

## FUTURE VALUE OF AN ANNUITY

An annuity is a form of investment involving equal, regular contributions to an account earning compound interest. Each contribution can be thought of as a different investment, over a different length of time. The individual investments are then added together to give the final total amount. Superannuation, where a worker 'saves' towards their retirement, is a common example of an annuity.

The formulae for dealing with annuities appear on the formula sheet so it is very important that students can apply these formulae correctly and accurately in a variety of situations.

**Future value of an annuity** - this is the final amount of money at the end of the investment period.

The formula  $A = M \left[ \frac{(1+r)^n - 1}{r} \right]$  can be used to calculate the future amount (i.e. total amount saved)  $A$  by making a regular contribution of  $\$M$  at  $r\%$  for  $n$  units of time.

Example (i): Sarah wants to be able to pay her car registration and insurance when they are due in twelve months time. Each month Sarah deposits \$250 in a special investment account. The account pays monthly compound interest at 8% p.a.. How much will be in her account after 12 months?

Solution:  $A = M \left[ \frac{(1+r)^n - 1}{r} \right]$  where  $M = \$250$ ,  $n = 12$  months  
and  $r = 8\% \div 12 = 0.00\bar{6}$

$$A = 250 \left[ \frac{(1+0.00\bar{6})^{12} - 1}{0.00\bar{6}} \right]$$

$$= 250 \times 12.449926\dots$$

$$= \$3112.48$$

← Make sure you can get this answer on your calculator. You may need to include additional brackets to ensure the calculator completes the operations in the correct order.

Sarah will have approximately \$3 112 in her investment account.  
The future value of her investment of \$250 a month is \$3 112.

## PRESENT VALUE OF AN ANNUITY – LUMP SUM INVESTMENT

The present value of an annuity is the single sum of money (principal) that you could invest today at the same compound interest rate to produce the same amount as you would obtain by investing a series of regular contributions over the same term.

In this course two different formulae (given on the formula sheet) are used to find the present value,  $N$ , of an annuity.

$$N = \frac{A}{(1+r)^n} \quad \text{and} \quad N = M \left[ \frac{(1+r)^n - 1}{r(1+r)^n} \right]$$

where  $A$  is the future value,  $M$  is the contribution per period, *paid at the end of the period*,  $r$  is the interest rate per period and  $n$  is the number of periods.

### Making a 'one-off' investment to produce a required future amount

Example (i): Troy invests a certain amount so that he will have \$10 000 in his account in 4 years' time. If the account earns 5.1% p.a. compounded monthly, what is the present value of the annuity?

Solution: We are told the future value of the annuity but not the payment.

$$N = \frac{A}{(1+r)^n} \quad \text{Substitute } A = 10\,000, \quad r = 0.051 \div 12 = 0.00425 \\ \text{and } n = 4 \times 12 = 48$$

$$N = \frac{10\,000}{(1+0.00425)^{48}}$$

$$\approx 8158.15 \quad \text{The present value of the annuity is } \$8\,158.15$$

Note: The present value ( $N$ ) is the principal ( $P$ ) in the compound interest formula  $A = P(1+r)^n$

This formula can also be used to find the present value of an annuity where the future value ( $A$ ) is known.

For example, with Troy's investment:

$$A = P(1+r)^n$$

$$10\,000 = P(1+0.00425)^{48}$$

$$10\,000 = P(1.00425)^{48}$$

$$P = 10\,000 \div (1.00425)^{48}$$

$$= \$8\,158.15 \quad \text{as before}$$

## STRAIGHT LINE METHOD OF DEPRECIATION

An article is said to be depreciating when it loses value year by year. Many businesses have to calculate the depreciation on their machinery, cars, office equipment, etc. for taxation purposes. The amount of depreciation is calculated using one of two methods – the straight line method and the declining balance method.

In the **straight line method** the value of the item decreases by the same amount each year. When the values of the item over a number of years are shown on a graph they lie in a straight line. The formula for straight-line depreciation is given on the examination formula sheet.

$$S = V_0 - Dn \quad \text{where } S = \text{salvage value of asset after } n \text{ periods}$$
$$V_0 = \text{original value (purchase price) of asset}$$
$$D = \text{amount of depreciation per period}$$
$$n = \text{number of periods}$$

Note: The depreciated value of an item after  $n$  periods can also be referred to as current value, book value, resale value or salvage value.

Example (i): For taxation purposes a computer system costing \$4 600 depreciates at 20% of its original value each year.

- (a) Calculate the salvage value after 2 years;
- (b) After how many years will the system be valued at \$0?

Solution: Depreciation = 20% of \$4 600 = \$920 per year

- (a) Using  $S = V_0 - Dn$  and  $V_0 = \$4\,600$ ,  $D = \$920$  and  $n = 2$

$$S = \$4\,600 - \$920 \times 2$$
$$= \$1\,840$$

$\therefore$  After 2 years the computer system has a salvage value of \$1 840

- (b) Need to determine the value of  $n$  when  $S = \$0$

$$\text{i.e. } 0 = \$4\,600 - \$920 \times n$$

$$920 \times n = 4\,600$$

$$n = 5$$

$\therefore$  After 5 years the computer would be 'written off' (worth nothing)

Note: For 5 years, \$920 is the amount that can be claimed as a tax deduction on the computer system.

## DECLINING BALANCE METHOD OF DEPRECIATION

In this method the depreciation is calculated as a percentage of the value of the item at the start of each year. The amount of depreciation reduces each year and the formula is similar to that for reducible interest.

The declining balance formula:

$$S = V_0(1 - r)^n$$

where  $S$  = salvage value of asset after  $n$  periods  
 $V_0$  = original value (purchase price) of asset  
 $r$  = percentage interest rate per period  
 $n$  = number of periods

Example (i): A printing machine originally costing \$14 000 depreciates at 20% p.a.  
Find (a) its value after 4 years, (b) the depreciation after 4 years

Solution: (a) Using  $S = V_0(1 - r)^n$   $V_0 = \$14\ 000$ ,  $r = 0.20$  and  $n = 4$

$$\therefore S = \$14\ 000 (1 - 0.20)^4$$

$$\therefore S = \$14\ 000 \times 0.8^4$$

$$\therefore S = \$5\ 734.40$$

Its value after 4 years is \$5 734.40

(b) The depreciation is  $\$14\ 000 - \$5\ 734.40 = \$8\ 265.60$

Example (ii): A car now worth \$12 000 has been depreciating at a rate of 12½ % p.a for the last 3 years. What was its original value 3 years ago?

Solution: Using  $S = V_0(1 - r)^n$  and  $S = \$12\ 000$ ,  $r = 0.125$ ,  $n = 3$

$$\$12\ 000 = V_0(1 - 0.125)^3$$

$$\$12\ 000 = V_0(0.875)^3$$

$$V_0 = \$12\ 000 \div 0.875^3$$

$$= \$17\ 912.54$$

The original value was approximately \$17 913 (to nearest dollar)

## REVIEW EXERCISE – LEVEL 1

1. Daniela earns \$19.40 per hour for a normal 35 hour week. Overtime is calculated at time-and-a-half for the first 6 hours and double-time for any hours after that. What is her gross pay for a week when she works 45 hours?
2. Paul's normal pay rate at Starving Jake's Burger Joint is \$12.60 per hour. He gets time-and-a-quarter for Saturdays and double-time-and-a-half for Sundays. Last week, Paul worked 8 hours during the week, 4 hours on Saturday and 5 hours on Sunday.
  - (a) Calculate his wage for the week.
  - (b) Paul's boss makes a deduction of 25% of gross pay for tax. What is Paul's take-home pay?
  - (c) At the end of the financial year, Paul's gross income was \$19 400. His employer had deducted \$2367 in tax instalments. Paul's allowable deductions came to \$274. If the medicare levy was 1.25%, calculate the amount of tax he should have paid and his refund. [Use the table on page 107].
3. Wayne earns a salary of \$48 479 per annum. Calculate:
  - (a) his weekly salary. (Based on 52.18 weeks per year)
  - (b) how much tax he should pay on his annual income. [Use the table on page 107]
  - (c) what his weekly tax should be.
  - (d) his weekly take-home pay.
4. Kylie works in a clothing store. Her wage is \$320 per week plus a commission of 8% on sales.
  - (a) How much does she earn in a week where she sells \$1980 in clothes?
  - (b) One week, Kylie was paid \$523.20. How much did she sell that week?
5. Trevor received \$4352.20 for his 4 week holiday (this included the 17½% loading). What is his regular weekly wage?
6. Rodney receives time-and-a-half for any hours worked in excess of the normal 38 hours per week. The normal hourly rate is \$18.40. In a particular week Rodney received \$627.90 after \$209.30 had been deducted for tax. How many hours overtime did he work in that week?

7. S-Mark Department Store is having a discount sale of 15% off the marked price.
- (a) How much would be paid for:
    - (i) a table tennis table marked \$189?
    - (ii) a lampshade marked \$25.80?
  - (b) Employees get a further 5% discount on any purchase. How much would an employee pay for an item marked \$218?
8. Calculate the simple interest on these investments
- (a) \$700 at 4.3% p.a. for 3 years
  - (b) \$70 000 at 6.95% p.a. for 5 months
9. On retirement Victor was offered the option of receiving monthly payments of \$2695.60 or a lump-sum payment of \$300 900.
- (a) What are the respective annual incomes of each option if the lump sum can be invested at 9.8% p.a.?
  - (b) At what interest rate must the lump sum be invested in order for the yearly interest to exceed the yearly superannuation payment?
10. Janet wants to buy a new sound system marked \$1599. She pays a deposit of \$600 and the remainder in instalments of \$98.95 per month over 12 months.
- (a) How much in total does Janet pay for the sound system?
  - (b) What rate of interest is she charged?
11. Karen is buying a lounge room furniture package priced at \$2775. She has four payment options:
- (i) Pay cash and receive a discount of 10%.
  - (ii) Pay a deposit of \$750 and 12 monthly payments of \$183.
  - (iii) Pay \$250 per month for 12 months.
  - (iv) Purchase it on finance at 9.25% p.a. over 2 years.
- (a) How much will Karen pay under each option?
  - (b) If Karen pays a deposit of \$775 and borrows the remainder at 9.25% p.a. for 1 year, will she be better off than if she chose option (ii)?
12. Find the future value of an annuity of:
- (a) \$2000 per year for 5 years invested at 4.25% p.a.
  - (b) \$12.70 per week for 3 years earning 5.6% p.a.

## REVIEW EXERCISE – LEVEL 2

1. Colin has 15 000 shares with a market value of 80 cents per share. The percentage yield is 7.3% p.a.
  - (a) What income does Colin receive from these shares?
  - (b) If the face value of the shares is 50 cents, what is the percentage dividend?
  
2. Charles borrows \$2400 to buy a notebook computer. The finance company charges 9% p.a. simple interest over 2 years. How much does Charles pay each month?
  
3. Justin Bootles earns an annual salary of \$78 653. His employer contributes 7% of the gross salary to a superannuation, while Justin makes a 6% contribution.
  - (a) What are the annual and fortnightly total contributions?
  - (b) Find the future value of the superannuation after 3 years of contributions if the fund earns 8.5% p.a. and contributions are made fortnightly for the 3 years.
  
4. Bill Masterton anticipates that he will need \$45 000 in 3 years time to build an extension to his house. He intends paying a set amount into a cash management fund which offers 6.5% interest p.a. calculated weekly. What amount must he invest each week to pay for the extensions in 3 years time?
  
5. Kymm intends borrowing \$120 000. She must decide between a 10 year loan and a 15 year loan. Both can be taken at a fixed rate of 12% p.a.
  - (a) Calculate the loan repayments for both options.
  - (b) How much extra per month will Kymm pay for the 10 year loan compared to the 15 year loan?
  - (c) How much extra will Kymm pay in total for the 15 year loan over the 10 year loan?
  
6.
  - (a) A new car depreciates by 30% in the first year and 20% of the previous year's value in each subsequent year. Calculate the depreciated value of a car, purchased for \$65 000, after 3 years.
  - (b) Calculate the depreciated value of the same car if straight line depreciation of 25% had been used for the 3 years.