

# General Mathematics

## General Instructions

- Reading time – 5 minutes
- Working time –  $2\frac{1}{2}$  hours
- Write using black or blue pen  
Black pen is preferred
- Calculators may be used
- A formulae sheet is provided at the back of this paper
- In Questions 26–30, show relevant mathematical reasoning and/or calculations
- Write your Centre Number and Student Number on the Question 28 Writing Booklet

## Total marks – 100

### Section I Pages 2–13

#### 25 marks

- Attempt Questions 1–25
- Allow about 35 minutes for this section

### Section II Pages 14–27

#### 75 marks

- Attempt Questions 26–30
- Allow about 1 hour and 55 minutes for this section

## Section I

25 marks

Attempt Questions 1–25

Allow about 35 minutes for this section

Use the multiple-choice answer sheet for Questions 1–25.

- 1 A set of 15 scores is displayed in a stem-and-leaf plot.

5	3	4		
6	2	6	7	
7	7	7	8	9
8	2	4		
9	1	3	5	7

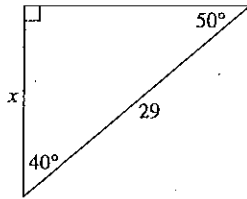
What is the median of these scores?

- (A) 7  
(B) 8  
(C) 77  
(D) 78
- 2 Handmade chocolates are checked for size and shape. Every 30th chocolate is sampled.

Which term best describes this type of sampling?

- (A) Census  
(B) Random  
(C) Stratified  
(D) Systematic
- 3 A pair of players is to be selected from 6 people.
- How many different pairs of players can be selected?
- (A) 6  
(B) 12  
(C) 15  
(D) 30

4

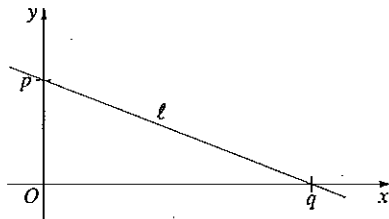


NOT TO SCALE

Which expression could be used to calculate the value of  $x$  in this triangle?

- (A)  $29 \times \cos 40^\circ$
- (B)  $29 \times \cos 50^\circ$
- (C)  $\frac{\cos 40^\circ}{29}$
- (D)  $\frac{\cos 50^\circ}{29}$

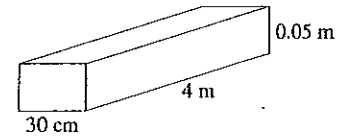
5 The line  $\ell$  has intercepts  $p$  and  $q$ , where  $p$  and  $q$  are positive integers.



What is the gradient of line  $\ell$ ?

- (A)  $-\frac{p}{q}$
- (B)  $-\frac{q}{p}$
- (C)  $\frac{p}{q}$
- (D)  $\frac{q}{p}$

6

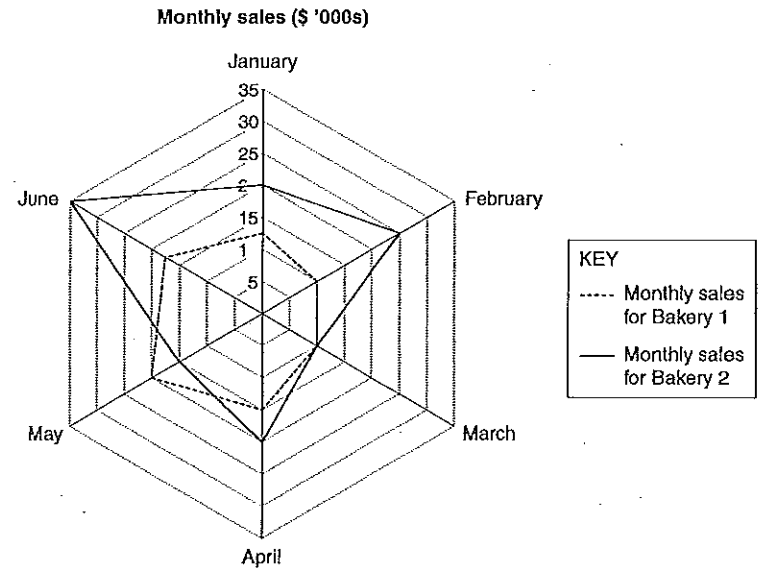


NOT TO SCALE

What is the volume of this rectangular prism in cubic centimetres?

- (A)  $6 \text{ cm}^3$
- (B)  $600 \text{ cm}^3$
- (C)  $60\,000 \text{ cm}^3$
- (D)  $6\,000\,000 \text{ cm}^3$

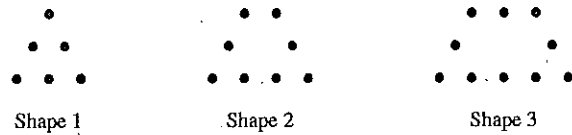
7 The Pi Company has two bakeries. The radar chart displays the monthly sales for the bakeries.



What was the difference in sales in June between the two bakeries?

- (A) \$7.50
- (B) \$17.50
- (C) \$7500
- (D) \$17 500

- 8 Dots were used to create a pattern. The first three shapes in the pattern are shown.



The number of dots used in each shape is recorded in the table.

Shape ( $S$ )	1	2	3
Number of dots ( $N$ )	6	8	10

How many dots would be required for Shape 156?

- (A) 316  
 (B) 520  
 (C) 624  
 (D) 936
- 9 Tracy invests some money for 2 years at 4% per annum, compounded quarterly.

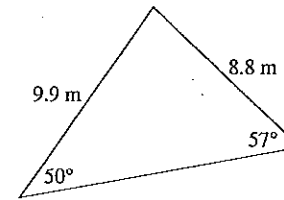
Compounded values of \$1

Period	Interest rate per period				
	1%	2%	3%	4%	5%
1	1.010	1.020	1.030	1.040	1.050
2	1.020	1.040	1.061	1.082	1.103
3	1.030	1.061	1.093	1.125	1.158
4	1.041	1.082	1.126	1.170	1.216
5	1.051	1.104	1.159	1.217	1.276
6	1.062	1.126	1.194	1.265	1.340
7	1.072	1.149	1.230	1.316	1.407
8	1.083	1.172	1.267	1.369	1.477

Which figure from the table should Tracy use to calculate the value of her investment at the end of 2 years?

- (A) 1.020  
 (B) 1.082  
 (C) 1.083  
 (D) 1.369

- 10



NOT TO SCALE

What is the area of this triangle, to the nearest square metre?

- (A) 33 m<sup>2</sup>  
 (B) 37 m<sup>2</sup>  
 (C) 42 m<sup>2</sup>  
 (D) 44 m<sup>2</sup>

- 11 Which of the following relationships would most likely show a negative correlation?

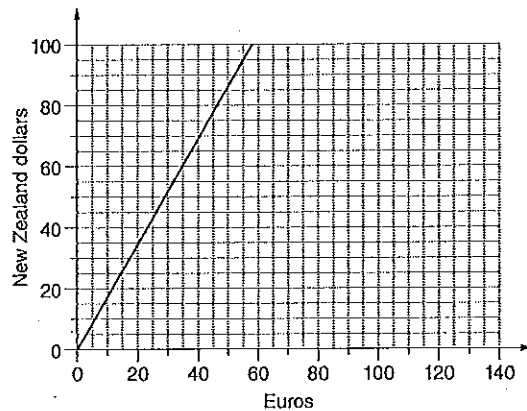
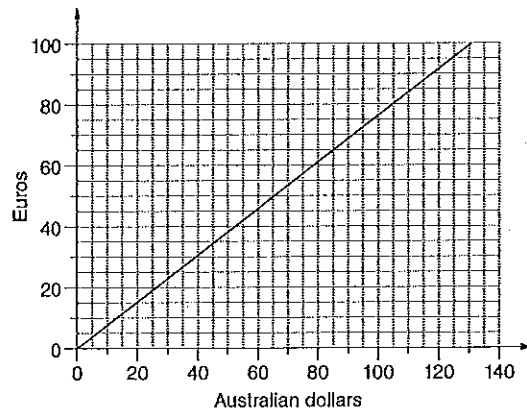
- (A) The population of a town and the number of hospitals in that town.  
 (B) The hours spent training for a race and the time taken to complete the race.  
 (C) The price per litre of petrol and the number of people riding bicycles to work.  
 (D) The number of pets per household and the number of computers per household.

- 12 Two unbiased dice, each with faces numbered 1, 2, 3, 4, 5 and 6, are rolled.

What is the probability of a 6 appearing on at least one of the dice?

- (A)  $\frac{1}{6}$   
 (B)  $\frac{11}{36}$   
 (C)  $\frac{25}{36}$   
 (D)  $\frac{5}{6}$

- 13 Conversion graphs can be used to convert from one currency to another.



Sarah converted 60 Australian dollars into Euros. She then converted all of these Euros into New Zealand dollars.

How much money, in New Zealand dollars, should Sarah have?

- (A) \$26
- (B) \$45
- (C) \$78
- (D) \$135

- 14 Which of the following expresses  $2x^2(5-x) - x(x-2)$  in its simplest form?

- (A)  $-2x^3 + 9x^2 + 2x$
- (B)  $-2x^3 - 9x^2 - 2x$
- (C)  $9x^2 - x + 2$
- (D)  $9x^2 - x - 2$

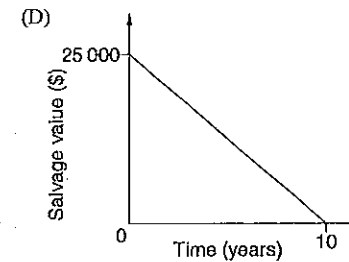
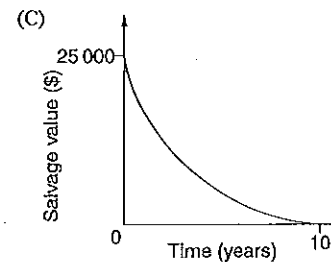
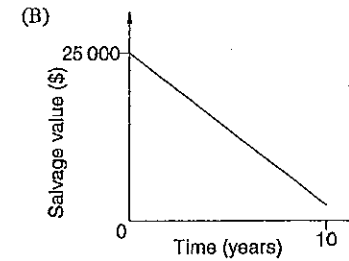
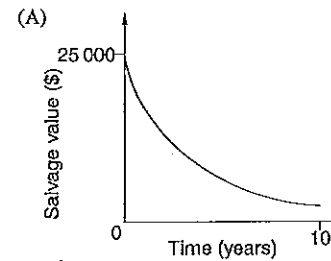
- 15 The time taken to complete a journey varies inversely with the speed of a car. A car takes 6 hours to complete a journey when travelling at 60 km/h.

How long would the same journey take if the car were travelling at 100 km/h?

- (A) 36 minutes
- (B) 1 hour and 40 minutes
- (C) 3 hours and 6 minutes
- (D) 3 hours and 36 minutes

- 16 A machine was bought for \$25 000.

Which graph best represents the salvage value of the machine over 10 years using the declining balance method of depreciation?



- 17 A spinner with different coloured sectors is spun 40 times. The results are recorded in the table.

Colour obtained	Frequency
Red	2
Yellow	4
Blue	6
Orange	<input type="text"/>
Green	10
Purple	12

What is the relative frequency of obtaining the colour orange?

- (A)  $\frac{3}{20}$
- (B)  $\frac{1}{5}$
- (C) 6
- (D) 8
- 18 Jo qualifies for both Rent Assistance and Youth Allowance and receives a fortnightly payment from the government.

Rent Assistance is \$119.40 per fortnight.

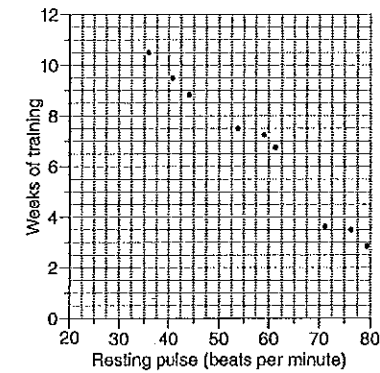
The maximum Youth Allowance is \$402.70 per fortnight. It is reduced by 50 cents in the dollar for any income earned over \$236 per fortnight.

Jo earns \$300 per fortnight from a part-time job.

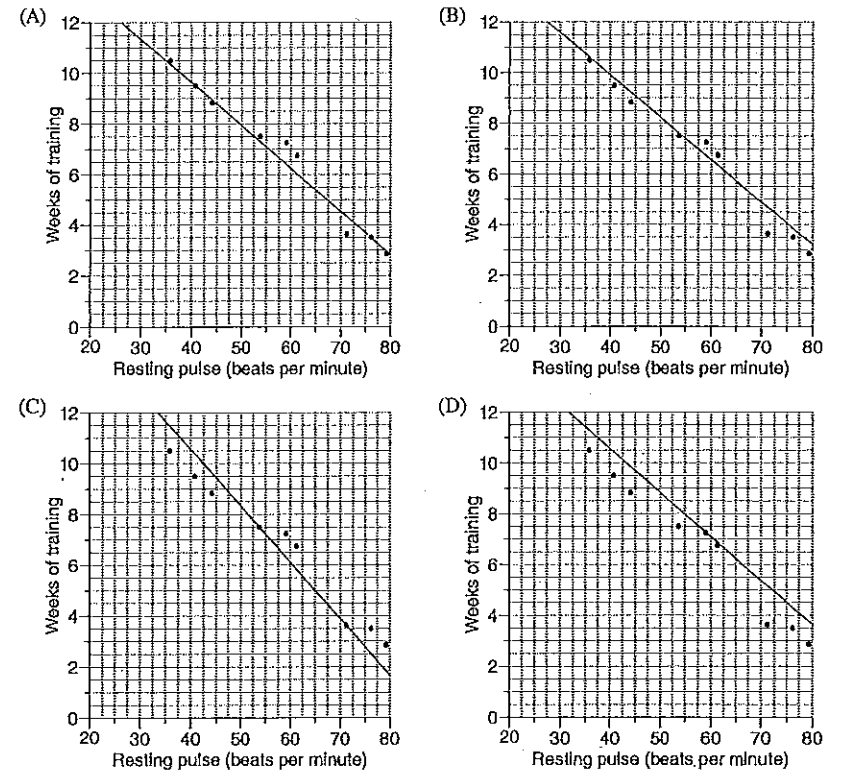
What is the total payment Jo receives each fortnight from the government?

- (A) \$370.70
- (B) \$372.10
- (C) \$458.60
- (D) \$490.10

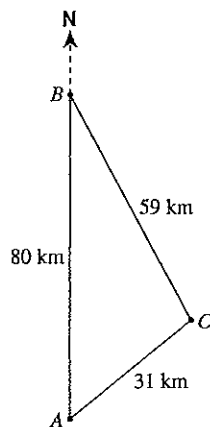
- 19 A coach compares the resting pulse and the number of weeks of training for nine cyclists. The information is graphed in order to draw a median regression line.



Which of the following graphs best shows the median regression line for the data?



- 20 Town  $B$  is 80 km due north of Town  $A$  and 59 km from Town  $C$ .  
Town  $A$  is 31 km from Town  $C$ .



NOT TO  
SCALE

What is the bearing of Town  $C$  from Town  $B$ ?

- (A)  $019^\circ$   
(B)  $122^\circ$   
(C)  $161^\circ$   
(D)  $341^\circ$
- 21 Which of the following correctly expresses  $c$  as the subject of  $E = mc^2 + p$ ?

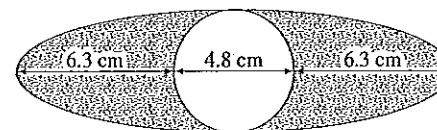
(A)  $c = \pm \sqrt{\frac{E-p}{m}}$

(B)  $c = \pm \sqrt{\frac{E-p}{m}}$

(C)  $c = \pm \sqrt{\frac{E}{m} - p}$

(D)  $c = \pm \sqrt{\frac{E}{m} - p}$

- 22 A label is designed with a circle inside an ellipse as shown.



NOT TO  
SCALE

What is the area of the shaded part of the label, to the nearest square centimetre?

- (A)  $29 \text{ cm}^2$   
(B)  $48 \text{ cm}^2$   
(C)  $113 \text{ cm}^2$   
(D)  $244 \text{ cm}^2$
- 23 A football club knows that at the end of 10 years it will need to replace goal posts and other equipment. It is estimated that the replacement cost will be \$12 000. For 10 years, the club will invest an amount at the end of each month at 6% per annum, compounded monthly.

Which equation should the club use to calculate the amount,  $M$ , it will need to deposit each month in order to have \$12 000 at the end of 10 years?

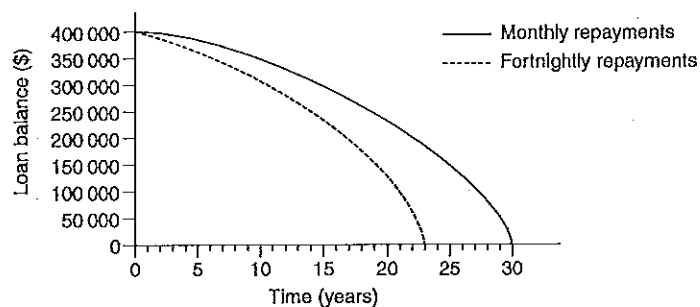
(A)  $12\,000 = M \left\{ \frac{(1+0.06)^{10} - 1}{0.06} \right\}$

(B)  $12\,000 = M \left\{ \frac{(1+0.06)^{10} - 1}{0.06(1+0.06)^{10}} \right\}$

(C)  $12\,000 = M \left\{ \frac{(1+0.005)^{120} - 1}{0.005(1+0.005)^{120}} \right\}$

(D)  $12\,000 = M \left\{ \frac{(1+0.005)^{120} - 1}{0.005} \right\}$

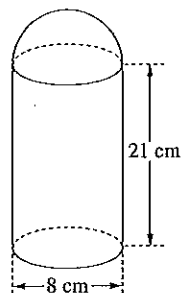
- 24 A \$400 000 loan can be repaid by making either monthly or fortnightly repayments. The graph shows the loan balances over time using these two different methods of repayment.



The monthly repayment is \$2796.86 and the fortnightly repayment is \$1404.76.

What is the difference in the total interest paid using the two different methods of repayment, to the nearest dollar?

- (A) \$51 596  
 (B) \$166 823  
 (C) \$210 000  
 (D) \$234 936
- 25 The solid shown is made of a cylinder with a hemisphere (half a sphere) on top.



NOT TO SCALE

What is the total surface area of the solid, to the nearest square centimetre?

- (A) 628 cm<sup>2</sup>  
 (B) 679 cm<sup>2</sup>  
 (C) 729 cm<sup>2</sup>  
 (D) 829 cm<sup>2</sup>

## Section II

75 marks

Attempt Questions 26–30

Allow about 1 hour and 55 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In Questions 26–30, your responses should include relevant mathematical reasoning and/or calculations.

Question 26 (15 marks) Use the Question 26 Writing Booklet.

- (a) Postcodes in Australia are made up of four digits eg 2040.
- (i) How many different postcodes beginning with a 2 are possible? 1
- (ii) Peta remembers that the first two digits of a town's postcode are 2 and then 4. She is unable to remember the rest of the postcode. 1

2 4 ? ?

What is the probability that Peta guesses the correct postcode?

- (b) Jim buys a photocopier for \$22 000. Its value is depreciated using the declining balance method at the rate of 15% per annum. 2

What is its value at the end of 3 years?

- (c) Heather used her credit card to purchase a plane ticket valued at \$1990 on 28 January 2011. She made no other purchases on her credit card account in January. She paid the January account in full on 19 February 2011. 2

The credit card account has no interest free period. Simple interest is charged daily at the rate of 20% per annum, including the date of purchase and the date the account is paid.

How much interest did she pay, to the nearest cent?

Question 26 continues on page 15

Question 26 (continued)

- (d) Greg needs to conduct a statistical inquiry into how much time people aged 18–25 years have spent accessing social media websites in the last two weeks. He has decided to survey a sample of students from his university.

The process of statistical inquiry includes the following steps, which are NOT in order.

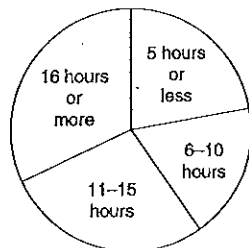
- A Writing a report
- B Posing questions
- C Organising data
- D Analysing data and drawing conclusions
- E Collecting data
- F Summarising and displaying data

- (i) Using the letters A, B, C, D, E and F, list the steps in the most appropriate order for Greg to conduct his statistical inquiry. 2

- (ii) Greg conducts his statistical inquiry. 1

At which step in the process would he have drawn this graph?

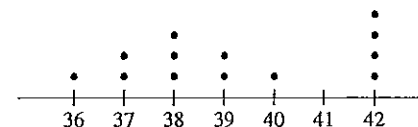
Time spent accessing social media websites (In hours)



Question 26 continues on page 16

Question 26 (continued)

- (e) The dot plot shows the number of push-ups that 13 members of a fitness class can do in one minute.



- (i) What is the probability that a member selected at random from the class can do more than 38 push-ups in one minute? 1

- (ii) A new member who can do 32 push-ups in one minute joins the class. Does the addition of this new member to the class change the probability calculated in part (e) (i)? Justify your answer. 1

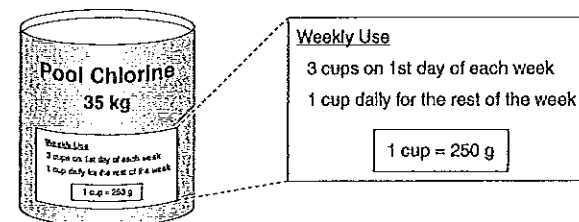
- (f) The capture-recapture technique was used to estimate a population of seals in 2012. 2

- 60 seals were caught, tagged and released.
- Later, 120 seals were caught at random.
- 30 of these 120 seals had been tagged.

The estimated population of seals in 2012 was 11% less than the estimated population for 2008.

What was the estimated population for 2008?

- (g) Bhawana purchases pool chlorine in a new container which holds 35 kg. 2



She begins using this new container on the first day of a week.

How many full weeks should this container last?

End of Question 26



Question 27 (15 marks) Use the Question 27 Writing Booklet.

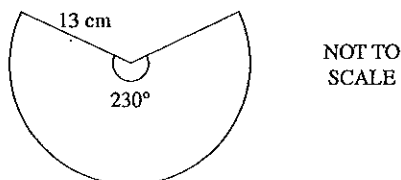
- (a) Tai earns a gross weekly wage of \$1024. Each week her deductions are: 3

- tax instalment of \$296.40
- health fund contribution of \$24.50
- union fees of \$15.80.

She also pays \$3640 over the year as her share of the household expenses.

What percentage of her net wage does Tai pay for household expenses?

- (b) The sector shown has a radius of 13 cm and an angle of  $230^\circ$ . 2



What is the perimeter of the sector to the nearest centimetre?

- (c) A map has a scale of 1:500 000. 1

- (i) Two mountain peaks are 2 cm apart on the map.

What is the actual distance between the two mountain peaks, in kilometres?

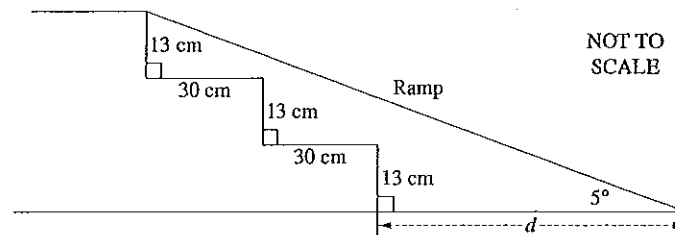
- (ii) Two cities are 75 km apart. 1

How far apart are the two cities on the map, in centimetres?

Question 27 continues on page 18

Question 27 (continued)

- (d) A disability ramp is to be constructed to replace steps, as shown in the diagram. The angle of inclination for the ramp is to be  $5^\circ$ . 3

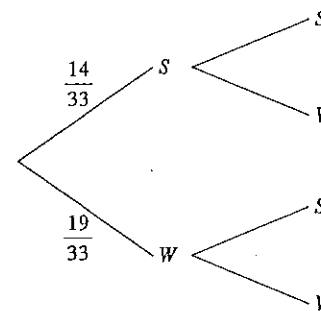


Calculate the extra distance,  $d$ , that the ramp will extend beyond the bottom step. Give your answer to the nearest centimetre.

- (e) A box contains 33 scarves made from two different fabrics. There are 14 scarves made from silk ( $S$ ) and 19 made from wool ( $W$ ).

Two girls each select, at random, a scarf to wear from the box.

- (i) Copy and complete the probability tree diagram in your answer booklet. 2



- (ii) Calculate the probability that the two scarves selected are made from silk. 1

- (iii) Calculate the probability that the two scarves selected are made from different fabrics. 2

End of Question 27

Question 28 (15 marks) Use the Question 28 Writing Booklet.

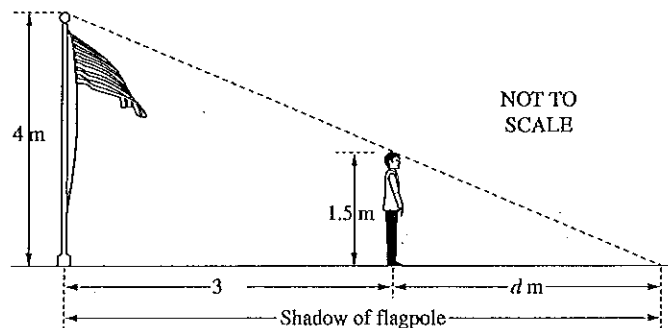
- (a) The solid shown in the diagram on page 1 of the Question 28 Writing Booklet is a cube. 2

Complete a sketch of the cube using the vanishing points  $V_1$  and  $V_2$ .

Leave all construction lines on your diagram and label the vertices.

- (b) Simplify fully  $\frac{18ab}{3a^2} \times \frac{c}{b}$ . 2

- (c) Jacques and a flagpole both cast shadows on the ground. The difference between the lengths of their shadows is 3 metres. 3

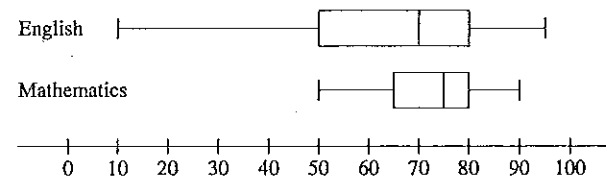


What is the value of  $d$ , the length of Jacques' shadow?

Question 28 continues on page 20

Question 28 (continued)

- (d) The test results in English and Mathematics for a class were recorded and displayed in the box-and-whisker plots.



- (i) What is the interquartile range for English? 1
- (ii) Compare and contrast the two data sets by referring to the skewness of the distributions and the measures of location and spread. 3
- (e) Matthew bought a laptop priced at \$2800. He paid a 10% deposit and made monthly repayments of \$95.20 for 3 years. 4

What annual flat rate of interest was Matthew charged? Justify your answer with suitable calculations.

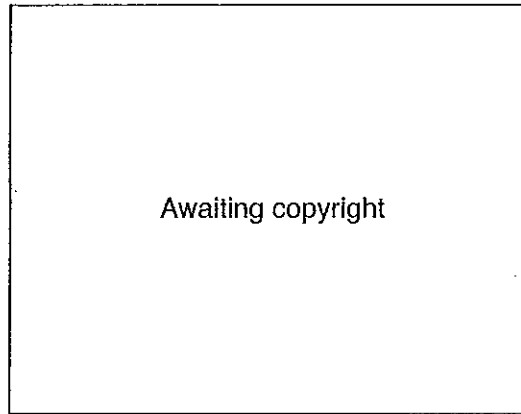
End of Question 28

Question 29 (15 marks) Use the Question 29 Writing Booklet.

- (a) Tourists visit a park where steam erupts from a particular geyser.

The brochure for the park has a graph of the data collected for this geyser over a period of time.

The graph shows the duration of an eruption and the time until the next eruption, timed from the end of one eruption to the beginning of the next.



- (i) Tony sees an eruption that lasts 4 minutes. Based on the data in the graph, what is the minimum time that he can expect to wait for the next eruption? 1
- (ii) Julia saw two consecutive eruptions, one hour apart. Based on the data in the graph, what was the longest possible duration of the first eruption that she saw? 1
- (iii) What does the graph suggest about the relationship between the duration of an eruption and the time to the next eruption? 1

Question 29 continues on page 22

Question 29 (continued)

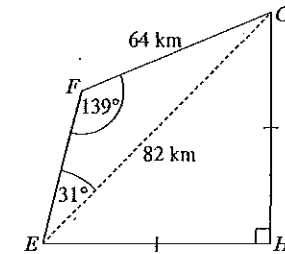
- (b) A machine produces nails. When the machine is set correctly, the lengths of the nails are normally distributed with a mean of 6.000 cm and a standard deviation of 0.040 cm. 2

To confirm the setting of the machine, three nails are randomly selected. In one sample the lengths are 5.950, 5.983 and 6.140.

The setting of the machine needs to be checked when the lengths of two or more nails in a sample lie more than 1 standard deviation from the mean.

Does the setting on the machine need to be checked? Justify your answer with suitable calculations.

- (c) Raj cycles around a course. The course starts at  $E$ , passes through  $F$ ,  $G$  and  $H$  and finishes at  $E$ . The distances  $EH$  and  $GH$  are equal. 2



NOT TO SCALE

- (i) What is the length of  $EF$ , to the nearest kilometre? 2
- (ii) What is the total distance that Raj cycles, to the nearest kilometre? 3
- (d) Su-Lin pays a monthly contribution of 5% of her salary into a superannuation fund. Her salary is \$81 600 per annum. The fund pays interest of 6.6% per annum, compounded monthly. 2
- (i) What is the amount of Su-Lin's monthly contribution? 2
- (ii) Su-Lin made a contribution at the end of each month, starting at the end of January 2000. 3

What will be the accumulated value of her superannuation at the end of December 2012 after making the December contribution?

End of Question 29

Question 30 (15 marks) Use the Question 30 Writing Booklet.

- (a) A ship located at  $4^{\circ}\text{N } 160^{\circ}\text{E}$  needs assistance.

A rescue boat leaves Honiara  $9^{\circ}\text{S } 160^{\circ}\text{E}$  and travels due north at a speed of 30 knots to reach the ship.

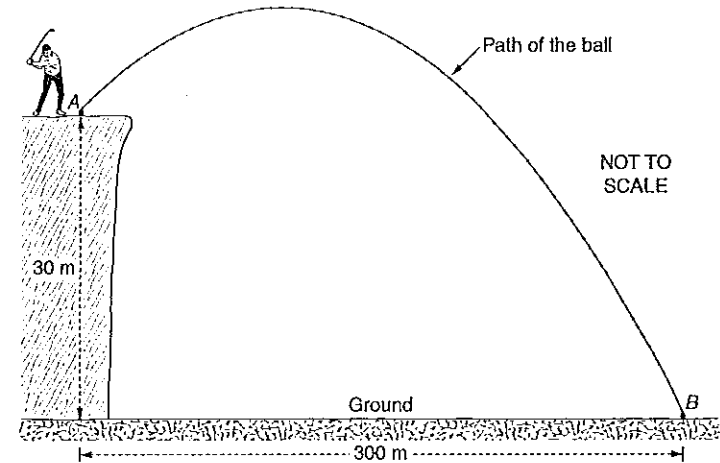
How long will it take the rescue boat to reach the ship? (You may assume that  $1^{\circ}$  on a great circle equals 60 nautical miles.)

Question 30 continues on page 24

3

Question 30 (continued)

- (b) A golf ball is hit from point  $A$  to point  $B$ , which is on the ground as shown. Point  $A$  is 30 metres above the ground and the horizontal distance from point  $A$  to point  $B$  is 300 m.



The path of the golf ball is modelled using the equation

$$h = 30 + 0.2d - 0.001d^2$$

where

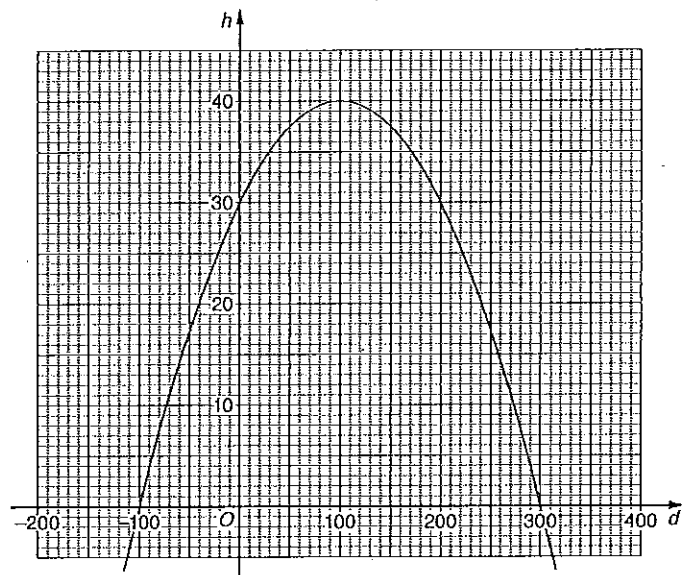
$h$  is the height of the golf ball above the ground in metres, and

$d$  is the horizontal distance of the golf ball from point  $A$  in metres.

Question 30 continues on page 25

Question 30 (continued)

The graph of this equation is drawn below.



- (i) What is the maximum height the ball reaches above the ground? 1
- (ii) There are two occasions when the golf ball is at a height of 35 metres. 1  
 What horizontal distance does the ball travel in the period between these two occasions?
- (iii) What is the height of the ball above the ground when it still has to travel a horizontal distance of 50 metres to hit the ground at point *B*? 1
- (iv) Only part of the graph applies to this model. 2  
 Find all values of *d* that are not suitable to use with this model, and explain why these values are not suitable.

Question 30 continues on page 26

Question 30 (continued)

- (c) In 2010, the city of Thagoras modelled the predicted population of the city using the equation

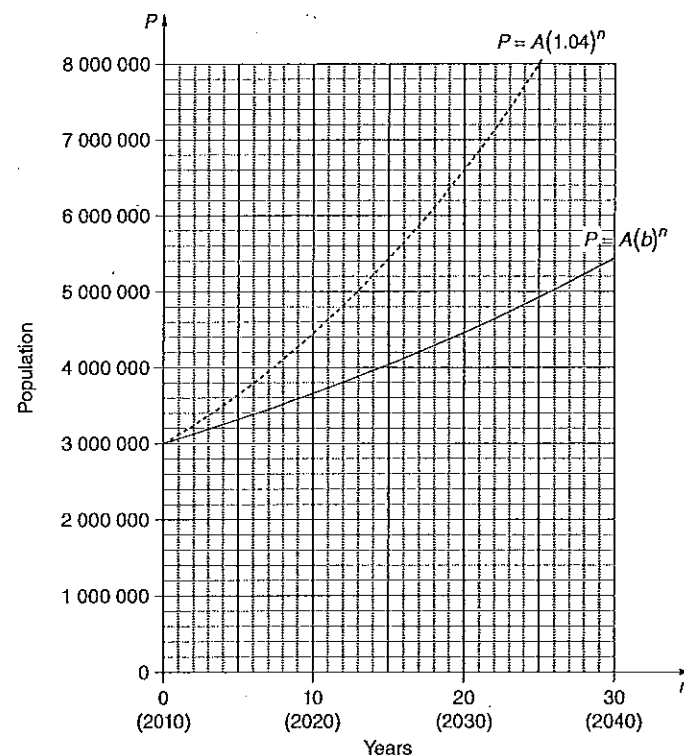
$$P = A(1.04)^n.$$

That year, the city introduced a policy to slow its population growth. The new predicted population was modelled using the equation

$$P = A(b)^n.$$

In both equations, *P* is the predicted population and *n* is the number of years after 2010.

The graph shows the two predicted populations.



- Predicted population if the policy had not been introduced  
 ——— Predicted population with the policy introduced

Question 30 continues on page 27

Question 30 (continued)

- (i) Use the graph to find the predicted population of Thagoras in 2030 if the population policy had NOT been introduced. 1
- (ii) In each of the two equations given, the value of  $A$  is 3 000 000. 1  
What does  $A$  represent?
- (iii) The guess-and-check method is to be used to find the value of  $b$ , in  $P = A(b)^n$ .
- (1) Explain, with or without calculations, why 1.05 is not a suitable first estimate for  $b$ . 1
- (2) With  $n = 20$  and  $P = 4\,460\,000$ , use the guess-and-check method and the equation  $P = A(b)^n$  to estimate the value of  $b$  to two decimal places. Show at least TWO estimate values for  $b$ , including calculations and conclusions. 2
- (iv) The city of Thagoras was aiming to have a population under 7 000 000 in 2050. Does the model indicate that the city will achieve this aim? Justify your answer with suitable calculations. 2

End of paper

# 2012 Higher School Certificate Solutions General Mathematics

## SECTION I

### Summary

1	D	8	A	14	A	20	C
2	D	9	C	15	D	21	A
3	C	10	C	16	A	22	B
4	A	11	B	17	A	23	D
5	A	12	B	18	D	24	B
6	C	13	C	19	B	25	B
7	D						

- 1 (D) For 15 scores, the median is the 8<sup>th</sup> score and that is 78.
- 2 (D) Systematic sampling is where items are selected at regular intervals.
- 3 (C) The order of the two players is not important.  

$$\text{Number of pairs} = \frac{6 \times 5}{2 \times 1}$$

$$= \frac{30}{2}$$

$$= 15.$$
- 4 (A) From the diagram:  

$$\cos 40^\circ = \frac{x}{29}$$

$$x = 29 \cos 40^\circ.$$
- 5 (A) The gradient of the line through  $(0, p)$  and  $(q, 0)$  is given by:  

$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{0 - p}{q - 0}$$

$$= -\frac{p}{q}.$$

- 6 (C) Using centimeters throughout:  
 $4 \text{ m} = 400 \text{ cm}, \quad 0.05 \text{ m} = 5 \text{ cm}$   

$$\text{Volume} = 30 \times 400 \times 5$$

$$= 60\,000 \text{ cm}^3.$$
- 7 (D) From the graph for June:  
 Bakery 2 sales = \$35 000  
 Bakery 1 sales = \$17 500  

$$\text{Difference} = \$35\,000 - \$17\,500$$

$$= \$17\,500.$$
- 8 (A) The number of dots increases by 2 each time. There will be 155 increases on top of the original 6.  

$$\text{Number of dots} = 155 \times 2 + 6$$

$$= 316.$$
  
 OR  

$$N = 2s + 4$$

$$= 2 \times 156 + 6$$

$$= 316.$$
- 9 (C) All figures to be used must be quarterly.  
 Interest rate = 4% pa  

$$= 1\% \text{ per quarter}$$
  

$$\text{Number of periods} = 2 \text{ years} \times 4 \text{ quarters}$$

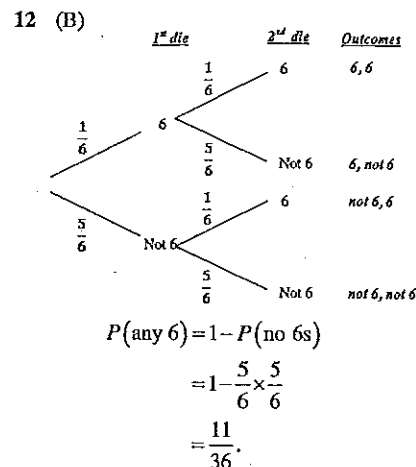
$$= 8 \text{ quarters}$$
  
 From the table, the value is 1.083.
- 10 (C) 
$$\text{Area} = \frac{1}{2} ab \sin C$$
  
 But  $C$  is not known.  

$$\angle C = 180^\circ - 50^\circ - 57^\circ$$

$$= 73^\circ$$

$$\begin{aligned} \text{Area} &= \frac{1}{2} ab \sin C \\ &= \frac{1}{2} \times 9 \times 8 \times \sin 73^\circ \\ &\approx 42 \text{ m}^2. \end{aligned}$$

- 11 (B) A positive correlation is where as one variable increases then the other variable also increases. A negative correlation is where as one variable increases the other variable decreases. This could only be applied to (B): the more hours spent training for a race, the shorter the time taken to complete the race.



- 13 (C) Using the first graph:  
 AU \$60 = 45 Euros  
 Using the second graph:  
 45 Euros = NZ \$78.
- 14 (A) 
$$2x^2(5-x) - x(x-2) = 10x^2 - 2x^3 - x^2 + 2x$$

$$= -2x^3 + 9x^2 + 2x.$$
- 15 (D) Use  $t$  for time in hours,  
 Use  $s$  for speed in km/h.  

$$t = \frac{k}{s}$$

$$6 = \frac{k}{60}$$

$$k = 6 \times 60$$

$$= 360$$

$$\therefore t = \frac{k}{s} = \frac{360}{s}$$

When  $s = 100$ :

$$t = \frac{360}{100}$$

$$= 3.6 \text{ hours}$$

$$= 3 \text{ h } 36 \text{ min.}$$

- 16 (A) The declining balance depreciation means that the value of the machine decreases at a constant percentage and not by a constant amount. This means that the graph will be a curve and not a straight line. In addition, the curve can never reach zero.
- 17 (A) The total of the frequency column should be 40. The total of the values given is 34. Thus the value for orange is  $40 - 34 = 6$ .  

$$\text{Relative frequency} = \frac{6}{40}$$

$$= \frac{3}{20}.$$
- 18 (D) Allowance =  $\$402.70 - (\$300 - \$236) \times 0.50$   

$$= \$402.70 - \$32$$

$$= \$370.70$$
  
 Payment =  $\$370.70 + \$119.40$   

$$= \$490.10.$$
- 19 (B) All the points are divided into 3 groups. The medians for group 1 and group 3 are joined. Answer (A) has that line. The line is then shifted by one third of the distance toward the median for group 2. In general, there should be a good balance between points above and below the line.
- 20 (C) Using the cosine rule to find an angle:  

$$\cos B = \frac{80^2 + 59^2 - 31^2}{2 \times 80 \times 59}$$

$$\angle B = 19.1058\dots$$

$$= 19^\circ$$
  
 Bearing of  $C$  from  $B = 180^\circ - 19^\circ$   

$$= 161^\circ.$$

21 (A)  $E = mc^2 + p$   
 $mc^2 = E - p$   
 $c^2 = \frac{E - p}{m}$   
 $c = \pm \sqrt{\frac{E - p}{m}}$

22 (B) For the ellipse:  
 Semi-major axis  $= \frac{1}{2}(6 \cdot 3 + 4 \cdot 8 + 6 \cdot 3)$   
 $= 8 \cdot 7$  cm  
 Semi-minor axis  $= \frac{1}{2} \times 4 \cdot 8$   
 $= 2 \cdot 4$  cm  
 Area<sub>shaded</sub> = Area<sub>ellipse</sub> - Area<sub>circle</sub>  
 $= \pi ab - \pi r^2$   
 $= \pi \times 8 \cdot 7 \times 2 \cdot 4 - \pi \times 2 \cdot 4^2$   
 $= 47 \cdot 50088 \dots$   
 $\approx 48 \text{ cm}^2$

23 (D) The interest is compounded monthly so:  
 $r = \frac{0 \cdot 06}{12}$   
 $= 0 \cdot 005$  per month  
 $n = 10 \times 12$   
 $= 120$  months  
 The question requires a *future value* calculation.  
 $\therefore$  it must be answer (D) which is a substitution into the future value formula.

24 (B) For monthly repayments:  
 Term<sub>monthly</sub> = 30 years (from graph)  
 $= 30 \times 12$   
 $= 360$  months  
 Total<sub>monthly</sub> =  $\$2796 \cdot 86 \times 360$   
 $= \$1006\ 869 \cdot 60$   
 For fortnightly repayments:  
 Term<sub>fortnightly</sub> = 23 years (from graph)  
 $= 23 \times 26$   
 $= 598$  fortnights

Total<sub>fortnightly</sub> =  $\$1404 \cdot 76 \times 598$   
 $= \$840\ 046 \cdot 48$   
 Difference =  $\$1006\ 869 \cdot 60 - \$840\ 046 \cdot 48$   
 $= \$166\ 823 \cdot 12$   
 $\approx \$166\ 823$

25 (B) radius  $= \frac{1}{2} \times 8 = 4$  cm  
 $SA_{\text{hemisphere}} = \frac{1}{2} \times 4\pi r^2$   
 $= \frac{1}{2} \times (4 \times \pi \times 4^2)$   
 $= 100 \cdot 53 \dots \text{cm}^2$   
 $SA_{\text{open cylinder}} = 2\pi rh + \pi r^2$   
 $= 2 \times \pi \times 4 \times 21 + \pi \times 4^2$   
 $= 578 \cdot 05 \dots \text{cm}^2$   
 Total SA =  $100 \cdot 53 \dots + 578 \cdot 05 \dots$   
 $= 678 \cdot 58 \dots$   
 $\approx 679 \text{ cm}^2$

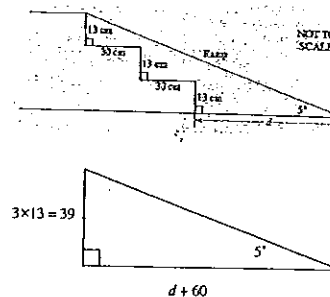
Question 26

(a) (i) Number of possibilities =  $10 \times 10 \times 10$   
 $= 1000$ .  
 (ii) Number of possibilities =  $10 \times 10$   
 $= 100$   
 $\therefore$  Probability of correct guess =  $\frac{1}{100}$ .  
 (b) Using the declining balance formula:  
 $S = V_0(1 - r)^n$   
 $= 22000(1 - 0 \cdot 15)^3$   
 $= \$13510 \cdot 75$   
 After 3 years, the value of the photocopier is \$13 510.75.  
 (c) The number of days from 28 January 2011 to 19 February 2011 inclusive is  $4 + 19 = 23$ .  
 $I = Prn$   
 $= 1990 \times 0 \cdot 20 \times \frac{23}{365}$   
 $= \$25 \cdot 08$ .

(d) (i) B Posing questions  
 E Collecting data  
 C Organising data  
 F Summarising and displaying data  
 D Analysing data and drawing conclusions  
 A Writing a report.  
 (ii) F Summarising and displaying data.  
 (e) (i) The number of students is 13.  
 The number of students with scores more than 38 is 7.  
 $\therefore P(\text{more than } 38) = \frac{7}{13}$ .  
 (ii) Yes, the probability will change.  
 There are now 14 students but still only 7 with a score more than 38.  
 $P(\text{more than } 38) = \frac{7}{14}$   
 $= \frac{1}{2}$ .  
 (f) Sample: 30 tagged out of 120.  
 Population: 60 tagged out of  $P$ .  
 $\therefore \frac{30}{120} = \frac{60}{P}$   
 $P = 240$  seals in 2012.  
 The population in 2012 is 11% less than the 2008 population ( $N$ ).  
 i.e. the 2012 population is 89% of the 2008 population.  
 $0 \cdot 89 \times N = 240$   
 $N = \frac{240}{0 \cdot 89}$   
 $N \approx 270$   
 The estimated population for 2008 is 270 seals.  
 (g) Each week she uses  $3 + 6 \times 1 = 9$  cups.  
 Amount used per week =  $9 \times 250$  g  
 $= 2250$  g  
 $= 2 \cdot 25$  kg  
 Number of weeks =  $35 \div 2 \cdot 25$   
 $\approx 15 \cdot 6$  weeks  
 The container will last 15 full weeks.

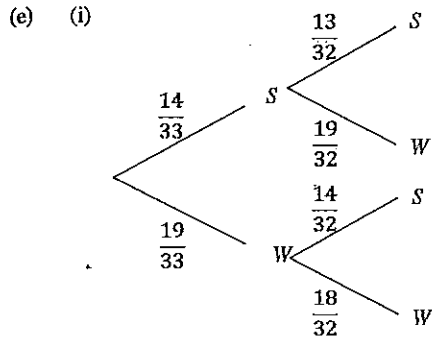
Question 27

(a) Total weekly deductions  
 $= \$296 \cdot 40 + \$24 \cdot 50 + \$15 \cdot 80$   
 $= \$336 \cdot 70$   
 Net weekly wage  
 $= \$1024 - \$336 \cdot 70$   
 $= \$687 \cdot 30$   
 Weekly household expenses  
 $= \frac{\$3640}{52}$   
 $= \$70$   
 % of net (weekly) wage paid in (w) household expenses  
 $= \frac{\$70}{\$687 \cdot 30} \times 100$   
 $\approx 10 \cdot 2\%$ .  
 (b)  $P = \frac{\theta^\lambda}{360^\theta} \times 2\pi r + 2r$   
 $= \frac{230}{360} \times 2 \times \pi \times 13 + 2 \times 13$   
 $\approx 52 \cdot 2 + 26$   
 $\approx 78$  cm.  
 (c) (i) Scale is 1 : 500 000  
 Actual distance =  $2 \times 500\ 000$  cm  
 $= 1\ 000\ 000$  cm  
 $= 10\ 000$  m  
 $= 10$  km.  
 (ii) Map distance =  $75 \div 500\ 000$  km  
 $= 0 \cdot 00015$  km  
 $= 0 \cdot 00015 \times 1000$  m  
 $= 0 \cdot 15$  m  
 $= 0 \cdot 15 \times 100$  cm  
 $= 15$  cm.





$$\begin{aligned} \tan 5^\circ &= \frac{39}{d+60} \\ d+60 &= \frac{39}{\tan 5^\circ} \\ &\approx 445.72 \\ d &= 445.72 - 60 \\ &= 386 \text{ cm.} \end{aligned}$$



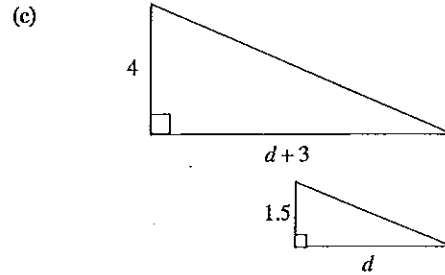
(ii)  $P(\text{both silk}) = \frac{14}{33} \times \frac{13}{32}$   
 $= \frac{91}{528}$

(iii)  $P(\text{diff. fabrics}) = \frac{14}{33} \times \frac{19}{32} + \frac{19}{32} \times \frac{14}{33}$   
 $= \frac{532}{1056}$   
 $= \frac{133}{264}$

**Question 28**

(a) Refer to the last page of these solutions as additional space is needed.

(b)  $\frac{18ab}{3a^2} \times \frac{c}{b} = \frac{18abc}{3a^2b}$   
 $= \frac{6c}{a}$



By similar triangles:  
 $\frac{d+3}{4} = \frac{d}{1.5}$   
 $4d = 1.5d + 4.5$   
 $2.5d = 4.5$   
 $d = 1.8 \text{ m.}$

OR

By scale factors:  
 Scale factor  $= \frac{4}{1.5}$   
 $= \frac{8}{3}$

$\therefore d+3 = \frac{8}{3} \times d$   
 $3(d+3) = 8d$   
 $3d+9 = 8d$   
 $5d = 9$   
 $d = 1.8 \text{ m.}$

(d) (i) From box-and-whisker plot for English:  
 Interquartile range  $= 80 - 50 = 30$ .

(ii) Students need to address the three components: skewness, location (median) and spread (range, interquartile range).  
 Answers could include:  
 Skewness:  
 • Both sets of scores are negatively skewed.  
 • English is more negatively skewed than Maths.

Location:  
 • Maths has a higher median (75)

than English (70).  
 • The lowest 25% of English scores are below the lowest Maths score.

Spread:

- Maths has a smaller range of scores (40) than English (85).
- Maths (15) has a smaller interquartile range than English (30).

(e) Deposit paid  $= 10\%$  of \$2800  
 $= \$280$   
 Balance owing  $= \$2800 - \$280$   
 $= \$2520$   
 Total repayments  $= \$95.20 \times 12 \times 3$   
 $= \$3427.20$   
 Interest paid  $= \$3427.20 - \$2520$   
 $= \$907.20$

Rate of simple interest:

Using:  $I = Prn$   
 $907.20 = 2520 \times r \times 3$   
 $r = \frac{907.2}{2520 \times 3}$   
 $r = 0.12$

$\therefore$  Interest rate paid was 12% pa.

**Question 29**

(a) (i) From the graph, the minimum time expected to wait after a 4 minute eruption (4 on horizontal axis) is 70 minutes.

(ii) From the graph, the longest possible duration of the first eruption of two that were 60 minutes apart (60 on vertical axis) is 3 minutes.

(iii) There is a positive correlation between the duration of the eruption and the time to the next eruption. i.e. the longer the eruption, the longer the time to the next eruption.

(b)  $\bar{x} = 6.000 \text{ cm}$ ,  $s = 0.040 \text{ cm}$ .  
 The range of 1 standard deviation from the mean is:  
 $6.400 - 0.040 \text{ cm}$  to  $6.400 + 0.040 \text{ cm}$   
 i.e.  $5.960 \text{ cm}$  to  $6.040 \text{ cm}$   
 Note that 2 lengths (5.950 cm and 6.050 cm) are outside this acceptable range. Therefore, the machine needs to be checked.

OR

Calculate the z-score for each length

$$z_1 = \frac{5.950 - 6.000}{0.040} = -1.25$$

$$z_2 = \frac{5.983 - 6.000}{0.040} = -0.425$$

$$z_3 = \frac{6.140 - 6.000}{0.040} = +3.50$$

Note that the two z-scores (-1.25 and -0.425) are outside the acceptable range of  $\pm 1$  standard deviation. Therefore, the machine needs to be checked.

(c) (i) Using the angle sum of a triangle:  
 $\angle EGF = 180^\circ - 139^\circ - 31^\circ$   
 $= 10^\circ$

By the sine rule:

$$\frac{EF}{\sin 10^\circ} = \frac{64}{\sin 31^\circ}$$

$$EF = \frac{64 \times \sin 10^\circ}{\sin 31^\circ}$$

$$EF \approx 22 \text{ km (nearest km)}$$

(ii) Using Pythagoras' Theorem in  $\triangle EBG$   
 Using  $EH = GH = x$

$$x^2 + x^2 = 82^2$$

$$2x^2 = 6724$$

$$x \approx 58.0 \text{ km}$$

Distance cycled  $= 64 + 22 + 58 + 58$   
 $d = 202 \text{ km (nearest km)}$

OR

Using trigonometry in  $\triangle EGH$ :

$$\sin 45^\circ = \frac{GH}{82}$$

$$GH = 82 \times \sin 45^\circ$$

$$GH \approx 58.0 \text{ km}$$

and  $BH \approx 58.0 \text{ km}$  (equal side length)

Distance cycled =  $64 + 22 + 58 + 58$

$$d = 202 \text{ km (nearest km).}$$

- (d) (i) Annual = 5% of \$81 000

$$= \frac{5}{100} \times 81000$$

$$= \$4080$$

Monthly =  $\$4080 \div 12$

$$= \$340.$$

- (ii) Using the future value formula, where

$$M = 340$$

$$r = 0.066 \div 12$$

$$= 0.0055$$

$$n = 13 \times 12$$

$$= 156$$

$$A = M \left\{ \frac{(1+r)^n - 1}{r} \right\}$$

$$= 340 \left\{ \frac{(1+0.0055)^{156} - 1}{0.0055} \right\}$$

$$= \$83633.89.$$

Question 30

- (a) Difference in latitude =  $4^\circ + 9^\circ = 13^\circ$

Using:  $1^\circ = 60$  nautical miles

Distance travelled =  $13 \times 60 = 780 \text{ M}$

Speed = 30 knots (nautical miles per hour)

$\therefore$  Time taken:

$$t = \frac{d}{v}$$

$$t = \frac{780}{30}$$

$$= 26 \text{ hours.}$$

- (b) (i) From the graph of the equation, the maximum height is 40 m.

- (ii) From the graph of the equation, when  $h = 35 \text{ m}$ ,  $d = 30 \text{ m}$  and  $170 \text{ m}$ .

Therefore, the ball travels a horizontal distance of  $170 - 30 = 140 \text{ metres}$ .

- (iii) When the ball still has 50 metres to travel it has already travelled  $300 - 50 = 250 \text{ metres}$ .

From the graph of the equation, when  $d = 250 \text{ m}$  the height is 17.5 metres.

OR

From the formula:

$$h = 30 + 0.2(250) - 0.001(250^2)$$

$$= 17.5 \text{ m.}$$

- (iv)  $d < 0$  is unsuitable because the ball commences its flight when  $d = 0$ . Also these values result in a negative distance which is impossible.  
 $d > 300$  is unsuitable because the ball concludes its flight when  $d = 300$ .

- (c) (i) From the graph, the predicted population in 2030 is 6 600 000.  
 (ii) A represents the initial population for this model. i.e. the population in 2010.  
 (iii) 1.05 is unsuitable because 1.05 is larger than 1.04 and therefore would yield a higher population growth when used for  $b$  in the formula  $P = A(b)^n$ .

OR

By substituting  $n = 25$  into

$$P = 3000000(1.05)^n \text{ the result is}$$

10 159 065 which is higher than the value of 8 000 000 read from the graph

$$\text{of } P = A(1.04)^n.$$

Therefore, 1.05 is unsuitable.

- (iii) (2) Using  $P = A(b)^n$  and substituting  $n = 20$  years,  $P = 4\,460\,000$ .

$$4460000 = 3000000(b)^{20}$$

$$(b)^{20} = \frac{4460000}{3000000}$$

$$\approx 1.487$$

Try  $b = 1.03$ :

$$(b)^{20} = (1.03)^{20}$$

$$\approx 1.806, \text{ too high}$$

Try  $b = 1.02$ :

$$(b)^{20} = (1.02)^{20}$$

$$\approx 1.486$$

Try  $b = 1.01$ :

$$(b)^{20} = (1.01)^{20}$$

$$\approx 1.220, \text{ too small}$$

$b = 1.02$  is the best estimate.

- (iv) In 2050,  $n = 40$  years.

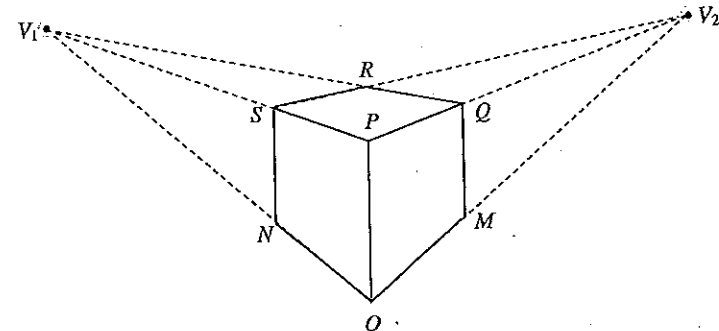
$$P = A(b)^n$$

$$= 3\,000\,000(1.02)^{40}$$

$$P \approx 6\,624\,000$$

This is below 7 000 000, so the aim is achieved.

Question 28 (a):



End of General Mathematics solutions