Functions

(Preliminary Course)

1. Sketch the graph of each of the relations $y=|x|$ and $|y|=x$. In each case explain whether or not the relation is a function.

2. Explain why the relation $x^2+y^2=5$ is not a function.

3. Find the largest possible domain of each of the following functions.
   (i) $y=\frac{1}{x^2+1}$ (ii) $y=\frac{1}{(x+1)^2}$
   (iii) $y=\frac{1}{\sqrt{x}+1}$ (iv) $y=\frac{1}{\sqrt{x+1}}$

4. Find the largest possible domain of the function $y=\sqrt{1-2x}+\sqrt{2+x}$.

5. Find the largest possible domain and the range of each of the following functions.
   (i) $y=x^2-2x$
   (ii) $y=4x-x^2$

6. Use the graph of $y=\sqrt{x}$ to sketch the graphs of
   (i) $y=\sqrt{x-2}$
   (ii) $y=\sqrt{x}-2$

7. On the same axes sketch the graphs of $y=4-x^2$ and $y^2=4-x^2$.

8. Sketch the graph of $y=\sqrt{4+x}$ and find its domain and range.

9. Show that the function $f(x)=|x|-2$ is even. Sketch its graph and use the graph to find the values of x for which $|x|>2$.

10. Show that the function $f(x)=4x-x^3$ is odd. Sketch its graph and use the graph to find the values of x for which $x^3>4x$.

11. Sketch the graph of the function
$$f(x)=\begin{cases}\sqrt{25-x^2}, & -5\le x\le 3\\ 4, & 3<x\le 5\end{cases}$$
Find the value of $f(4)-f(0)$.

12. Sketch the graph of the function
$$f(x)=\begin{cases}2^x, & x\le 2\\ \frac{8}{x}, & x>2\end{cases}$$
Find the range of the function.

13. Sketch the graph of the function
$$f(x)=\begin{cases}x^2-4, & x\le 0\\ x-4, & x>0\end{cases}$$
Find the values of x for which $f(x)$ is negative.

14. Sketch the graph of the function
$$f(x)=\begin{cases}x^2+1, & x\le 0\\ |x-1|, & 0<x\le 2\\ 1, & x>2\end{cases}$$

15. Find the centre and radius of the circle $x^2+y^2-6x+2y+6=0$. Sketch its graph.

16. Sketch the graph of the circle $(x-4)^2+(y-4)^2=16$. Find the area of the region in the first quadrant bounded by the circle and the coordinate axes.

17. On the same axes sketch the graphs of $y=13-x^2$ and $y=\dfrac{12}{x}$. By inspection of the graph, state the number of solutions of the equation $13-x^2=\dfrac{12}{x}$. (There is no need to find these solutions nor the coordinates of the intersection points of the graphs.)

18. On the same axes sketch the graphs of $x^2+y^2=4$ and $x^2+y^2=16$. Shade in the region where $4 \le x^2+y^2 \le 16$, and find its area.

19. On the same axes sketch the graphs of $y=|x|$ and $xy=1$. Shade in the region where $y<|x|$ and $xy \ge 1$.

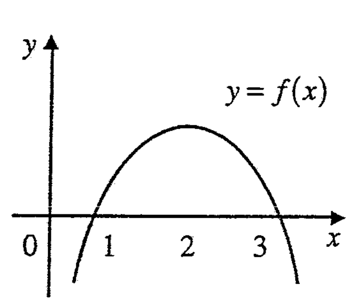
20. On the same axes sketch the graphs of $y=\sqrt{4-x^2}$ and $x-y+2=0$. Shade the region where $y \le \sqrt{4-x^2}$ and $x-y+2 \le 0$, and find its area.

21. Show that the function $f(x)=\sqrt{x^2-4}$ is even and find its domain and range.

22. Sketch the graph of the function $y=\dfrac{x+2}{x+1}$ and state its domain and range.

23.

Figure 4.19



The diagram shows the graph of the function $y=f(x)$. Find the domain of each of the functions

(i) $y=\dfrac{1}{f(x)}$ **(ii)** $y=\sqrt{f(x)}$

24. Sketch the graph of the function

$$f(x)=\begin{cases} 2 & , \quad 2<|x|\le 4 \\ |x| & , \quad |x|\le 2 \end{cases}$$

25. **(i)** If $x^2+y^2+2fx+2gy+c=0$ is the equation of a circle, find its centre and radius.

(ii) Find the condition on f, g and c so that $x^2+y^2+2fx+2gy+c=0$ is the equation of a circle.

ANSWERS

1.

Figure 16.1

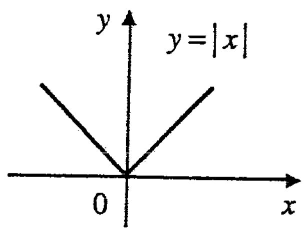
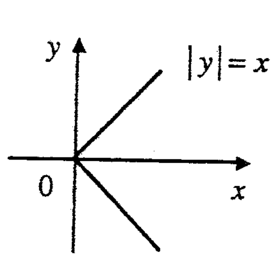


Figure 16.2



Applying the vertical line test, $y=|x|$ is a function but $|y|=x$ is not a function.

2. Both the points $(1,2)$ and $(1,-2)$ lie on the circle $x^2+y^2=5$, hence the relation is not a function.

3. (i) All real x (ii) $\{x: x\neq -1\}$
(iii) $\{x: x\geq 0\}$ (iv) $\{x: x>-1\}$

4. $2x\leq 1$ and $x\geq -2 \Rightarrow \{x: -2\leq x\leq \frac{1}{2}\}$

5. (i) $y=x(x-2)$ is concave up parabola with vertex $(1,-1)$.
$\therefore$ Domain: all real x;
Range: $\{y: y\geq -1\}$

(ii) $y=-x(x-4)$ is concave down parabola with vertex $(2,4)$.

$\therefore$ Domain: all real x;
Range: $\{y: y\leq 4\}$

6. (i)

Figure 16.3

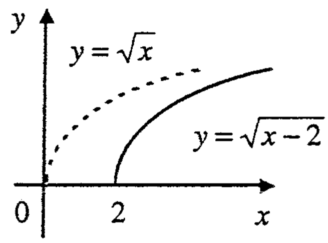
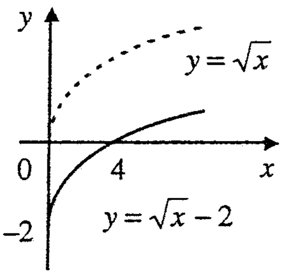
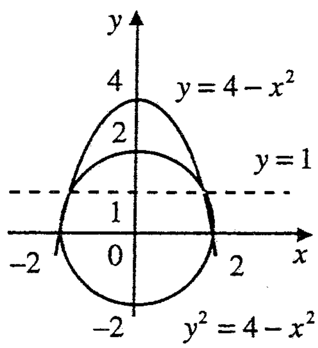


Figure 16.4



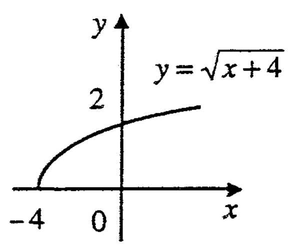
7.

Figure 16.5



8.

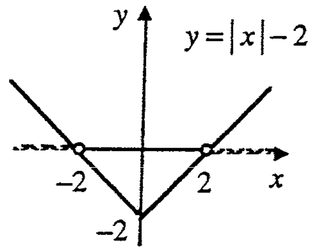
Figure 16.6



Domain: $\{x: x\geq -4\}$;
Range: $\{y: y\geq 0\}$

9.

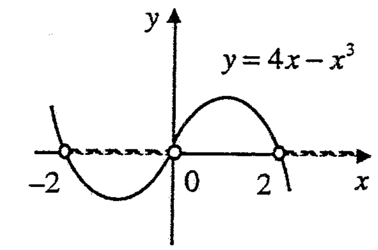
Figure 16.7



Require x values for which graph lies above x-axis. $\therefore\ x<-2$ or $x>2$

10.

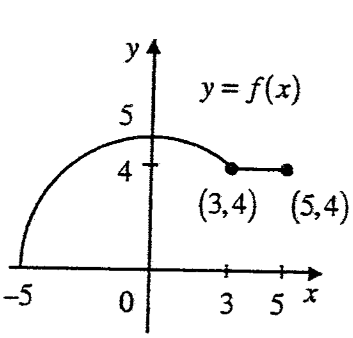
Figure 16.8



Require values of x for which graph lies below the x-axis.
$\therefore\ -2<x<0$ or $x>2$

11.

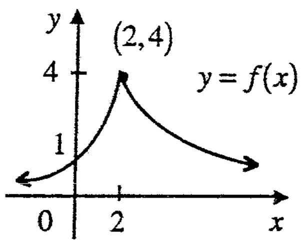
Figure 16.9



$f(4)-f(0)=4-\sqrt{25}=-1$

12.

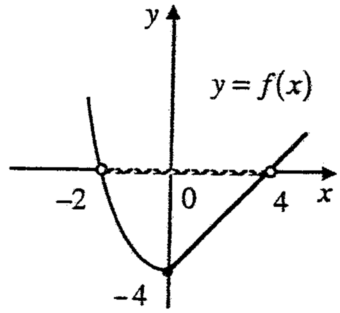
Figure 16.10



Range: $\{y:0<y\le 4\}$

13.

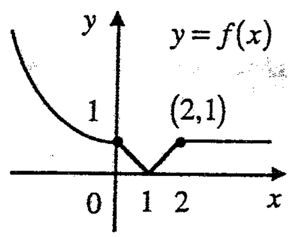
Figure 16.11



Require values of x for which graph lies below the x-axis. $\therefore\ -2<x<4$

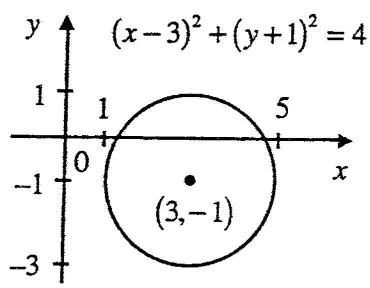
14.

Figure 16.12



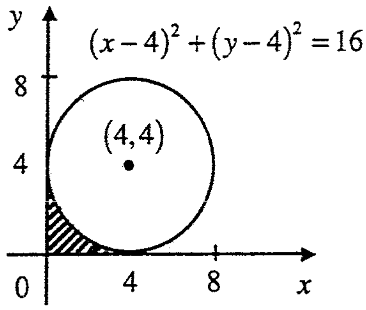
15. $(x-3)^2+(y+1)^2=4$ is circle with centre $(3,-1)$ and radius 2.

Figure 16.13



16.

Figure 16.14

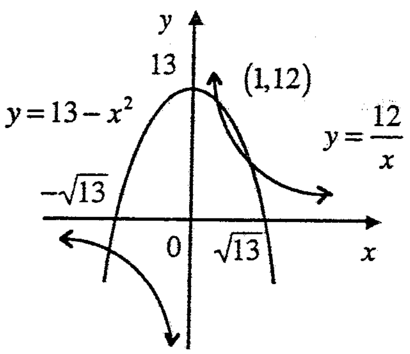


Area square – Area $\frac{1}{4}$ circle

$=16-\frac{1}{4}\pi\times4^2=16-4\pi$

17.

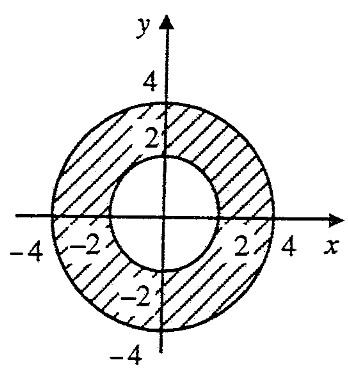
Figure 16.15



Three intersection points and hence three solutions.

18.

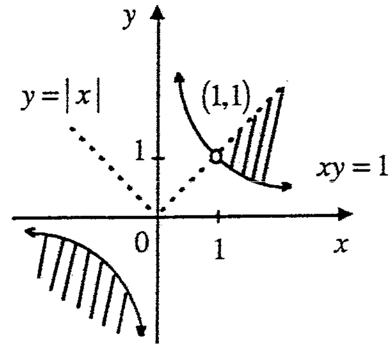
Figure 16.16



Area is $\pi\left(4^2-2^2\right)=12\pi$

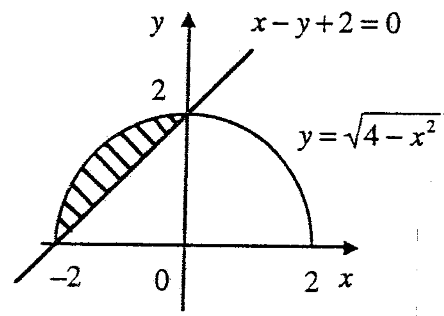
19.

Figure 16.17



20.

Figure 16.18



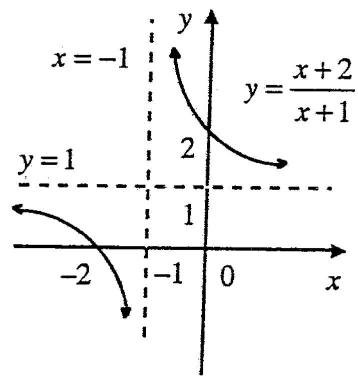
Area $\frac{1}{4}$ circle – Area Δ

$=\frac{1}{4}\pi\times2^2-\frac{1}{2}\times2\times2=\pi-2$

21. Domain: $\{x: x\le -2 \text{ or } x\ge 2\}$
Range: $\{y: y\ge 0\}$

22.

Figure 16.19



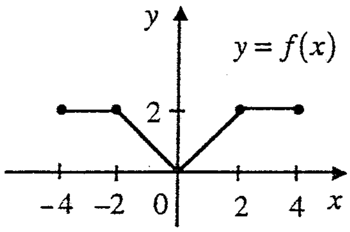
Domain: $\{x: x\ne -1\}$;
Range: $\{y: y\ne 1\}$

23. **(i)** $\{x : x \neq 1,\ x \neq 3\}$

(ii) $\{x : 1 \leq x \leq 3\}$

24.

Figure 16.20



25. $(x+f)^2 + (y+g)^2 = f^2 + g^2 - c$

(i) Centre $(-f, -g)$;

radius $\sqrt{f^2 + g^2 - c}$

(ii) $f^2 + g^2 \geq c$