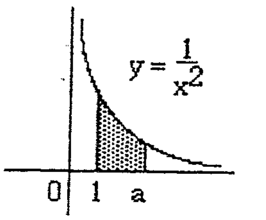
Areas and Volumes 1

1 The area between $y = 6x - x^2$ and the x - axis between $x = 0$ and $x = 6$, is rotated about the x-axis through one complete revolution. Find the volume of the solid generated.

2 The area bounded by the curve $y = x^2 - 4$ and the x - axis is rotated about the y- axis. Find the volume so formed.

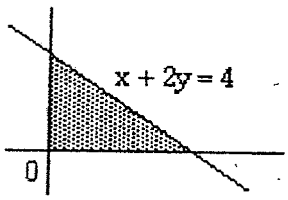
3 Find the area between the curve $y = x^3$, the x-axis and the ordinates at $x = 1$ and $x = 3$.

4



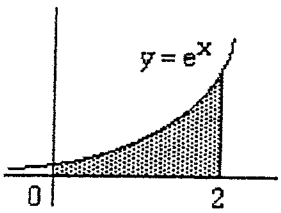
The shaded area in the diagram equals $\frac{2}{3}u^2$.
Find the value of a.

5



The shaded area in the diagram is rotated about
i) the x - axis
ii) the y - axis.
Find the volume generated in each case.

6



Find the area shaded in the diagram.

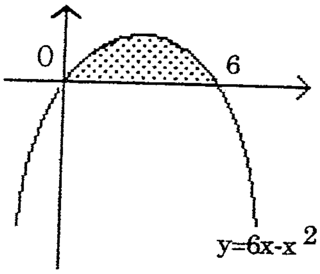
7 The curve $y = \sqrt{\cos x}$ between 0 and $\frac{\pi}{2}$ is rotated about the x - axis. Find the volume generated.

8 The area bounded by the curve $y = \sqrt{x}$, the y - axis and $y = 2$ is rotated about the y-axis. Find the volume so formed.

9 i) Sketch the curve $y = \sin x$, $0 \le x \le 2\pi$.
ii) Find the area of one arch of this curve.

Areas and Volumes 1

1



$V = \pi\int y^2\,dx$

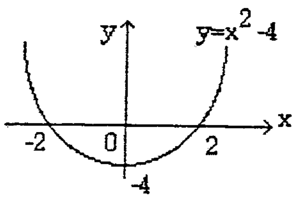
$= \pi\int_0^6 (6x - x^2)^2\,dx$

$= \pi\int_0^6 [36x^2 - 12x^3 + x^4]dx$

$= \pi\,[12x^3 - 3x^4 + \frac{x^5}{5}]_0^6$

$= 259.2\,\pi\,u^3$

2



$V = \pi\int x^2\,dy$

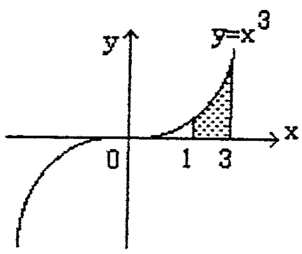
$= \pi\int_{-4}^0 (y + 4)\,dy$

$= \pi\,[\frac{y^2}{2} + 4y]_{-4}^0$

$= \pi\,[0 - (\frac{16}{2} - 16)]$

$= 8\,\pi\,u^3$

3



$A = \int y\,dx$

$= \int_1^3 x^3\,dx$

$= [\frac{x^4}{4}]_1^3$

$= \frac{81}{4} - \frac{1}{4}$

$= 20u^2$

4 $A = \int y\,dx$

i.e. $\frac{2}{3} = \int_1^a \frac{1}{x^2}\,dx$

$\frac{2}{3} = [\frac{-1}{x}]_1^a$

$\frac{2}{3} = \frac{-1}{a} + 1$

$\frac{1}{a} = \frac{1}{3} \quad \therefore \; a = 3$

5 i) $V = \pi\int (\frac{4 - x}{2})^2\,dx$

$= \frac{\pi}{4}\int_0^4 (16 - 8x + x^2)\,dx$

$= \frac{\pi}{4}[16x - 4x^2 + \frac{x^3}{3}]_0^4$

$= \frac{\pi}{4}[64 - 64 + \frac{64}{3} - 0]$

$= \frac{16\pi}{3}$

ii) $V = \pi\int x^2\,dy$

$= \pi\int_0^2 (4 - 2y)^2\,dy$

$= \pi\int_0^2 (16 - 16y + 4y^2)\,dy$

$= \pi\,[16y - 8y^2 + \frac{4y^3}{3}]_0^2$

$= \frac{32\pi}{3}\,u^3$

6 $A = \int y\,dx$

$= \int_0^2 e^x\,dx$

$= [\,e^x\,]_0^2$

$= e^2 - e^0$

$= (e^2 - 1)\,u^2$

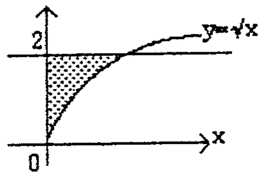
7 $V = \pi\int y^2\,dx$

$= \pi\int_0^{\frac{\pi}{2}} \cos x\,dx$

$= \pi\,[\sin x]_0^{\pi/2}$

$= \pi\,u^3$

8



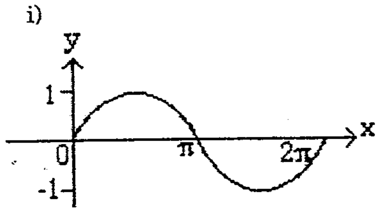
$V = \pi\int x^2\,dy$

$= \pi\int_0^2 y^4\,dy$

$= \pi\left[\frac{y^5}{5}\right]_0^2$

$= \frac{32\pi}{5}\,u^3$

9 i)



ii) $A = \int y\,dx$

$= \int_0^{\pi} \sin x\,dx$

$= [-\cos x]_0^{\pi}$

$= -\cos\pi + \cos 0$

$= 2\,u^2$