Areas and Volumes 2

1 Find the area bounded by the parabola $y = x^2 - 6x$ and the line $y = 2x$.

2 That portion of the curve $y = 4(1 + e^x)$ between $x = 0$ and $x = 1$ is rotated about the x - axis. Show that the volume of the solid of revolution so formed is $\pi[8e^2 + 32e - 24]\ u^3$.

3 i) Find the area bounded by the parabola $y = x^2 + 2$ and the line $y = 6$.
ii) This area is rotated about the y - axis. Find the volume so formed.

4 i) Give a neat sketch of the curve $y = \sqrt{4 - x^2}$
ii) The area bounded by this curve and the x - axis is rotated about the x - axis. Find the volume so formed.
iii) Name the shape formed by this rotation.
iv) The same area is rotated about the y - axis. Find the volume formed.

5 i) Solve the equation $\sin x = \frac{-1}{2}$, $0 \le x \le 2\pi$.
ii) Sketch the curve $y = \frac{1}{2} + \sin x$, $0 \le x \le 2\pi$.
iii) Find the area bounded by the portion of the graph below the x - axis, and the x - axis.

6 i) Sketch the curves $y = (x - 1)^2$ and $y = 10x - x^2 - 9$ carefully on the same diagram, showing their turning points and point(s) of intersection.
ii) Find the area bounded by these two curves.

7 i) Sketch the curve $y = \frac{1}{\sqrt{x + 2}}$ in the domain $x = 0$ to $x = 4$.
ii) The area bounded by this curve and the x - axis between $x = 0$ and $x = 4$, is rotated about the x - axis. Find the volume generated.

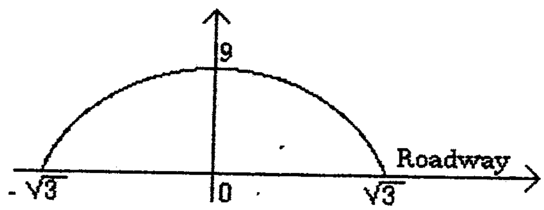
8 A function is defined by the equation: $f(x) = 1, \quad x \le 0$
$= 1 + x^2, \quad x > 0.$

Find the value of $\int_{-2}^{4} f(x)\,dx$

9 Find the area which is completely bounded by the curve $y = 3x^2 - x^3$ and the x - axis.

10 i) Find the indefinite integral $\int \sec^2 x\,dx$
ii) The area bounded by the curve $y = \sec x$ and the x - axis, between $x = 0$ and $x = \frac{\pi}{3}$ is rotated about the x - axis. Find the volume so formed.

11 The arch of a bridge is shown in the diagram. If the arch is known to be a parabola, find the area bounded by it and the roadway.



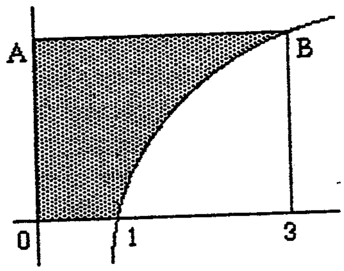
12 Evaluate $\int_0^1 2\sin\frac{\pi x}{2}\,dx$

13 The diagram shows the graph of $y = \log_e x$.

i) Find the co-ordinates of the point A in exact form.

ii) Find the shaded area.

iii) Evaluate $\int_1^3 \log_e x \, dx$



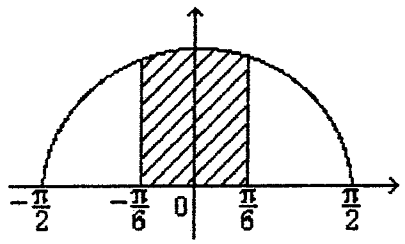
14 The equation $x^2 + 8y^2 = 16$ is the equation of an ellipse.

i) Find where the ellipse cuts the x - axis.

ii) Find where the ellipse cuts the y - axis.

iii) If the ellipse is rotated about the x - axis find the volume so formed.

15



The diagram shows part of the curve $y = \cos x$

i) Show that the shaded area is 1 square unit.

ii) Copy and complete the following table of values for $y = \cos x$ in your writing booklet. (Give exact answers)

x	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$
y			

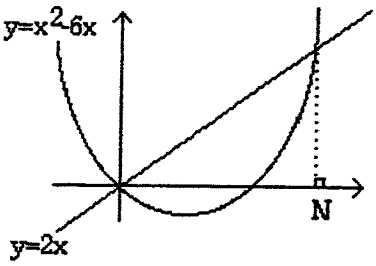
iii) Use Simpson's rule with 3 function values to estimate the integral $\int_{-\pi/6}^{\pi/6} \cos x \, dx$

iv) By comparing parts i) and iii) show that an approximate value for π would be $\pi \approx \frac{18}{\sqrt{3} + 4}$

v) By correcting this approximation to 4 significant figures, and taking the true value of π to be 3.142, find the percentage error in the approximation, correct to 3 significant figures.

Areas and Volumes 2

1



Co-ordinates of N :

$y = 2x$

$y = x^2 - 6x$

$\therefore\ 2x = x^2 - 6x$

$0 = x^2 - 8x$

$0 = x(x - 8)$

$\therefore\ x = 0, 8$

$\therefore\ N = (8,0)$

Area between 2 curves =

$$\int f(x)\,dx - \int g(x)\,dx$$

$$\therefore\ A = \int_0^8 2x\,dx - \int_0^8 (x^2 - 6x)\,dx$$

$$= [x^2]_0^8 - [\frac{x^3}{3} - 3x^2]_0^8$$

$$= 64 - [\frac{512}{3} - 3(64)]$$

$$= 85\frac{1}{3}\ u^2$$

2 $V = \pi \int y^2\,dx$

$$= \pi \int_0^1 [4(1 + e^x)]^2\,dx$$

$$= 16\pi \int_0^1 (1 + 2e^x + e^{2x})\,dx$$

$$= 16\pi\,[x + 2e^x + \frac{1}{2}e^{2x}]_0^1$$

$$= 16\pi\,[-1\frac{1}{2} + 2e + \frac{1}{2}e^2]$$

$$= \pi\,[-24 + 32e + 8e^2]\ u^3$$

3 i) $A = \int x\,dy$

$$= 2\int_2^6 \sqrt{y - 2}\,dy$$

using symmetry

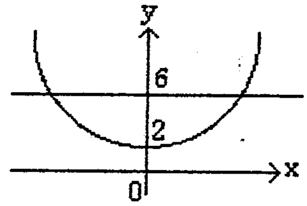
$$= 2\int_2^6 (y - 2)^{1/2}\,dy$$

$$= 2\,[\frac{(y - 2)^{3/2}}{3/2}]_2^6$$

$$= \frac{4}{3}[(y - 2)^{3/2}]_2^6$$

$$= \frac{4}{3}[4^{3/2} - 0]$$

$$= \frac{32}{3}\ u^3$$



ii) $V = \pi \int x^2\,dy$

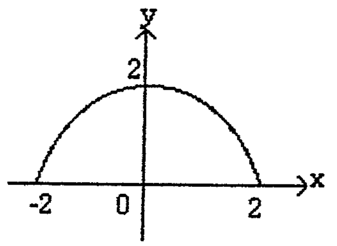
$$= \pi \int_2^6 (y - 2)\,dy$$

$$= \pi\,[\frac{y^2}{2} - 2y]_2^6$$

$$= \pi\,[18 - 12 - (2 - 4)]$$

$$= 8\pi\ u^3$$

4 i)



ii) $V = \pi \int y^2 dx$

$$= \pi \int_{-2}^2 (4 - x^2)\,dx$$

$$= 2\pi \int_0^2 (4 - x^2)\,dx,$$

using symmetry

$$= 2\pi\,[4x - \frac{x^3}{3}]_0^2$$

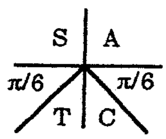
$$= \frac{32\pi}{3}\ u^3$$

iii) This would be a sphere of radius 2.

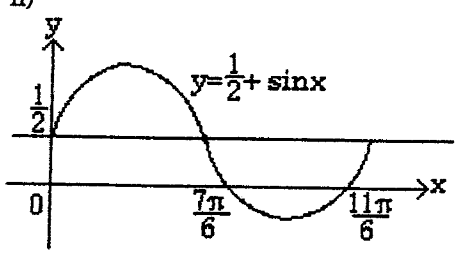
iv) This rotation would give a hemisphere. Hence, its volume would be $\frac{16\pi}{3}\ u^3$

5 i) $\sin x = -\frac{1}{2},\ 0 \le x \le 2\pi$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$



ii)



iii) $A = \int y\,dx$

$$= \left| \int_{\frac{7\pi}{6}}^{\frac{11\pi}{6}} \left(\frac{1}{2} + \sin x\right) dx \right|$$

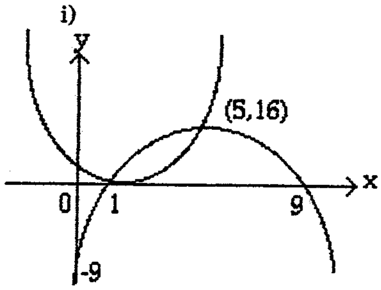
since area is below x-axis

$$= \left[\frac{x}{2} - \cos x\right]_{7\pi/6}^{11\pi/6}$$

$$= \frac{11\pi}{12} - \frac{\sqrt{3}}{2} - \left(\frac{7\pi}{12} + \frac{\sqrt{3}}{2}\right)$$

$$= \left(\frac{\pi}{3} - \sqrt{3}\right) u^2$$

6 i)



ii) $A =$

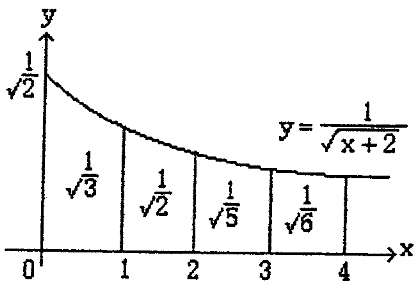
$$\int_1^5 (10x - x^2 - 9)\,dx - \int_1^5 (x-1)^2\,dx$$

$$= \left[5x^2 - \frac{x^3}{3} - 9x\right]_1^5 - \left[\frac{(x-1)^3}{3}\right]_1^5$$

$$= 38\frac{1}{3} + 4\frac{1}{3} - \left[\frac{64}{3} - 0\right]$$

$$= 21\frac{1}{3}\ u^2$$

7 i)



ii) $V = \pi \int y^2 dx$

$$= \pi \int_0^4 \frac{1}{x+2}\,dx$$

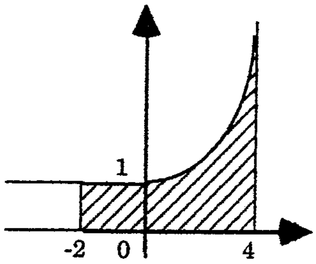
$$= \pi\left[\ln(x+2)\right]_0^4$$

$$= \pi\,.\,[\ln 6 - \ln 2]$$

$$= \pi\,.\,\ln\frac{6}{2}$$

$$= \pi \ln 3\ u^3$$

8



$$A = \text{area rectangle} + \int_0^4 (1 + x^2)\,dx$$

$$= 2(1) + \left[x + \frac{x^3}{3}\right]_0^4$$

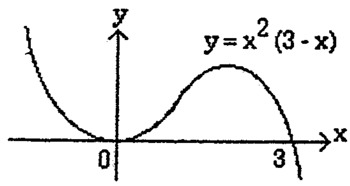
$$= 2 + \left[4 + \frac{64}{3}\right]$$

$$= 27\frac{1}{3}\,u^2$$

9 $y = x^2(3 - x)$

When $y = 0$: $0 = x^2(3 - x)$

$\therefore\ x = 0, 3$



$A = \int y\,dx$

$$= \int_0^3 (3x^2 - x^3)\,dx$$

$$= \left[x^3 - \frac{x^4}{4}\right]_0^3$$

$$= 6\frac{3}{4}\,u^2$$

10 i) $\int \sec^2 x\,dx = \tan x + c$

ii) $V = \pi \int y^2\,dx$

$$= \pi \int_0^{\pi/3} \sec^2 dx$$

$$= \pi\,[\tan x]_0^{\pi/3}$$

$$= \pi\sqrt{3}$$

11 Equation of arch : $y = ax^2 + b$

When $y = 0$, $x = \sqrt{3}$

$\therefore\ 0 = 3a + b$

When $x = 0$, $y = 9$

$\therefore\ 9 = b$

$\therefore\ a = -3$ $\therefore\ y = 9 - 3x^2$

$A = \int y\,dx$

$$= 2\int_0^{\sqrt{3}} (9 - 3x^2)\,dx,$$

by symmetry

$$= 2\left[9x - x^3\right]_0^{\sqrt{3}}$$

$$= 12\sqrt{3}\,u^2$$

12 $\int_0^1 2\sin\frac{\pi x}{2}\,dx = 2\cdot\frac{2}{\pi}\int_0^1 \frac{\pi}{2}\sin\frac{\pi x}{2}\,dx$

$= \frac{4}{\pi}[-\cos\frac{\pi x}{2}]_0^1$

$= \frac{4}{\pi}[-\cos\frac{\pi}{2} + \cos 0]$

$= \frac{4}{\pi}$

13 i) $y = \log_e x$

When $x = 3$, $y = \log_e 3$

$\therefore$ A = $(0, \log_e 3)$

ii) $A = \int x\,dy$

$= \int_0^{\ln 3} e^y\,dy$

$= [e^y]_0^{\log 3}$

$= e^{\log 3} - e^0$

$= 2\ u^2$

iii) This integral could measure the unshaded area.

i.e. $\int_1^3 \log x\,dx$ = unshaded area

= area rectangle - shaded area

= $(3\log 3 - 2)\ u^2$

14 i) Any curve cuts the x-axis when $y = 0$

i.e. $x^2 + 0 = 16$

$x = \pm 4$

$\therefore$ cuts x-axis at (-4,0) and (4,0)