

BOARD OF STUDIES
NEW SOUTH WALES

2013

HIGHER SCHOOL CERTIFICATE
EXAMINATION

General Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – $2\frac{1}{2}$ hours
- Write using black or blue pen
Black pen is preferred
- Calculators may be used
- A formulae sheet is provided at the back of this paper
- In Questions 26–30, show relevant mathematical reasoning and/or calculations

Total marks – 100

Section I Pages 2–12

25 marks

- Attempt Questions 1–25
- Allow about 35 minutes for this section

Section II Pages 13–26

75 marks

- Attempt Questions 26–30
- Allow about 1 hour and 55 minutes for this section

Section I

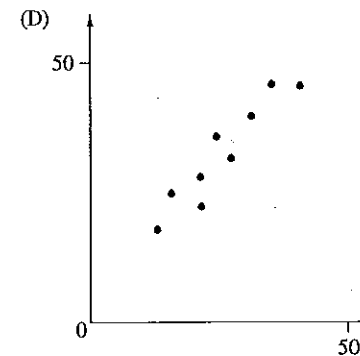
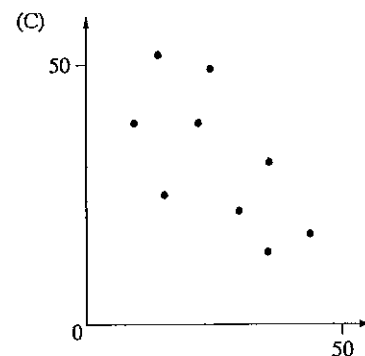
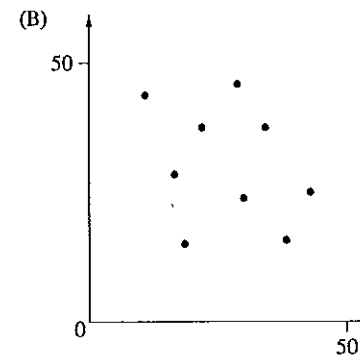
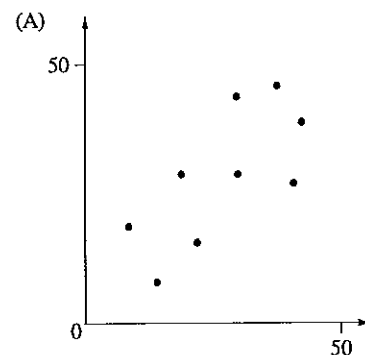
25 marks

Attempt Questions 1–25

Allow about 35 minutes for this section

Use the multiple-choice answer sheet for Questions 1–25.

- 1 Which of the following events would be LEAST likely to occur?
 - (A) Tossing a fair coin and obtaining a head
 - (B) Rolling a standard six-sided die and obtaining a 3
 - (C) Randomly selecting the letter 'G' from the 26 letters of the alphabet
 - (D) Winning first prize in a raffle of 100 tickets in which you have 4 tickets
- 2 Which graph best shows data with a correlation closest to 0.3?

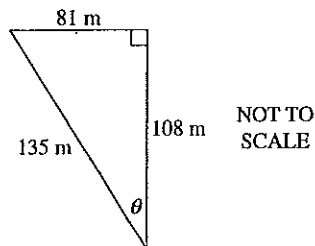


- 3 Luke's normal rate of pay is \$24.80 per hour. In one week he worked 14 hours at the normal rate, 4 hours at time-and-a-half, and $3\frac{1}{2}$ hours at double time. He was also paid a wet weather allowance of \$50 for the week.

What was his pay for the week?

- (A) \$583.20
- (B) \$620.40
- (C) \$669.60
- (D) \$719.60

- 4 What is the value of θ , to the nearest degree?



- (A) 31°
- (B) 37°
- (C) 49°
- (D) 53°

- 5 Which of the following is $\frac{12T^2}{3T \times 2W}$ in its simplest form?

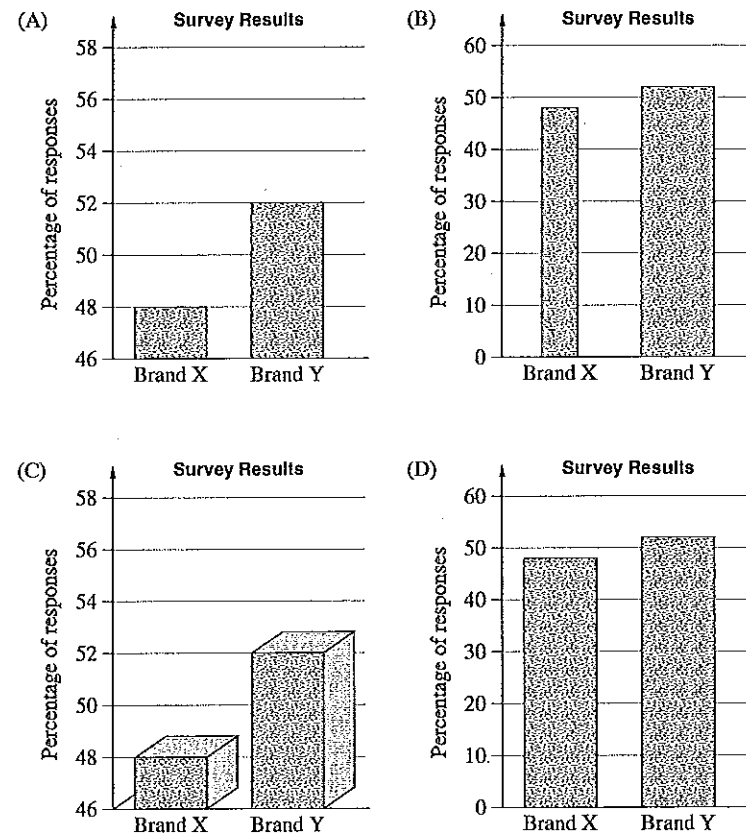
- (A) $\frac{2T}{W}$
- (B) $2TW$
- (C) $\frac{8T}{W}$
- (D) $8TW$

- 6 A survey was conducted where people were asked which of two brands of smartphones they preferred. The results were:

- 48% preferred Brand X
- 52% preferred Brand Y.

A graph displaying the data is to be included in a magazine article. The editor of the magazine wishes to ensure that the graph is not misleading in any way.

Which graph should the editor choose to include in the article?



- 7 In an experiment, a standard six-sided die was rolled 72 times. The results are shown in the table.

<i>Number on die</i>	<i>Frequency</i>
1	6
2	12
3	10
4	20
5	9
6	15

Which number on the die was obtained the expected number of times?

- (A) 1
 (B) 2
 (C) 3
 (D) 6
- 8 A high school has 100 students in each year group, Year 7 to Year 12. A survey is to be conducted to determine the average number of text messages sent per month by students at the school.
- Which of the following would provide the most representative sample for this survey?
- (A) All Year 7 students
 (B) All Physics students in Years 11 and 12
 (C) 20 students chosen at random from each year group
 (D) 120 students chosen at random from the school roll
- 9 Lynne invests \$1000 for a term of 15 months. Interest is paid at a flat rate of 3.75% per annum.

How much will Lynne's investment be worth at the end of the term?

- (A) \$1046.88
 (B) \$1047.09
 (C) \$1296.88
 (D) \$1468.75

- 10 Students studying vocational education courses were surveyed about their living arrangements.

	<i>Females</i>	<i>Males</i>	<i>Totals</i>
<i>Living with parent(s)</i>	46	155	201
<i>Not living with parent(s)</i>	182	122	304
<i>Totals</i>	228	277	505

One of these students is selected at random.

What is the probability that this student is male and living with his parent(s)?

- (A) 31%
 (B) 40%
 (C) 56%
 (D) 77%

- 11 An enterprise agreement has the following annual salary arrangements:

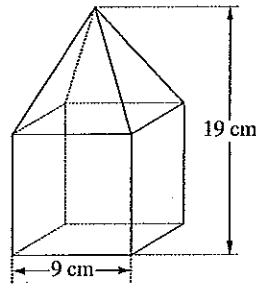
<i>Base Salary</i>		<i>Leadership Allowance</i>	
Step 1	\$35 000	Leader 1	\$5000
Step 2	\$40 000	Leader 2	\$7500
Step 3	\$45 000	Leader 3	\$10 000

George's employer pays 6% more than the enterprise agreement. He is on Step 3 and receives an allowance for Leader 2.

What is George's gross monthly pay?

- (A) \$4375.00
 (B) \$4412.50
 (C) \$4600.00
 (D) \$4637.50

- 12 A square pyramid fits exactly on top of a cube to form a solid.



NOT TO SCALE

What is the volume of the solid?

- (A) 513 cm^3
 (B) 999 cm^3
 (C) 1242 cm^3
 (D) 1539 cm^3
- 13 Polly borrowed \$11 000. She repaid the loan in full at the end of two years with a lump sum of \$12 000.

What annual simple interest rate was she charged?

- (A) 4.17%
 (B) 4.55%
 (C) 8.33%
 (D) 9.09%
- 14 The July sale prices for properties in a suburb were:

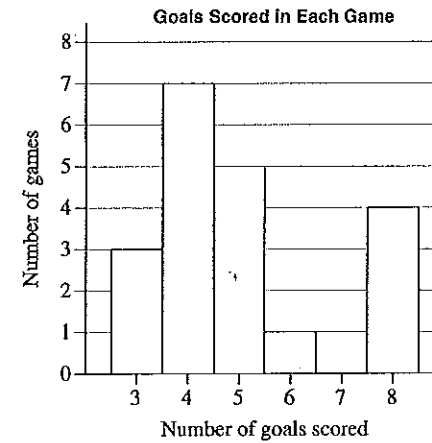
\$552 000, \$595 000, \$607 000, \$607 000, \$682 000 and \$685 000.

On 1 August, another property in the same suburb was sold for over one million dollars.

If this property had been sold in July, what effect would it have had on the mean and median sale prices for July?

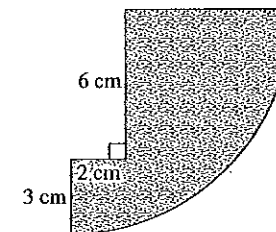
- (A) Both the mean and the median would have changed.
 (B) Neither the mean nor the median would have changed.
 (C) The mean would have changed and the median would have stayed the same.
 (D) The mean would have stayed the same and the median would have changed.

- 15 The frequency histogram shows the number of goals scored by a football team in each game in a season.



What was the mean number of goals scored per game by this team?

- (A) 4
 (B) 4.5
 (C) 5
 (D) 5.5
- 16 The shaded region shows a quadrant with a rectangle removed.

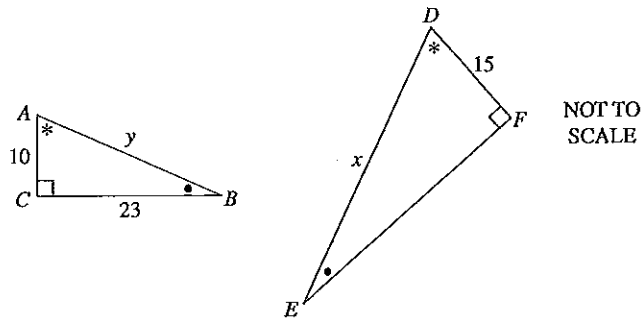


NOT TO SCALE

What is the area of the shaded region, to the nearest cm^2 ?

- (A) 38 cm^2
 (B) 52 cm^2
 (C) 61 cm^2
 (D) 70 cm^2

- 17 Triangles ABC and DEF are similar.



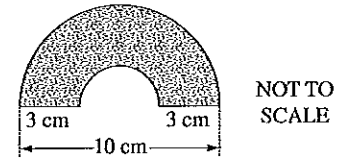
Which expression could be used to find the value of x ?

- (A) $y \times \frac{10}{15}$
 (B) $y \times \frac{10}{23}$
 (C) $y \times \frac{15}{10}$
 (D) $y \times \frac{23}{15}$
- 18 Two unbiased dice, each with faces numbered 1, 2, 3, 4, 5 and 6, are rolled.

What is the probability of obtaining a sum of 6?

- (A) $\frac{1}{6}$
 (B) $\frac{1}{12}$
 (C) $\frac{5}{12}$
 (D) $\frac{5}{36}$

- 19 A logo is designed using half of an annulus.



What is the area of the logo, to the nearest cm^2 ?

- (A) 25 cm^2
 (B) 33 cm^2
 (C) 132 cm^2
 (D) 143 cm^2
- 20 There are 60 000 students sitting a state-wide examination. If the results form a normal distribution, how many students would be expected to score a result between 1 and 2 standard deviations above the mean?

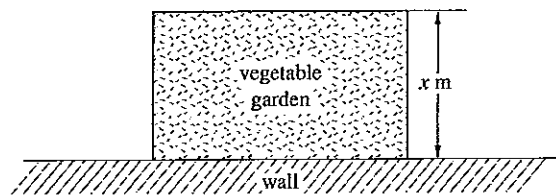
You may assume for normally distributed data that:

- 68% of scores have z-scores between -1 and 1
- 95% of scores have z-scores between -2 and 2
- 99.7% of scores have z-scores between -3 and 3 .

- (A) 8100
 (B) 16 200
 (C) 20 400
 (D) 28 500
- 21 Which equation correctly shows r as the subject of $S = 800(1 - r)$?

- (A) $r = \frac{800 - S}{800}$
 (B) $r = \frac{S - 800}{800}$
 (C) $r = 800 - S$
 (D) $r = S - 800$

- 22 Leanne wants to build a rectangular vegetable garden in her backyard. She has 20 metres of fencing and will use a wall as one side of the garden. The plan for her garden is shown, where x metres is the width of her garden.



NOT TO SCALE

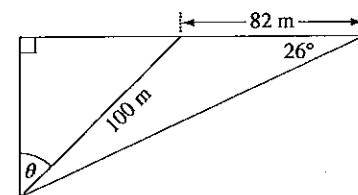
Which equation gives the area, A , of the vegetable garden?

- (A) $A = 10x - x^2$
 (B) $A = 10x - 2x^2$
 (C) $A = 20x - x^2$
 (D) $A = 20x - 2x^2$
- 23 Zina opened an account to save for a new car. Six months after opening the account, she made her first deposit of \$1200 and continued depositing \$1200 at the end of each six month period. Interest was paid at 3% per annum, compounded half-yearly.

How much was in Zina's account two years after first opening it?

- (A) \$4909.08
 (B) \$4982.72
 (C) \$5018.16
 (D) \$5094.55

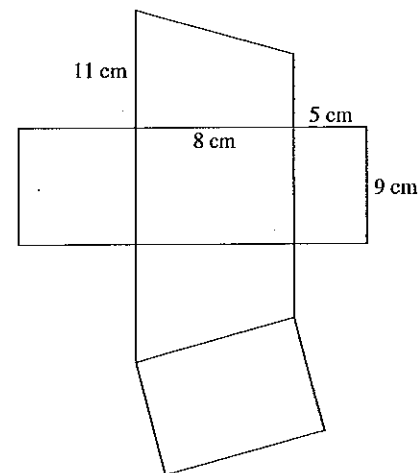
- 24 What is the value of θ , to the nearest degree?



NOT TO SCALE

- (A) 21°
 (B) 32°
 (C) 43°
 (D) 55°

- 25 A net is made using four rectangles and two trapeziums. It is folded to form a solid.



NOT TO SCALE

What is the volume of the solid, in cm^3 ?

- (A) 360 cm^3
 (B) 434 cm^3
 (C) 440 cm^3
 (D) 576 cm^3

Section II

75 marks

Attempt Questions 26–30

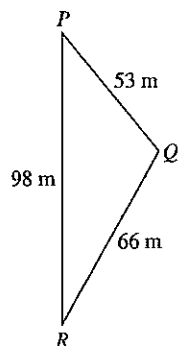
Allow about 1 hour and 55 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In Questions 26–30, your responses should include relevant mathematical reasoning and/or calculations.

Question 26 (15 marks) Use the Question 26 Writing Booklet.

(a) Triangle PQR is shown. 2



NOT TO SCALE

Find the size of angle Q , to the nearest degree.

(b) Write down a set of six data values that has a range of 12, a mode of 12 and a minimum value of 12. 2

Question 26 continues on page 14

Question 26 (continued)

(c) The probability that Michael will score more than 100 points in a game of bowling is $\frac{31}{40}$.

(i) A commentator states that the probability that Michael will score less than 100 points in a game of bowling is $\frac{9}{40}$. 1

Is the commentator correct? Give a reason for your answer.

(ii) Michael plays two games of bowling. What is the probability that he scores more than 100 points in the first game and then again in the second game? 1

(d) A section of Jim's electricity bill is shown.

Energy Used and Costs					
METER ID	THIS READING	LAST READING	ENERGY USED	RATE (per kWh)	COST
Peak Energy Charge					
TMV04221/01	531.2	274.8	256.4 kWh	47.7700c	\$122.48
Shoulder Energy Charge					
TMV04221/02	A	560.9	523.5 kWh	19.4000c	\$101.56
Off-peak (Night) Energy Charge					
TMV04221/03	242.5	0.0	242.5 kWh	9.6000c	\$23.28

(i) What is the value of A ? 1

(ii) How much will Jim save if he uses 154 kWh of energy at the Off-peak rate rather than at the Peak rate? 2

Question 26 continues on page 15

Question 26 (continued)

- (e) Kimberley has invested \$3500.

Interest is compounded half-yearly at a rate of 2% per half-year.

Compounded values of \$1

Period	Interest rate per period					
	1%	2%	3%	4%	5%	6%
1	1.010	1.02	1.03	1.04	1.05	1.06
2	1.020	1.040	1.061	1.082	1.103	1.124
3	1.030	1.061	1.093	1.125	1.158	1.191
4	1.041	1.082	1.126	1.170	1.216	1.262
5	1.051	1.104	1.159	1.217	1.276	1.338
6	1.062	1.126	1.194	1.265	1.340	1.419
7	1.072	1.149	1.230	1.316	1.407	1.504
8	1.083	1.172	1.267	1.369	1.477	1.594

Use the table to calculate the value of her investment at the end of 4 years.

- (f) Jason travels to work by car on all five days of his working week, leaving home at 7 am each day. He compares his travel times using roads without tolls and roads with tolls over a period of 12 working weeks.

He records his travel times (in minutes) in a back-to-back stem-and-leaf plot.

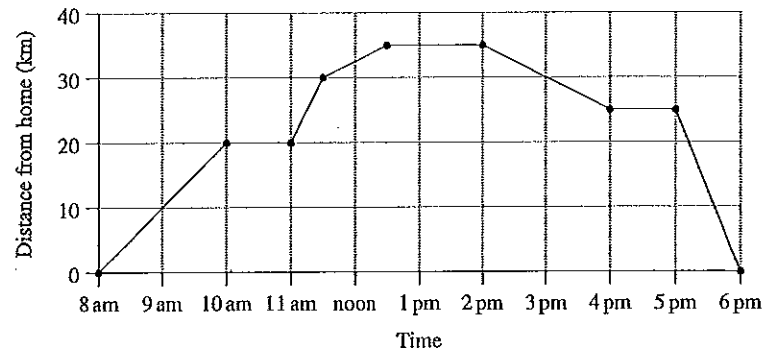
Travel time (minutes)		
Without tolls		With tolls
	9	3 5 8 9 9
9 9 8 7 7 6 5 5 4 4 3 2 0	4	0 1 2 6 7 7 8 8 8 9
9 8 7 5 4 3 3 3 2 2 2 1 1 0	5	2 4 4 5 6 8 9
	1	6 1 3 5 7
		7 0 2 8
		8 2
		9 0

- (i) What is the modal travel time when he uses roads without tolls? 1
- (ii) What is the median travel time when he uses roads without tolls? 1
- (iii) Describe how the two data sets differ in terms of the spread and skewness of their distributions. 2

End of Question 26

Question 27 (15 marks) Use the Question 27 Writing Booklet.

- (a) Lucy went for a bike ride. She left home at 8 am and arrived back at home at 6 pm. A graph representing her journey is shown.



- (i) What was the total distance that she rode during the day? 1
- (ii) How much time did Lucy spend riding her bike during the day? 1
- (b) The table shows the tax payable to the Australian Taxation Office for different taxable incomes. 4

Taxable income	Tax on this income
\$0 – \$18 200	Nil
\$18 201 – \$37 000	19c for each \$1 over \$18 200
\$37 001 – \$80 000	\$3572 plus 32.5c for each \$1 over \$37 000
\$80 001 – \$180 000	\$17 547 plus 37c for each \$1 over \$80 000
\$180 001 and over	\$54 547 plus 45c for each \$1 over \$180 000

Acknowledgment: © Australian Taxation Office for the Commonwealth of Australia

Peta has a gross annual salary of \$84 000. She has tax deductions of \$1000 for work-related travel and \$500 for stationery. The Medicare levy that she pays is calculated at 1.5% of her taxable income.

Peta has already paid \$18 500 in tax.

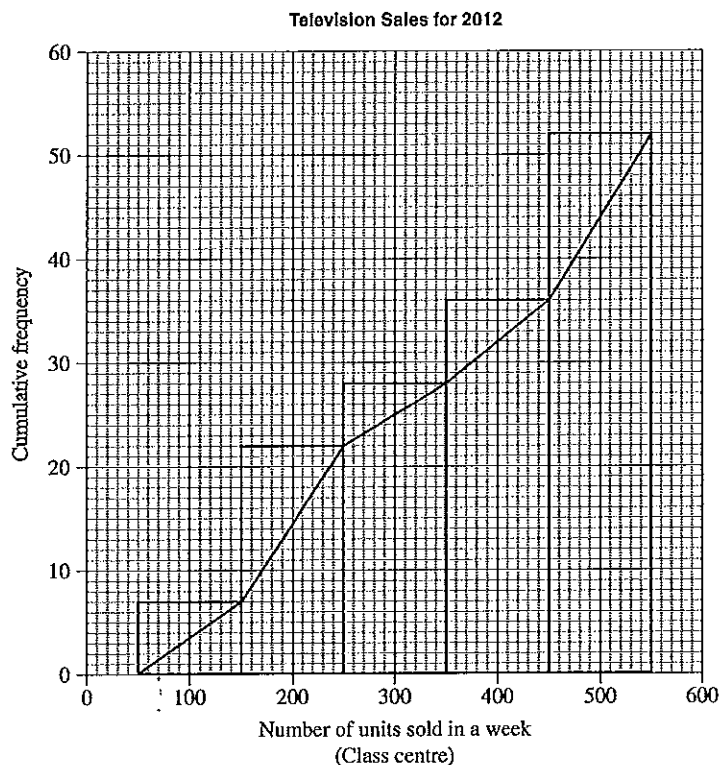
Will Peta receive a tax refund or will she owe money to the Australian Taxation Office? Justify your answer by calculating the refund or amount owed.

Question 27 continues on page 17

Question 27 (continued)

- (c) A retailer has collected data on the number of televisions that he sold each week in 2012.

He grouped the data into classes and displayed the data using a cumulative frequency histogram and polygon (ogive).



- (i) Use the cumulative frequency polygon to determine the interquartile range. 2
- (ii) Oscar said that the retailer sold 300 televisions in 6 of the weeks in 2012. 1
Is he correct? Give a reason for your answer.

Question 27 continues on page 18

Question 27 (continued)

- (d) A rectangular wooden chopping board is advertised as being 17 cm by 25 cm, with each side measured to the nearest centimetre.

- (i) Calculate the percentage error in the measurement of the longer side. 1
- (ii) Between what lower and upper limits does the actual area of the top of the chopping board lie? 2

- (e) Karin is in Athens, which is two hours ahead of Greenwich Mean Time. Marco is in New York, which is five hours behind Greenwich Mean Time.

- (i) Karin is going to ring Marco at 10 pm on Tuesday, Athens time. 1

What day and time will it be in New York when she rings?

- (ii) Marco is going to fly from New York to Athens. His flight will leave on Wednesday at 9 am, New York time, and will take 11 hours. 2

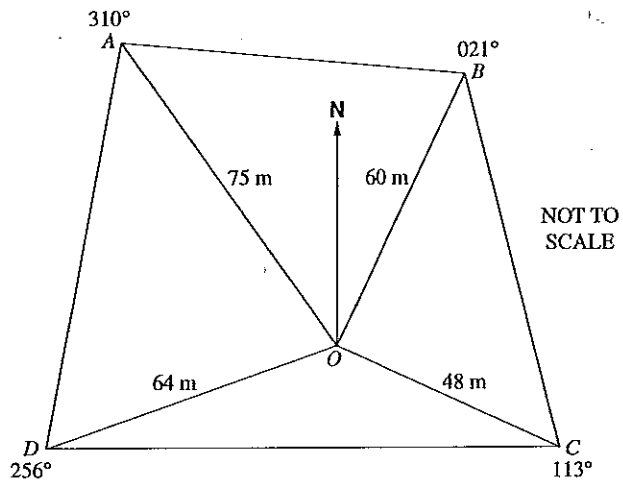
What day and time will it be in Athens when he arrives?

End of Question 27

Question 28 (15 marks) Use the Question 28 Writing Booklet.

(a) A compass radial survey of the field $ABCD$ has been conducted from O .

2

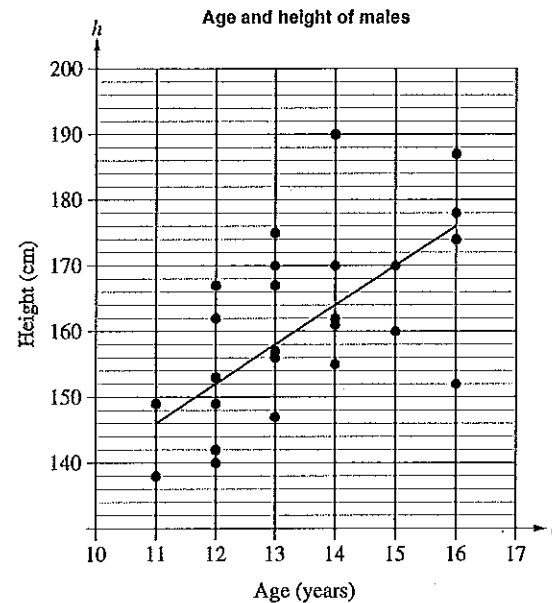


Find the area of the section ABO , to the nearest square metre.

Question 28 continues on page 20

Question 28 (continued)

(b) Ahmed collected data on the age (a) and height (h) of males aged 11 to 16 years. He created a scatterplot of the data and constructed a line of best fit to model the relationship between the age and height of males.



- (i) Determine the gradient of the line of best fit shown on the graph. 1
- (ii) Explain the meaning of the gradient in the context of the data. 1
- (iii) Determine the equation of the line of best fit shown on the graph. 2
- (iv) Use the line of best fit to predict the height of a typical 17-year-old male. 1
- (v) Why would this model not be useful for predicting the height of a typical 45-year-old male? 1

Question 28 continues on page 21

Question 28 (continued)

- (c) A ship sails due South from Channel-Port-aux-Basques, Canada, 47°N 59°W to Barbados, 13°N 59°W. 2

How far did the ship sail, to the nearest kilometre? Assume that the radius of Earth is 6400 km.

- (d) Adhele has 2000 shares. The current share price is \$1.50 per share. Adhele is paid a dividend of \$0.30 per share.

(i) What is the current value of her shares? 1

(ii) Calculate the dividend yield. 1

- (e) Zheng has purchased a computer for \$5000 for his company. He wants to compare two different methods of depreciation over two years for the computer. 3

Method 1: Straight-line with \$1250 depreciation per annum.

Method 2: Declining balance with 35% depreciation per annum.

Which method gives the greatest depreciation over the two years? Justify your answer with suitable calculations.

End of Question 28

Question 29 (15 marks) Use the Question 29 Writing Booklet.

- (a) Sarah tried to solve this equation and made a mistake in Line 2. 2

$$\frac{W+4}{3} - \frac{2W-1}{5} = 1 \dots\dots\dots \text{Line 1}$$

$$5W + 20 - 6W - 3 = 15 \dots\dots\dots \text{Line 2}$$

$$17 - W = 15 \dots\dots\dots \text{Line 3}$$

$$W = 2 \dots\dots\dots \text{Line 4}$$

Copy the equation in Line 1 into your writing booklet and continue your solution to solve this equation for W . Show all lines of working.

- (b) Ali's class sits two Geography tests. The results of her class on the first Geography test are shown.

58, 74, 65, 66, 73, 71, 72, 74, 62, 70

The mean was 68.5 for the first test.

- (i) Calculate the standard deviation for the first test. Give your answer correct to one decimal place. 1

- (ii) On the second Geography test, the mean for the class was 74.4 and the standard deviation was 12.4. 3

Ali scored 62 on the first test. Calculate the mark that she needed to obtain in the second test to ensure that her performance relative to the class was maintained.

Question 29 continues on page 23

Question 29 (continued)

- (c) Mary is designing a website that requires unique logins to be generated. 2

She plans to generate the logins using two capital letters from the alphabet followed by a series of numerals from 0 to 9 inclusive. All logins will have the same number of numerals. Repetition of letters and numerals is allowed.

What is the minimum number of numerals required for each login so that Mary can generate at least 3 million logins? Justify your answer with suitable calculations.

- (d) Jane plays a game which involves two coins being tossed. The amounts to be won for the different possible outcomes are shown in the table. 3

Win \$6 for two heads
Win \$1 for one head and one tail
Win \$2 for two tails

It costs \$4 to play one game. Will Jane expect a gain or a loss, and how much will it be? Justify your answer with suitable calculations.

- (e) Jack borrowed \$300 000 to buy a house. Interest is charged at the rate of 6% per annum, compounded monthly. His minimum monthly repayment is \$1798.65. 4

Jack decided to pay an extra \$250 each month from the start of the loan period.

Will he be able to pay off the loan in 20 years? Justify your answer with suitable calculations.

End of Question 29

Question 30 (15 marks) Use the Question 30 Writing Booklet.

- (a) Wind turbines, such as those shown, are used to generate power.



Acknowledgement: Stock Photo - Wind Turbines -- Image ID: 1051412 © Miguel Saavedra

In theory, the power that could be generated by a wind turbine is modelled using the equation

$$T = 20\,000w^3$$

where T is the theoretical power generated, in watts
 w is the speed of the wind, in metres per second.

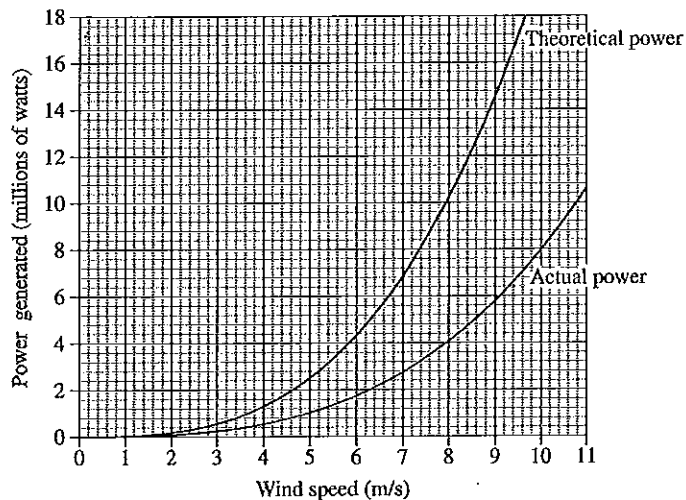
- (i) Using this equation, what is the theoretical power generated by a wind turbine if the wind speed is 7.3 m/s? 1
- (ii) In practice, the actual power generated by a wind turbine is only 40% of the theoretical power. 1

If A is the actual power generated, in watts, write an equation for A in terms of w .

Question 30 continues on page 25

Question 30 (continued)

The graph shows both the theoretical power generated and the actual power generated by a particular wind turbine.



- (iii) Using the graph, or otherwise, find the difference between the theoretical power and the actual power generated when the wind speed is 9 m/s. 1
- (iv) A particular farm requires at least 4.4 million watts of actual power in order to be self-sufficient. 1
 What is the minimum wind speed required for the farm to be self-sufficient?
- (v) A more accurate formula to calculate the power (P) generated by a wind turbine is 3

$$P = 0.61 \times \pi \times r^2 \times w^3$$

where r is the length of each blade, in metres
 w is the speed of the wind, in metres per second.

Each blade of a particular wind turbine has a length of 43 metres. The turbine operates at a wind speed of 8 m/s.

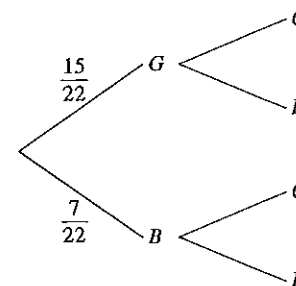
Using the formula above, if the wind speed increased by 10%, what would be the percentage increase in the power generated by this wind turbine?

Question 30 continues on page 26

Question 30 (continued)

- (b) In a class there are 15 girls (G) and 7 boys (B). Two students are chosen at random to be class representatives.

- (i) Copy and complete the tree diagram in your answer booklet. 2



- (ii) What is the probability that the two students chosen are of the same gender? 2

- (c) Joel mixes petrol and oil in the ratio 40:1 to make fuel for his leaf blower.

- (i) Joel pours 5 litres of petrol into an empty container to make fuel for his leaf blower. 1

How much oil should he add to the petrol to ensure that the fuel is in the correct ratio?

- (ii) Joel has 4.1 litres of fuel left in his container after filling his leaf blower. 3

He wishes to use this fuel in his lawnmower. However, his lawnmower requires the petrol and oil to be mixed in the ratio 25:1.

How much oil should he add to the container so that the fuel is in the correct ratio for his lawnmower?

End of paper

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FORMULAE SHEET

Area of an annulus

$$A = \pi(R^2 - r^2)$$

R = radius of outer circle

r = radius of inner circle

Area of an ellipse

$$A = \pi ab$$

a = length of semi-major axis

b = length of semi-minor axis

Area of a sector

$$A = \frac{\theta}{360} \pi r^2$$

θ = number of degrees in central angle

Arc length of a circle

$$l = \frac{\theta}{360} 2\pi r$$

θ = number of degrees in central angle

Simpson's rule for area approximation

$$A \approx \frac{h}{3}(d_f + 4d_m + d_l)$$

h = distance between successive measurements

d_f = first measurement

d_m = middle measurement

d_l = last measurement

Surface area

Sphere $A = 4\pi r^2$

Closed cylinder $A = 2\pi rh + 2\pi r^2$

r = radius

h = perpendicular height

Volume

Cone $V = \frac{1}{3}\pi r^2 h$

Cylinder $V = \pi r^2 h$

Pyramid $V = \frac{1}{3}Ah$

Sphere $V = \frac{4}{3}\pi r^3$

r = radius

h = perpendicular height

A = area of base

Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Area of a triangle

$$A = \frac{1}{2}ab \sin C$$

Cosine rule

$$c^2 = a^2 + b^2 - 2ab \cos C$$

or

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

FORMULAE SHEET

Simple Interest

$$I = Prn$$

P = initial quantity

r = percentage interest rate per period, expressed as a decimal

n = number of periods

Compound interest

$$A = P(1+r)^n$$

A = final balance

P = initial quantity

n = number of compounding periods

r = percentage interest rate per compounding period, expressed as a decimal

Future value (A) of an annuity

$$A = M \left\{ \frac{(1+r)^n - 1}{r} \right\}$$

M = contribution per period, paid at the end of the period

Present value (N) of an annuity

$$N = M \left\{ \frac{(1+r)^n - 1}{r(1+r)^n} \right\}$$

or

$$N = \frac{A}{(1+r)^n}$$

Straight-line formula for depreciation

$$S = V_0 - Dn$$

S = salvage value of asset after n periods

V_0 = purchase price of the asset

D = amount of depreciation apportioned per period

n = number of periods

Declining balance formula for depreciation

$$S = V_0(1-r)^n$$

S = salvage value of asset after n periods

r = percentage interest rate per period, expressed as a decimal

Mean of a sample

$$\bar{x} = \frac{\sum x}{n}$$

$$\bar{x} = \frac{\sum fx}{\sum f}$$

\bar{x} = mean

x = individual score

n = number of scores

f = frequency

Formula for a z-score

$$z = \frac{x - \bar{x}}{s}$$

s = standard deviation

Gradient of a straight line

$$m = \frac{\text{vertical change in position}}{\text{horizontal change in position}}$$

Gradient-intercept form of a straight line

$$y = mx + b$$

m = gradient

b = y-intercept

Probability of an event

The probability of an event where outcomes are equally likely is given by:

$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

2013 Higher School Certificate Solutions General Mathematics

SECTION I

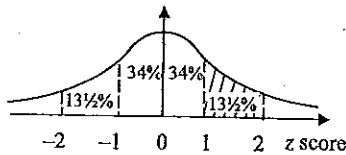
Summary

1	C	8	C	14	C	20	A
2	A	9	A	15	C	21	A
3	D	10	A	16	B	22	D
4	B	11	D	17	C	23	A
5	A	12	B	18	D	24	C
6	D	13	B	19	B	25	D
7	B						

SECTION I

- 1 C $P(\text{Letter 'G'}) = \frac{1}{26}$
This is less than A, B and D and therefore is the LEAST likely.
- 2 A B and C have negative correlation and D is very close to 1. A is the only answer that has a low positive correlation.
- 3 D $\text{Pay} = (14 + 4 \times 1.5 + 3.5 \times 2) \times 24.80 + 50$
 $= 27 \times 24.80 + 50$
 $= \$719.60.$
- 4 B $\cos \theta = \frac{108}{135}, \sin \theta = \frac{81}{135}, \tan \theta = \frac{81}{108}$
All ratios lead to $\theta = 37^\circ.$
- 5 A $\frac{2 \sqrt{T}^\lambda}{\sqrt{K} \times \sqrt{W}} = \frac{2T}{W}$
- 6 D A and C are distorted because the vertical axis starts at 46 and not 0. B is distorted because the Brand Y column is wider than Brand X.

- 7 B The expected value of any number on a standard 6 sided die rolled 72 times is $\frac{1}{6} \times 72 = 12.$
The value 2 occurred 12 times.
- 8 C Since there are equal numbers of students in each year then the most representative sample would be to take equal numbers from each year group.
- 9 A $\text{Total} = \frac{3.75}{100} \times 1000 \times \frac{15}{12} + 1000$
 $= \$1046.88.$
- 10 A $P(\text{male living with parents}) = \frac{155}{505}$
 $\approx 31\%.$
- 11 D $\text{Monthly pay} = \frac{(45000 + 7500) \times 1.06}{12}$
 $= \$4637.50.$
- 12 B $\text{Volume} = 9^3 + \frac{1}{3} \times 9 \times 9 \times (19 - 9)$
 $= 729 + 270$
 $= 999 \text{ cm}^3.$
- 13 B $\text{Interest rate} = \frac{1000}{11000} \times 100 + 2$
 $\approx 4.55\%.$
- 14 C The extra property is an outlier. The mean would increase but the median would be unaffected.

- 15 C $\text{Mean} = \frac{3 \times 3 + 4 \times 7 + 5 \times 5 + 6 \times 1 + 8 \times 4}{3 + 7 + 5 + 1 + 4}$
 $= 5.$
- 16 B $\text{Area} = \frac{1}{4} \pi \times 9^2 - 12$
 $\approx 52 \text{ cm}^2.$
- 17 C $\frac{x}{y} = \frac{15}{10}$
 $x = y \times \frac{15}{10}.$
- 18 D There are 5 favorable outcomes from the 36 possibilities (1,5), (2,4), (3,3), (4,2), (5,1).
- 19 B $A = \frac{1}{2} \pi (R^2 - r^2)$
 $= \frac{1}{2} \pi (5^2 - 2^2)$
 $= \frac{1}{2} \times \pi \times 21$
 $\approx 33 \text{ cm}^2.$
- 20 A **Method 1:**
Using a graph
- 
- 2 -1 0 1 2 z score
- No. of students $= 13\frac{1}{2}\% \times 60\,000$
 $= 8100.$
- OR
- Method 2:**
 $\left(\frac{95\% - 68\%}{2} \right) \times 60000 = 8100.$

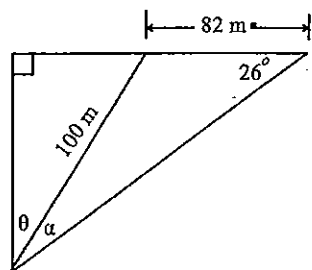
- 21 A **Method 1:**
 $S = 800(1-r)$
 $\frac{S}{800} = 1-r$
 $\frac{S}{800} + r = 1$
 $r = 1 - \frac{S}{800}$
 $r = \frac{800 - S}{800}.$

OR

- Method 2:**
 $S = 800(1-r)$
 $S = 800 - 800r$
 $S + 800r = 800$
 $800r = 800 - S$
 $r = \frac{800 - S}{800}.$

- 22 D Let the length be ℓ and from the diagram $\ell = 20 - 2x.$
 $A = \ell b$
 $= (20 - 2x)x$
 $= 20x - 2x^2.$
- 23 A $A = M \left[\frac{(1+r)^n + 1}{r} \right]$
 $= 1200 \left[\frac{(1+0.015)^4 + 1}{0.015} \right]$
 $\approx \$4909.08.$

24 C



$$\frac{\sin \alpha}{82} = \frac{\sin 26}{100}$$

$$\sin \alpha = \frac{82 \sin 26}{100}$$

$$= 0.35946\dots$$

$$\therefore \alpha = 21.07^\circ \text{ (2 dp)}$$

$$26^\circ + 90^\circ + (\theta + \alpha) = 180^\circ$$

$$\theta + \alpha = 180^\circ - 90^\circ - 26^\circ$$

$$= 64^\circ$$

$$\theta = 64 - 21$$

$$\approx 43^\circ$$

25 D Area of trapezium:

$$A = \frac{h}{2}(a+b)$$

$$= \frac{8}{2}(5+11)$$

$$= 64$$

Volume of a prism:

$$V = \text{area} \times \text{perpendicular height}$$

$$= 64 \times 9$$

$$= 576 \text{ cm}^3$$

SECTION II

Question 26

$$(a) \quad \cos Q = \frac{66^2 + 53^2 - 98^2}{2 \times 66 \times 53}$$

$$= -0.3486\dots$$

$$Q = 110^\circ \text{ (nearest degree)}$$

(b) One possible answer is:
12, 12, 12, 12, 13, 24.

Other answers are of the form 12, *a*, *b*, *c*, *d*, 24 where *a*, *b*, *c* and *d* are between 12 and 24 inclusive with 12 repeated to create the mode.

(c) (i) If it is possible to score exactly 100 points, the commentator is incorrect. The commentator should say less than or equal to 100 points.

$$(ii) \quad P(> 100 \text{ twice}) = \left(\frac{31}{40}\right)^2 = \frac{961}{1600}$$

$$(d) (i) \quad 523.5 + 560.9 = 1084.4$$

(ii) Values are rounded to nearest cent.
Off Peak = $154 \times 9.6c = \$14.78$ (2 dp)
Peak = $154 \times 47.77c = \$73.57$ (2 dp)
Saving = $\$73.57 - \$14.78 = \$58.78$ (2 dp).

$$(e) \quad \text{Total of investment} = 3500 \times 1.172 = \$4102$$

(f) (i) The mode is 52 minutes.

$$(ii) \quad \frac{50 + 51}{2} = 50.5$$

\therefore the median is 50.5 minutes.

(iii) **Spread:** The 'without tolls' data ranges from 39 to 61, which is very bunched, compared to 'with tolls', which ranges from 35 to 90.

Skewness: The 'without tolls' data is almost symmetrical, whereas the 'with tolls' data shows positive skew.

Question 27

(a) (i) Lucy rides 35km out and 35km back. Total distance is 70 km.

(ii) **Method 1:**
Lucy took 3 breaks in the day of 1 hour, $1\frac{1}{2}$ hours and 1 hour.

$$\text{Time riding} = 10 - 3\frac{1}{2} = 6\frac{1}{2} \text{ hours}$$

OR

Method 2:
Use the sections where distance is changing with time.

$$\text{Time riding} = 2 + 1\frac{1}{2} + 2 + 1 = 6\frac{1}{2} \text{ hours}$$

$$(b) (i) \quad \text{Taxable income} = \$84\,000 - \left(\begin{array}{l} \$1000 \\ + \$500 \end{array} \right) = \$82\,500$$

From the table:

$$\begin{aligned} \text{Tax on taxable income} &= (82500 - 80000) \times 0.37 + 17547 \\ &= 925 + 17547 \\ &= \$18\,472 \end{aligned}$$

$$\begin{aligned} \text{Medicare levy} &= 1.5\% \text{ of } 82\,500 \\ &= 0.015 \times 82\,500 \\ &= \$1237.50 \end{aligned}$$

$$\begin{aligned} \text{Total tax payable} &= 18472 + 1237.50 \\ &= \$19\,709.50 \end{aligned}$$

Since this is greater than what Peta had already paid, she owes money (she has a balance payable):

$$\begin{aligned} \text{Amount owed} &= 19709.50 - 18500 \\ &= \$1209.50 \end{aligned}$$

(c) (i) The total number of weeks is 52. \therefore Each quarter is 13 weeks.

From the cumulative frequency polygon (ogive):

$$\begin{aligned} \text{Interquartile range} &= Q_3 - Q_1 \\ &= 470 - 190 \\ &= 280 \end{aligned}$$

(ii) From the graph:

Class Centre	Frequency
100	7
200	$22 - 7 = 15$
300	$27 - 22 = 5$
400	$36 - 28 = 8$
500	$52 - 36 = 16$

The number 300 in the table above is the class centre. It is used to represent data values from 250 up to (but not including) 350. It is possible that all 6 data values were 300, but this is unlikely, so Oscar is probably incorrect.

(d) (i) Since each side is measured to the nearest centimeter the absolute error is 0.5 cm.

$$\begin{aligned} \% \text{ error} &= \frac{\text{Absolute error}}{\text{Measurement}} \times 100 \\ &= \frac{0.5}{25} \times 100 \\ &= 2\% \end{aligned}$$

$$(ii) \quad \begin{aligned} \text{Lower limit of area} &= 16.5 \times 24.5 \\ &= 404.25 \text{ cm}^2 \end{aligned}$$

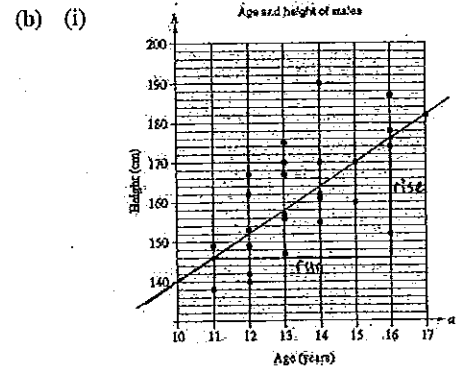
$$\begin{aligned} \text{Upper limit of area} &= 17.5 \times 25.5 \\ &= 446.25 \text{ cm}^2 \end{aligned}$$

The lower and upper limits for the actual area are 404.25 cm^2 and 446.25 cm^2 .

- (e) (i) Athens is 2 hours ahead of GMT and New York is 5 hours behind GMT.
 \therefore New York is 7 hours behind Athens.
 \therefore 10pm Tuesday in Athens is 3pm Tuesday in New York.
- (ii) When the plane leaves New York at 9 am Wednesday, it is 4 pm Wednesday in Athens.
 From that, 11 hours later, it will be 3 am Thursday in Athens.

Question 28

(a) $\angle AOB = 360 - 310 + 21 = 71^\circ$
 $A = \frac{1}{2} ab \sin C$
 $= \frac{1}{2} \times 75 \times 60 \times \sin 71^\circ$
 $= 2127.416\dots$
 $= 2127 \text{ m}^2$ (nearest m^2).



From the graph, use the points (11, 146) and (15, 170)

Gradient = $\frac{\text{rise}}{\text{run}}$
 $= \frac{170 - 146}{15 - 11}$
 $= 6.$

- (ii) The gradient represents the average rate of increase of the height of boys aged 11 to 16 years. This turns out to be 6 cm per year.

(iii) Using $y = mx + b$ with $m = 6$
 $y = 6x + b$
 Using the point (10, 140)
 $140 = 6(10) + b$
 $140 = 60 + b$
 $b = 140 - 60$
 $= 80$
 Thus the equation is of the form
 $y = 6x + 80$
 When using the correct variables, this becomes:
 $h = 6a + 80.$

- (iv) For a 17 year old male substitute
 $a = 17$
 $h = 6(17) + 80$
 $= 182 \text{ cm}.$
- (v) The scatterplot only includes males aged 11 to 16 years. Also, the rate of growth is not constant up to 45 years of age.

(c) The ship is sailing along a great circle. The difference in latitude is given by: $\theta = 47^\circ - 13^\circ$
 $= 34^\circ$
 The length of the arc is
 $l = \frac{\theta}{360} \times 2\pi r$
 $= \frac{34}{360} \times 2\pi \times 6400$
 $= 3798 \text{ km}$ (nearest km).

- (d) (i) Value of shares = 2000×1.50
 $= \$3000.$
- (ii) Dividend yield = $\frac{0.30}{1.50} \times 100$
 $= 20\%.$

(e) Straight-line method:
 $S = V_0 - Dn$
 $= 5000 - 1250 \times 2$
 $= \$2500$
 \therefore Depreciation = $\$5000 - \2500
 $= \$2500$

Declining balance method:
 $S = V_0(1 - r)^n$
 $= 5000(1 - 0.35)^2$
 $= \$2112.50$
 \therefore Depreciation = $\$5000 - \2112.50
 $= \$2887.50$
 The declining balance method gives a greater depreciation over 2 years.

Question 29

(a) $\frac{W+4}{3} - \frac{2W-1}{5} = 1$
 $\frac{5(W+4) - 3(2W-1)}{15} = 1$
 $\frac{5W+20-6W+3}{15} = 1$
 $-W+23=15$
 $-W=-8$
 $W=8.$

(b) (i) Using a calculator, the population standard deviation is 5.2 (1 dp).

(ii) For the first test:
 $z_1 = \frac{62 - 68.5}{5.2}$
 $= -1.25$

For the second test:
 $z_2 = \frac{x - 74.4}{12.4}$
 $-1.25 = \frac{x - 74.4}{12.4}$
 $-15.5 = x - 74.4$
 $x = 74.4 - 15.5$
 $= 58.9$

Ali needs to score 59 marks to maintain her relative performance.

- (c) **Method 1:**
 Using guess, check and refine:
 Each letter has 26 possibilities whereas each digit has 10 possibilities.
 2 letters and 2 numerals:
 $26 \times 26 \times 10 \times 10 = 67\ 600$
 \therefore too small.
 2 letters and 3 numerals:
 $26 \times 26 \times 10 \times 10 \times 10 = 676\ 000$
 \therefore too small.
 2 letters and 4 numerals:
 $26 \times 26 \times 10 \times 10 \times 10 \times 10 = 6\ 760\ 000$
 \therefore Mary needs to use at least 4 numerals to generate 3 million possibilities.

OR

- Method 2:**
 The total needs to exceed 3 million, that is 3×10^6 . So:
 $26^2 \times 10^n > 3 \times 10^6$
 $10^n > \frac{3 \times 10^6}{26^2}$
 $10^n > 4437\dots$
 By trial and error:
 $10^3 = 1000$ too small
 $10^4 = 10000$ sufficient
 $\therefore n = 4$
 \therefore 4 numerals are required.

(d)

Outcome	Bet	Collect	Profit	Probability
HH	\$4	\$6	+\$2	$\frac{1}{4}$
HT	\$4	\$1	-\$3	$\frac{1}{4}$
TH	\$4	\$1	-\$3	$\frac{1}{4}$
TT	\$4	\$2	-\$2	$\frac{1}{4}$

Method 1:

Financial expectation:

$$= \$2 \times \frac{1}{4} - \$3 \times \frac{1}{4} - \$3 \times \frac{1}{4} - \$2 \times \frac{1}{4}$$

$$= -\$1.50$$

In the long run, Jane will lose \$1.50, on average, every time she plays this game.

OR

Method 2:

Expected return

$$= \$6 \times \frac{1}{4} + \$1 \times \frac{1}{4} + \$1 \times \frac{1}{4} + \$2 \times \frac{1}{4} - \$4$$

$$= -\$1.50$$

∴ Jane expects a loss of \$1.50.

(e)

Method 1:

Using the present value formula one method is to see how much Jack can borrow using the given values and see if this is at least \$300 000.

$$N = 300\,000$$

$$r = \frac{0.06}{12} = 0.005$$

$$n = 12 \times 20 = 240$$

$$N = M \left\{ \frac{(1+r)^n - 1}{r(1+r)^n} \right\}$$

$$= (1798.65 + 250) \left\{ \frac{(1+0.005)^{240} - 1}{0.005(1+0.005)^{240}} \right\}$$

$$= 2048.65 \left\{ \frac{2.3102\dots}{0.005 \times 3.3102\dots} \right\}$$

$$= \$285\,952.15 \text{ (nearest cent)}$$

Since this value is less than \$300 000 then Jack cannot pay the loan off in 20 years.

OR

Method 2:

Use the present value formula determine if the required payment to pay the \$300 000 loan off in 20 years.

$$N = 300\,000$$

$$r = \frac{0.06}{12} = 0.005$$

$$n = 12 \times 20 = 240$$

$$N = M \left\{ \frac{(1+r)^n - 1}{r(1+r)^n} \right\}$$

$$300\,000 = M \left\{ \frac{(1+0.005)^{240} - 1}{0.005(1+0.005)^{240}} \right\}$$

$$300\,000 = M \{139.5807717\}$$

$$M = \frac{300\,000}{139.5807717}$$

$$= 2149.293175$$

$$= \$2149.30 \text{ (nearest cent)}$$

Jack can pay

$$\$1798.65 + \$250 = \$2048.65$$

$$\$2048.65 < \$2149.30$$

∴ Jack is unable to pay off the loan.

Question 30

(a) (i) $T = 20000 \times 7.3^3$
 $= 7\,780\,340 \text{ watts.}$

(ii) $A = 40\% \text{ of } T$
 $A = 0.4 \times 20000w^3$
 $A = 8000w^3$

(iii) **Method 1:**
 From graph (Note each small division is worth 0.4):
 Difference $= (14.4 - 5.6) \times 10^6$
 $= 8.8 \times 10^6 \text{ watts}$
 $= 8\,800\,000 \text{ watts}$
 $= 8.8 \text{ megawatts.}$

OR

Method 2:

Using the formula:

$$20\,000 \times 9^3 - 8000 \times 9^3$$

$$= 8\,748\,000 \text{ watts.}$$

(iv) **Method 1:**

From the graph of actual power 4.4 million watts requires a wind speed of 8.2 metres per second.

OR

Method 2:

Using the formula.

$$T = 8000 \times w^3$$

$$4400000 = 8000w^3$$

$$w^3 = 550$$

$$w = \sqrt[3]{550}$$

$$= 8.193\dots$$

$$= 8.2 \text{ (1 dp)}$$

∴ The required wind speed is 8.2 m/sec

(v) $P_{\text{before}} = 0.61 \times \pi \times 4.3^2 \times 8^3$
 $= 1814\,205.92 \text{ watts}$

We need to increase the wind speed by 10%.

New wind speed is $1.1 \times 8 = 8.8$

$$P_{\text{after}} = 0.61 \times \pi \times 4.3^2 \times 8.8^3$$

$$= 2\,414\,708.08 \text{ watts}$$

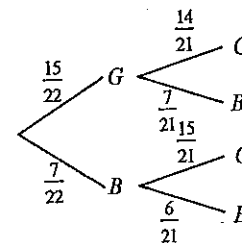
$$\% \text{ inc.} = \frac{\text{Increase}}{\text{Original}} \times 100$$

$$= \frac{2414708.08 - 1814205.92}{1814205.92} \times 100$$

$$= 33.1000003\%$$

$$= 33.1\% \text{ (1 dp).}$$

(b) (i)



(ii) $P(GG \text{ or } BB) = \frac{15}{22} \times \frac{14}{21} + \frac{7}{22} \times \frac{6}{21}$
 $= \frac{6}{11}$

(c) (i) **Method 1:**

Using ratio.

Petrol : Oil

$$40 : 1$$

$$5000 : x$$

$$x = \frac{5000}{40}$$

$$= 125 \text{ mL.}$$

OR

Method 2:

Using proportions

$$\frac{5}{40} = \frac{x}{1}$$

$$x = \frac{5}{40}$$

$$x = 0.125 \text{ L.}$$

$$= 125 \text{ mL.}$$

(ii) The remaining 4.1 litres (4100mL) is in the ratio:

Petrol : Oil

$$40 : 1$$

$$4000 : 100$$

Now we require a 25:1 mix. We know the petrol but we need to find the amount of oil.

Petrol : Oil

$$25 : 1$$

$$4000 : y$$

$$y = \frac{4000}{25}$$

$$= 160 \text{ mL.}$$

Since we have 100mL of oil already in the mix, we require an extra $160 - 100 = 60 \text{ mL}$ to bring the remaining mix to 25:1.