time zones. In **standard** time, Queensland, NSW, A.C.T., Victoria and Tasmania all share the same time known as **Eastern Standard Time (EST)**.

The Northern Territory and South Australia are on **Central Standard Time (CST)** and Western Australia is on **Western Standard Time (WST)**.

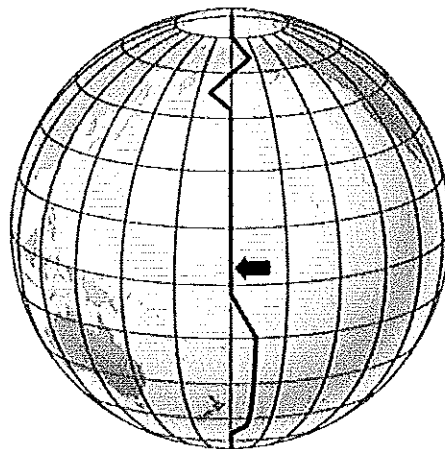
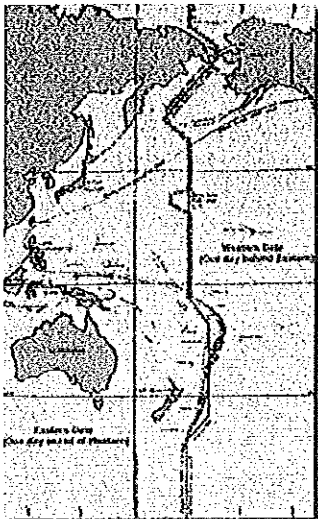
**CST** is half an hour behind **EST** and **WST** is two hours behind EST. For example, if it is 6.00 p.m. in NSW, it will be 5.30 p.m. in South Australia and 4.00 p.m. in Western Australia.

## Greenwich Mean Time

**Greenwich Mean Time (GMT)** is the time in Greenwich, London. The time is determined by the Greenwich meridian. All other times around the world can be given in terms of GMT. For example, if it is 12 noon in Greenwich, then in a place where the time is 5 hours ahead of GMT it will be 5 p.m. and in a place that is 3 hours behind GMT it will be 9 am.

## The International Date line

The International Date Line is an **imaginary** line, which roughly follows the 180° meridian. This meridian runs mainly through the Pacific Ocean. The date line **bends** around a few land masses and islands so that it always runs over water.



## Determining time from longitude

The difference in time between any two locations can be determined by considering the longitude of each location. This method ignores time zones.

Because it takes a day for the Earth to move through one rotation, we have the basis of the conversion between Longitude and time.

Every 1° in longitude is 4 minutes in time.

ie  $1^\circ = 4 \text{ minutes}$

As we move to the **RIGHT**, we are moving **FORWARD** in time.

As we move to the **LEFT**, we are moving **BACK** in time.

### Example 23:

If it is 7.15 am in Greenwich, what is the time in Manila (15° N, 120° E)?

ANS: 3.15 p.m

### Example 24:

If it is 4.30 am on Tuesday in Greenwich, what is the time and day in New Orleans (30° N, 90° W)?

ANS: 10.30 p.m on Monday



**Example 25:**

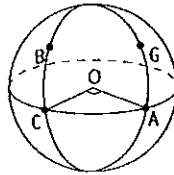
If it is 2.30 p.m. in Pretoria ( $26^\circ \text{ S}$ ,  $28^\circ \text{ E}$ ), what time is it in Greenwich?

ANS: 12.38 p.m

**Example 26:**

Given that G represents Greenwich and O is the centre of the Earth:

- (a) What time will it be at point A when it is 5 am at Greenwich?
- (b) What time will it be at point B when it is 2.00 p.m. at Greenwich?
- (c) What time will it be at Greenwich when it is noon at point C?



ANS: (a) 5 am (b) 8 am (c) 6 pm

**Example 27:**

At 8.00 am on Thursday, Kelly in Auckland ( $37^\circ \text{ S}$ ,  $175^\circ \text{ E}$ ), decides to phone her friend in Helsinki ( $60^\circ \text{ N}$ ,  $25^\circ \text{ E}$ ). What time and day is it in Helsinki? (Ignore time zones.)

ANS: 10.00 p.m on Wednesday

**Example 28:**

If it is 11.15 am on 27th February in Minneapolis ( $93^\circ \text{ W}$ ), what is the time and date in Mildura ( $142^\circ \text{ E}$ )? (Ignore time zones.)

ANS: 2.55 am on 28th February

**Example 29:**

While on holiday in Bali ( $8^{\circ}$  S,  $115^{\circ}$  E), Jake decides to phone his parents in Istanbul ( $41^{\circ}$  N,  $29^{\circ}$  E). Jake knows that the best times to reach his parents are around 8.30 am or 6.15 p.m. Istanbul time. When should Jake make the call? (Ignore time zones.)

ANS: (a) 2.14 p.m. or midnight

**Example 30:**

A plane leaves Seattle ( $48^{\circ}$  N,  $122^{\circ}$  W) at 11 am and flies to Mexico City ( $19^{\circ}$  N,  $99^{\circ}$  W). The time of flight is 5 hours. What is the local time when the plane arrives? (Ignore time zones.)

ANS: (a) 5.32 p.m.

**Example 31:**

A plane left Perth ( $32^{\circ}$  S,  $116^{\circ}$  E) at 11.50 pm Tuesday local time and arrived in Johannesburg, located at ( $26^{\circ}$  S,  $28^{\circ}$  E), at 5.35 am Wednesday local time. How long was the flight?

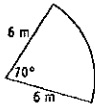
**Example 32:**

A plane flies from Brisbane ( $153^{\circ}$  E) to Honolulu ( $158^{\circ}$  W). If the flight departs at 4.15 p.m. Thursday and takes 11 hours and 19 minutes, what time and day will it land in Honolulu?

ANS: (a) 6.50 a.m. Thursday

## FURTHER PRACTICE

- 1 Find the arc length. Give the answer to the nearest metre.

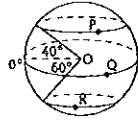


- 2 Find, to the nearest kilometre, the length of the arc of this sector.

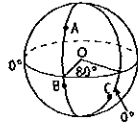


- 3 Find, to the nearest kilometre, the arc length of a circle with radius 83 km and angle subtended at the centre of  $282^\circ$ .

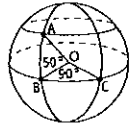
- 4 O is the centre of the Earth.  
What is the latitude of:  
a P  
b Q  
c R?



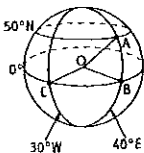
- 5 What is the longitude of:  
a A  
b B  
c C?



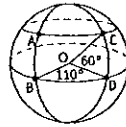
- 6 Points A and B lie on the Greenwich meridian. B and C lie on the equator.  
a What is the latitude of A?  
b What is the longitude of C?



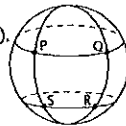
- 7 If O is the centre of the Earth what is the size of:  
a  $\angle BOC$   
b  $\angle AOB$



- 8 Points B and D lie on the equator. A has longitude  $50^\circ\text{E}$ . Give the position coordinates of:  
a A  
b B  
c C



- 9 In the diagram below, Q has position  $(20^\circ\text{N}, 130^\circ\text{W})$  and S is at  $(40^\circ\text{S}, 80^\circ\text{E})$ . Give the position of:  
a P  
b R



- 10 Convert 80 nautical miles to kilometres, given that  $1.852\text{ km} = 1\text{ M}$ .

- 11 Given that  $1.852\text{ km} = 1\text{ M}$ , how many kilometres have you travelled if you travel 500 nautical miles?

- 12 Find the number of nautical miles in 4630 km, given that  $1.852\text{ km} = 1\text{ M}$ .

- 13 Given that  $1.852\text{ km} = 1\text{ M}$ , convert 1207 km to nautical miles. Give the answer to the nearest nautical mile.

- 14 A boat is travelling at 18 km/h. How many nautical miles will it travel in 4 hours?

- 15 A plane is travelling at 150 km/h. Approximately how long will it take to travel 700 nautical miles?

- 16 Find the angular difference in longitude of two places, one with longitude  $27^\circ\text{E}$  and the other  $152^\circ\text{E}$ .

- 17 Find the angular difference in latitude of two places with latitudes  $61^\circ\text{N}$  and  $48^\circ\text{N}$ .

- 18 Find the angular difference in longitude between Beijing  $(40^\circ\text{N}, 116^\circ\text{E})$  and Ankara  $(40^\circ\text{N}, 33^\circ\text{E})$ .

- 19 Find the difference in latitude between Rabaul  $(4^\circ\text{S}, 152^\circ\text{E})$  and Bundaberg  $(25^\circ\text{S}, 152^\circ\text{E})$ .

- 20 Find the angular difference in latitude of two places, one with latitude  $54^\circ\text{N}$  and the other with latitude  $19^\circ\text{S}$ .

- 21 Find the angular difference in longitude of two places, one with longitude  $107^\circ\text{W}$  and the other with longitude  $38^\circ\text{E}$ .

- 22 Cape Town  $(34^\circ\text{S}, 18^\circ\text{E})$  and Stockholm  $(59^\circ\text{N}, 18^\circ\text{E})$  lie on the same meridian of longitude. What is their difference in latitude?

- 23 Find the difference in longitude between Denver  $(40^\circ\text{N}, 105^\circ\text{W})$  and Munich  $(48^\circ\text{N}, 12^\circ\text{E})$ .

- 24 Find the smallest angular difference in longitude between Dili, East Timor  $(9^\circ\text{S}, 126^\circ\text{E})$ , and Salt Lake City, USA  $(41^\circ\text{N}, 112^\circ\text{W})$ .

- 25 Two cities, lying on the same meridian of longitude, have a difference in latitude of  $34^\circ$ . Find the distance in kilometres between them, given that the radius of the Earth is approximately 6400 km.

- 26 Nauru (longitude  $167^\circ\text{E}$ ) and Nias (longitude  $97^\circ\text{E}$ ) both lie on the equator. Given that the radius of the Earth is approximately 6400 km, find the distance between them in kilometres.

- 27 Find the distance, in nautical miles, between Dublin  $(53^\circ\text{N}, 6^\circ\text{W})$  and Seville  $(37^\circ\text{N}, 6^\circ\text{W})$ . (The radius of the Earth is approximately 6400 km.  $1.852\text{ km} = 1\text{ M}$ )

- 28 Find the distance, in nautical miles, between two points lying on the same meridian of longitude with angular difference in latitude of  $70^\circ$ .

- 29 Find the distance, in nautical miles, between Tobruk  $(32^\circ\text{N}, 24^\circ\text{E})$  and Athens  $(38^\circ\text{N}, 24^\circ\text{E})$ .

- 30 Find the distance in nautical miles between Nagoya  $(35^\circ\text{N}, 137^\circ\text{E})$  and Woomera  $(31^\circ\text{S}, 137^\circ\text{E})$ .

- 31 Find the distance, in kilometres, between Djakarta  $(6^\circ\text{S}, 107^\circ\text{E})$  and Ho Chi Minh City  $(11^\circ\text{N}, 107^\circ\text{E})$ . ( $1.852\text{ km} = 1\text{ M}$ )

- 32 A ship is travelling at an average speed of 32 knots. How far will it travel in 6 hours?

- 33 A boat travels 187 nautical miles in 17 hours. What is its average speed in knots?

- 34 A yacht averaged 9 knots on a journey of 720 nautical miles. How long did the journey take?

- 35 A helicopter flies 825 km in 3 hours. What is its average speed in knots? ( $1.852\text{ km} = 1\text{ M}$ )

- 36 A plane is flying at an average speed of 260 knots. What is this speed in kilometres per hour? ( $1.852\text{ km} = 1\text{ M}$ )

- 37 A plane flies directly from Buenos Aires  $(35^\circ\text{S}, 58^\circ\text{W})$  to Georgetown  $(7^\circ\text{N}, 58^\circ\text{W})$ .  
a Find the distance covered in kilometres. (Use the Earth's radius as 6400 km.)  
b If the plane averaged 400 knots, how long did the journey take? ( $1.852\text{ km} = 1\text{ M}$ )

- 38 A boat sails from Havana  $(23^\circ\text{N}, 83^\circ\text{W})$  to Tampa, Florida  $(28^\circ\text{N}, 83^\circ\text{W})$ . Find the average speed in knots if the journey takes 2 days and 2 hours.

- 39 If it is 10.30 am (WST) in Perth, what is the time in Brisbane (EST)?

- 40 If it is 6 pm in Adelaide (CST) what is the standard time in Broome (WST)?

- 41 If Melbourne is on EDTST but Cairns is on EST, what time should a person in Cairns make a phone call to reach a person in Melbourne at 7.30 am?

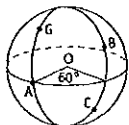
- 42 Perth is 8 hours ahead of GMT.  
a What time is it in Perth if it is 6 am in London?  
b What time will it be in London if it is 10.30 pm in Perth?

- 43 When it is 12 noon in Greenwich it is 7 am in Santiago and 6 pm in Calcutta.  
a If it is 3 am on April 13th in Greenwich, what is the time and date in Santiago?  
b If it is 11 pm on 17th October in Santiago, what is the time and date in Calcutta?

- 44 Fiji and Tahiti are both in the Pacific Ocean. Fiji lies to the west and Tahiti lies to the east of the International Date Line. If it is Friday afternoon in Tahiti, what day will it be in Fiji?

- 45 It is 2.15 pm on 19th January on an island just to the west of the International Date Line. It is 3.45 pm on another island to the east of the  $180^\circ$  meridian. What is the date on the second island?

- 46** If it is 3.45 pm in Greenwich, what is the time in Ottawa (45°N, 75°W). (Ignore time zones.)
- 47** If it is 11.30 am on Thursday in Greenwich, what is the time and day in Mogadishu (2°N, 45°E)? (Ignore time zones.)
- 48** If it is 6.30 am in Colombo (7°N, 80°E), what time is it in Greenwich? (Ignore time zones.)
- 49** Given that G represents Greenwich and O is the centre of the Earth:



- a What time will it be at point A when it is 9 pm at Greenwich?
- b What time will it be at point B when it is 2.00 am at Greenwich?
- c What time will it be at Greenwich when it is 1 am at point C?
- 50** At 9.00 pm on Tuesday, Mary in Copenhagen (56°N, 13°E), decides to phone her friend in Port Arthur (43°S, 148°E). What time and day is it at Port Arthur? (Ignore time zones.)
- 51** If it is 5.15 am on 25th July in San Francisco (122°W), what is the time and date in Sydney (151°E)? (Ignore time zones.)
- 52** While on business in New York (41°N, 74°W), Wasim decides to phone his partner in Karachi (22°N, 67°E). Wasim knows that the best times to reach his partner are around 9.00 am or 5.00 pm Karachi time. When should Wasim make the call? (Ignore time zones.)
- 53** If it is 8.45 am daylight-saving time on Wednesday in Narrabri (30°S, 150°E), what is the standard time and day in St Petersburg (60°N, 30°E). (Ignore time zones.)
- 54** At 6.00 am on Christmas Day, Bernadette in Great Falls, Montana (48°N, 111°W) decides to phone her friend on Lord Howe Island (31°S, 159°E). What time will it be on Lord Howe Island (daylight-saving time)? (Ignore time zones.)

- 55** A plane leaves Singapore (1°N, 105°E) at 7.15 pm and flies to Brisbane (27°N, 153°E). The flight takes  $6\frac{3}{4}$  hours. What is the local time when the plane arrives? (Ignore time zones.)
- 56** A plane is due to arrive at Auckland (longitude 175°E) at 9.30 am after an eleven-hour flight from Shanghai, which has longitude 121°E. What was the local time in Shanghai when the plane departed? (Ignore time zones.)
- 57** A plane left Rome (42°N, 13°E) at 11.42 am Monday local time, and arrived in New York, located at (41°N, 74°W), at 7.48 pm Monday local time. How long was the flight? (Ignore time zones.)
- 58** A plane flies from Sydney (151°E) to Los Angeles (118°W). If the flight departs at 9.45 am Saturday and takes 14 hours and 16 minutes, what time and day will it land in Los Angeles? (Ignore time zones.)

## ANSWERS

**1** 
$$l = \frac{\theta}{360} 2\pi r$$

$$= \frac{70}{360} \times 2 \times \pi \times 6$$

$$= 7.330\ 382\ 858 \dots$$

$$= 7 \text{ (nearest unit)}$$
The length of the arc is 7 m, correct to the nearest metre.

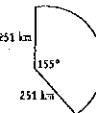


**2** 
$$l = \frac{\theta}{360} 2\pi r$$

$$= \frac{155}{360} \times 2 \times \pi \times 251$$

$$= 679.020\ 3455 \dots$$

$$= 679 \text{ (nearest unit)}$$
The length of the arc is 679 km, to the nearest kilometre.

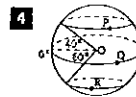


**3** 
$$l = \frac{\theta}{360} 2\pi r$$

$$= \frac{282}{360} \times 2 \times \pi \times 83$$

$$= 408.511\ 7647 \dots$$

$$= 409 \text{ (nearest unit)}$$
The length of the arc is 409 km, correct to the nearest kilometre.



- a 40°N  
b 0° [It lies on the equator.]  
c 60°S

**5**

a 80°W  
b 80°W  
c 0° [It lies on the prime meridian.]

**6**

a The latitude of A is 50°N.  
b The longitude of C is 90°E.

**7**

a  $\angle BOC = 70^\circ$  [ $30^\circ + 40^\circ$ ]  
b  $\angle AOB = 50^\circ$

**8**

a A is at (60°N, 50°E)  
b B is at (0°, 50°E).  
c C is at (60°N, 160°E)

**9**

a P(20°N, 80°E)  
b R(40°S, 130°W)

**10**  $80\text{ M} = 80 \times 1.852\text{ km}$   
 $= 148.16\text{ km}$

**11**  $500\text{ M} = 500 \times 1.852\text{ km}$   
 $= 926\text{ km}$

**12**  $4630\text{ km} = (4630 + 1.852)\text{ M}$   
 $= 2500\text{ M}$

**13**  $1207\text{ km} = (1207 + 1.852)\text{ M}$   
 $= 651.727\ 8618 \dots\text{ M}$   
 $= 652\text{ M}$  (nearest M)

**14** Distance travelled =  $18 \times 4\text{ km}$   
 $= 72\text{ km}$   
 $= (72 + 1.852)\text{ M}$   
 $= 38.876\ 889\ 85 \dots$   
 $= 39\text{ M}$  (nearest M)  
The boat will travel approximately 39 nautical miles in that time.

**15**  $700\text{ M} = 700 \times 1.852\text{ km}$   
 $= 1296.4\text{ km}$   
Time =  $(1296.4 + 150)\text{ hours}$   
 $= 8.642\ 6666\text{ hours}$   
 $= 8\text{ h } 40\text{ min}$  (nearest 5 min)  
The plane will take approximately 8 hours and 40 minutes to travel 700 nautical miles.

**16** Angular difference =  $152^\circ - 27^\circ$   
 $= 125^\circ$   
The angular difference in longitude is  $125^\circ$ .

**17** Angular difference =  $61^\circ - 48^\circ$   
 $= 13^\circ$   
The angular difference in latitude is  $13^\circ$ .

**18** Angular difference =  $116^\circ - 33^\circ$   
 $= 83^\circ$

**19** Angular difference =  $25^\circ - 4^\circ$   
 $= 21^\circ$

**20** Angular difference =  $54^\circ + 19^\circ$   
 $= 73^\circ$   
The difference in latitude is  $73^\circ$ .

**21** Angular difference =  $107^\circ + 38^\circ$   
 =  $145^\circ$   
 The angular difference is  $145^\circ$ .

**22** Angular difference =  $34^\circ + 59^\circ$   
 =  $93^\circ$   
 The difference in latitude is  $93^\circ$ .

**23** Angular difference =  $105^\circ + 12^\circ$   
 =  $117^\circ$   
 The difference in longitude is  $117^\circ$ .

**24** Angular difference  
 =  $360^\circ - (126^\circ + 112^\circ)$   
 =  $122^\circ$

**25** 
$$l = \frac{\theta}{360} \times 2\pi r$$

$$= \frac{34}{360} \times 2 \times \pi \times 6400$$

$$= 3797.836\ 452 \dots$$

$$\approx 3800$$
 The distance between the cities is approximately 3800 kilometres.

**26** Angular difference =  $167^\circ - 97^\circ$   
 =  $70^\circ$

$$l = \frac{\theta}{360} \times 2\pi r$$

$$= \frac{70}{360} \times 2 \times \pi \times 6400$$

$$= 7819.075\ 049 \dots$$

$$\approx 7820$$
 The distance between the localities is approximately 7820 kilometres.

**27** Angular difference =  $53^\circ - 37^\circ$   
 =  $16^\circ$

$$l = \frac{\theta}{360} \times 2\pi r$$

$$= \frac{16}{360} \times 2 \times \pi \times 6400$$

$$= 1787.217 \dots \text{ km}$$

$$= (1787.217 \dots + 1.852) \text{ M}$$

$$= 965.020\ 0616 \dots \text{ M}$$

$$\approx 965 \text{ M}$$
 The distance between the localities is approximately 965 nautical miles.

**28** Distance =  $70 \times 60 \text{ M}$   
 =  $4200 \text{ M}$   
 The distance between the points is 4200 nautical miles.

**29** Angular difference =  $38^\circ - 32^\circ$   
 =  $6^\circ$

Distance =  $6 \times 60 \text{ M}$   
 =  $360 \text{ M}$   
 The distance between the cities is 360 nautical miles.

**30** Angular difference =  $35^\circ + 31^\circ$   
 =  $66^\circ$   
 Distance =  $66 \times 60 \text{ M}$   
 =  $3960 \text{ M}$

The distance between the cities is 3960 nautical miles.

**31** Angular difference =  $6^\circ + 11^\circ$   
 =  $17^\circ$

Distance =  $17 \times 60 \text{ M}$   
 =  $1020 \text{ M}$   
 =  $1020 \times 1.852 \text{ km}$   
 =  $1889.04 \text{ km}$   
 =  $1890 \text{ km}$

The distance between the two cities is approximately 1890 kilometres.

**32** Distance =  $(32 \times 6) \text{ M}$   
 =  $192 \text{ M}$

The ship will travel 192 nautical miles in that time.

**33** Speed =  $187 \text{ M} \div 17 \text{ h}$   
 =  $11 \text{ nautical miles per hour}$   
 The average speed is 11 knots.

**34** Time =  $(720 \div 9) \text{ hours}$   
 =  $80 \text{ hours}$   
 =  $3 \text{ days and } 8 \text{ hours}$

**35** Speed =  $(825 \div 3) \text{ km/h}$   
 =  $275 \text{ km/h}$   
 =  $(275 \div 1.852) \text{ M per hour}$   
 =  $148.488\ 121 \dots \text{ knots}$   
 The average speed is approx. 148 knots.

**36** Speed =  $260 \text{ knots}$   
 =  $260 \text{ M per hour}$   
 =  $(260 \times 1.852) \text{ km per hour}$   
 =  $481.52 \text{ km/h}$   
 The average speed is approx. 480 km/h.

**37** a Angular difference =  $35^\circ + 7^\circ$   
 =  $42^\circ$

$$l = \frac{\theta}{360} \times 2\pi r$$

$$= \frac{42}{360} \times 2 \times \pi \times 6400$$

$$= 4691.445\ 029 \dots$$

$$\approx 4690$$
 The distance covered was approx. 4690 kilometres.

b  $400 \text{ knots} = (400 \times 1.852) \text{ km/h}$   
 =  $740.8 \text{ km/h}$   
 Time = distance  $\div$  speed  
 =  $4690 \div 740.8$   
 =  $6.330\ 993\ 521 \dots$   
 The journey took approximately 6.33 hours, or approximately 6 hours and 20 minutes.

**38** Angular difference =  $28^\circ - 23^\circ$   
 =  $5^\circ$

Distance =  $5 \times 60 \text{ M}$   
 =  $300 \text{ M}$   
 Time = 2 days and 2 hours  
 = 50 hours

Speed = distance  $\div$  time  
 =  $300 \text{ M} \div 50 \text{ hours}$   
 = 6 knots

The average speed of the boat was 6 knots.

**39** Perth is two hours behind EST.  
 The time in Brisbane is 12.30 pm.

**40** CST is  $1\frac{1}{2}$  hours ahead of WST.  
 Time in Broome is 4.30 pm.

**41** When it is 7.30 am in Melbourne it will be one hour earlier in Queensland. The call should be made at 6.30 am.

**42** a The time will be 8 hours ahead in Perth. It will be 2 pm.  
 b It will be 8 hours before 10.30 pm. The time in London will be 2.30 pm.

**43** a Santiago is 5 hours behind Greenwich. It will be 10 pm on 12th April.  
 b Calcutta is 11 hours ahead of Santiago. It will be 10 am on the 18th of October.

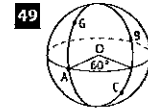
**44** It will be Saturday in Fiji.

**45** 18th January

**46** Angular difference =  $75^\circ - 0^\circ$   
 =  $75^\circ$   
 Time difference =  $(75 \div 15) \text{ hours}$   
 = 5 hours  
 Ottawa is 5 hours behind Greenwich.  
 Time in Ottawa is 10.45 am.

**47** Angular difference =  $45^\circ - 0^\circ$   
 =  $45^\circ$   
 Time difference =  $(45 \div 15) \text{ hours}$   
 = 3 hours  
 Mogadishu is 3 hours ahead of Greenwich.  
 Time in Mogadishu is 2.30 pm on Thursday.

**48** Angular difference =  $80^\circ - 0^\circ$   
 =  $80^\circ$   
 Time difference =  $80 \div 4 \text{ minutes}$   
 = 320 minutes  
 = 5 h 20 min  
 Greenwich is 5 hours and 20 minutes behind Colombo.  
 Time in Greenwich is 1.10 am.



a 9 pm

b Angular difference =  $60^\circ$   
 Time difference =  $(60 \div 15) \text{ hours}$   
 = 4 hours

Point B is east of Greenwich so the time will be 4 hours ahead of Greenwich.

Time at B = 6 am

c Time difference = 4 hours  
 Greenwich is 4 hours behind point C.  
 Time in Greenwich = 9 pm the previous day.

**50** Angular difference =  $148^\circ - 13^\circ$   
 =  $135^\circ$   
 Time difference =  $(135 \div 15) \text{ hours}$   
 = 9 hours  
 Port Arthur is 9 hours ahead of Copenhagen. Time in Port Arthur is 6 am on Wednesday.

**51** Angular difference =  $122^\circ + 151^\circ$   
 =  $273^\circ$   
 Time difference =  $(273 \div 15) \text{ hours}$   
 = 18.2 hours  
 = 18 h 12 min  
 Sydney is 18 hours and 12 minutes ahead of San Francisco.  
 Time in Sydney is 11.27 pm on 25th July.

**52** Angular difference =  $74^\circ + 67^\circ$   
 =  $141^\circ$   
 Time difference =  $(141 \div 15) \text{ hours}$   
 = 9.4 hours  
 = 9 h 24 min

New York is 9 hours and 24 minutes behind Karachi.  
 When it is 9.00 am in Karachi, it is 11.36 pm in New York.  
 When it is 5.00 pm in Karachi, it is 7.36 am in New York.  
 Wasim should call at around 11.36 pm or 7.36 am.

**53** Angular difference =  $150^\circ - 30^\circ$   
 =  $120^\circ$   
 Time difference =  $(120 \div 15) \text{ hours}$   
 = 8 hours

On standard time, St Petersburg would be 8 hours behind Narrabri. On daylight-saving time, St Petersburg will be 9 hours behind.  
 Time in St Petersburg is 11.45 pm Tuesday.

**54** Angular difference =  $111^\circ + 159^\circ$   
 =  $270^\circ$   
 Time difference =  $(270 \div 15) \text{ hours}$   
 = 18 hours  
 Lord Howe Island is normally 18 hours ahead of Great Falls. With daylight-saving, Lord Howe Island will be 19 hours ahead.  
 Time on Lord Howe Island is 1 am on Boxing Day.

**55** Angular difference =  $153^\circ - 105^\circ$   
 =  $48^\circ$   
 Time difference =  $48 \times 4 \text{ minutes}$   
 = 192 minutes  
 = 3 h 12 min  
 Brisbane is 3 hours and 12 minutes ahead of Singapore.  
 The plane leaves at 10.27 pm Brisbane time.

It arrives  $6\frac{1}{2}$  hours later. The plane arrives at 5.12 am the next day.

**56** Angular difference =  $175^\circ - 121^\circ$   
 =  $54^\circ$   
 Time difference =  $(54 \div 15) \text{ hours}$   
 = 3.6 hours  
 = 3 h 36 min  
 Shanghai is 3 hours and 36 minutes behind Auckland.  
 The time in Shanghai when the plane is due to arrive in Auckland is 5.54 am.  
 The plane left 11 hours earlier. It left at 6.54 pm the previous day.

**57** Angular difference =  $13^\circ + 74^\circ$   
 =  $87^\circ$   
 Time difference =  $(87 \div 15) \text{ hours}$   
 = 5.8 hours  
 = 5 h 48 min  
 New York is 5 hours and 48 minutes behind Rome.  
 The plane left Rome at 5.54 am Monday, New York time.  
 The flight took 13 hours and 54 minutes.

**58** Angular difference =  $151^\circ + 118^\circ$   
 =  $269^\circ$   
 Time difference =  $(269 \div 15) \text{ hours}$   
 = 17.9333 ... hours  
 = 17 h 56 min  
 Los Angeles is 17 hours and 56 minutes behind Sydney.  
 Time in Los Angeles when the plane departs is 3.49 pm Friday.  
 The plane will land 14 h 16 min later.  
 The plane lands at 6.05 am Saturday.