Geometrical applications of differentiation

TOPIC TEST

Time allowed: 1 hour **Total marks = 100**

SECTION I Multiple-choice questions

10 marks

Instructions
- This section consists of 10 multiple-choice questions
- Each question is worth 1 mark
- Fill in only ONE CIRCLE
- Calculators may be used

1 If a curve is always increasing which MUST occur for all values of x.

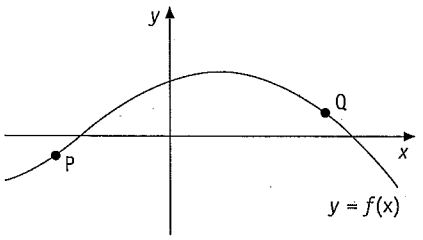
Ⓐ $\frac{dy}{dx} > 0$ Ⓑ $\frac{dy}{dx} < 0$ Ⓒ $\frac{d^2y}{dx^2} > 0$ Ⓓ $\frac{d^2y}{dx^2} < 0$

2 If $f'(a) < 0$ and $f''(a) > 0$, then at $x = a$, $y = f(x)$ is:

Ⓐ increasing and concave up
Ⓑ increasing and concave down
Ⓒ decreasing and concave up
Ⓓ decreasing and concave down

3 For the curve $y = f(x)$ which is correct?

Ⓐ $f'(x) > 0$ at P and $f'(x) < 0$ at Q
Ⓑ $f'(x) < 0$ at P and $f'(x) < 0$ at Q
Ⓒ $f'(x) > 0$ at P and $f'(x) > 0$ at Q
Ⓓ $f'(x) < 0$ at P and $f'(x) > 0$ at Q



4 Which statement is correct?

Ⓐ All stationary points are turning points.
Ⓑ All turning points are stationary points.
Ⓒ An absolute maximum must be a maximum turning point.
Ⓓ A local minimum is the least value of the function over the given domain.

5 If $f(x) = x^2 + 5x + 4$ then $f''(3) = ?$

Ⓐ 2 Ⓑ 3 Ⓒ 4 Ⓓ 5

6 At a horizontal point of inflexion, which is correct?

Ⓐ $\frac{dy}{dx} \neq 0$ and $\frac{d^2y}{dx^2} = 0$
Ⓑ $\frac{dy}{dx} \neq 0$ and $\frac{d^2y}{dx^2} \neq 0$
Ⓒ $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} = 0$
Ⓓ $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} \neq 0$

7 Which is NOT a primitive function of $3x^2$?

Ⓐ $x^3 + 7$ Ⓑ $x^3 - 4$ Ⓒ $x^3 + 1$ Ⓓ $x^3 + 7x$

8 If $f'(a) = 0$ and $f''(a) = 0$, then at $x = a$, $y = f(x)$ has:

Ⓐ a maximum turning point

Ⓑ a minimum turning point

Ⓒ a horizontal point of inflexion

Ⓓ There is insufficient information to determine the nature of the stationary point.

9 At a point of inflexion a curve MUST have:

Ⓐ $\frac{dy}{dx} = 0$ Ⓑ $\frac{d^2y}{dx^2} = 0$ Ⓒ $\frac{dy}{dx} \neq 0$ Ⓓ $\frac{d^2y}{dx^2} \neq 0$

10 The equation of a tangent to a curve $y = f(x)$ at the point P(2, 4) is $y = 10x - 16$. Which is the only possible equation of the normal to $y = f(x)$ at the point P?

Ⓐ $10x - y - 16 = 0$ Ⓑ $10x - y + 16 = 0$ Ⓒ $x + 10y - 42 = 0$ Ⓓ $x + 10y + 42 = 0$

SECTION II

90 marks

Show all necessary working

11 Determine whether the curve $y = x^3 - 5x + 4$ is increasing or decreasing at the point (1, 0). **2 marks**

12 For what values of x is the curve $y = x^3 + 9x - 3$ increasing? **2 marks**

13 Find the stationary points of the curve $y = \frac{x^3}{3} - x^2 - 8x + 11$ and use the first derivative to determine their nature. **6 marks**

14 Find the second derivative of:

2 marks each

a $y = x^3 - 9x^2 + 5x - 4$

b $y = 3x - 5$

c $y = (2x + 3)^4$ **3 marks**

d $y = \frac{7}{x^2} - \frac{4}{x^3}$ **3 marks**

15 $f(x) = 8x^2 - 3x^5$ Find $f''(-1)$ **3 marks**

16 Determine whether the curve $y = x^3 - 2x^2 - 7x + 8$ is concave up or concave down at the point where $x = 5$ **3 marks**

17 For what values of x is the curve $y = 7x^3 - 9x^2$ concave down? **4 marks**

18 Find the stationary points and determine their nature:

a $y = 2x^3 + 15x^2 - 84x + 13$ **6 marks**

b $y = 4x^3 - 5$ **4 marks**

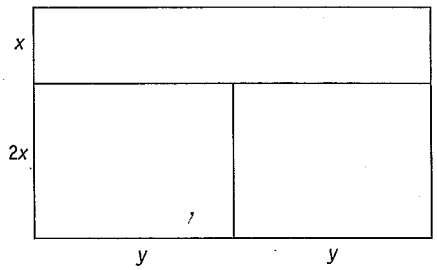
19 Find the point of inflexion of the curve $y = x^3 + 9x^2 - 4x + 3$ **4 marks**

20 Find the equation of the tangent to the curve $y = 7 - x^3$ at the point (1, 6) **3 marks**

21 Find the equation of the normal to the curve $y = x^2 + 4x$ at $x = \frac{1}{2}$ **4 marks**

22 Sketch the curve $y = x^4 - 8x^2 + 7$, $-3 \leq x \leq 3$ **16 marks**

23 A farmer wishes to build three new calf yards, as shown in the diagram. She has enough materials to fence 800 m.



a Show that the total area (A m^2) is given by $A = 800x - 8x^2$ **3 marks**

b Find the maximum total area (in hectares) **6 marks**

24 Find primitive functions of:

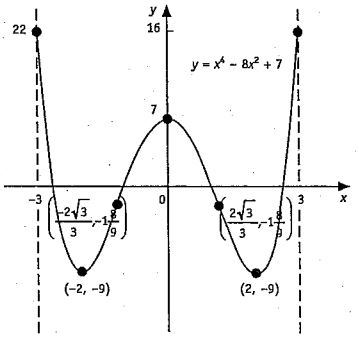
a x^5 **3 marks**

b $10x^4 - 6x^2 + 5$ **3 marks**

c $\frac{1}{x^3}$ **3 marks**

25 A curve $y = f(x)$ has gradient function $f'(x) = 3x^2 - 8x + 5$. The curve passes through the point (1, 4). Find the equation of the curve. **5 marks**

Pages 28-33 **1** A **2** C **3** A **4** B **5** A **6** C **7** D **8** D **9** B **10** C **11** decreasing **12** all values of x **13** minimum at $\left(4, -15\frac{2}{3}\right)$, maximum at $\left(-2, 20\frac{1}{3}\right)$ **14** **a** $6x - 18$ **b** 0 **c** $48(2x + 3)^2$ **d** $\frac{42}{x^4} - \frac{48}{x^5}$ **15** 76 **16** concave up **17** $x < \frac{3}{7}$ **18** **a** maximum at $(-7, 650)$, minimum at $(2, -79)$, **b** horizontal point of inflexion at $(0, -5)$ **19** $(-3, 69)$ **20** $y = -3x + 9$ **21** $4x + 20y - 47 = 0$ **22** maximum at $(0, 7)$ minimum at $(2, -9)$ and at $(-2, -9)$, points of inflexion at $\left(\frac{2\sqrt{3}}{3}, -1\frac{8}{9}\right)$ and $\left(-\frac{2\sqrt{3}}{3}, -1\frac{8}{9}\right)$, endpoints of domain $(-3, 16)$, $(3, 16)$ (below)



23 **b** 2 ha **24** **a** $\frac{x^6}{6} + C$ **b** $2x^5 - 2x^3 + 5x + C$ **c** $-\frac{1}{2x^2} + C$ **25** $y = x^3 - 4x^2 + 5x + 2$