J.M.J.

MARCELLIN COLLEGE RANDWICK

HALF-YEARLY ASSESSMENT

MATHEMATICS ADVANCED

2009 – Task 1

Weighting: 30% (Preliminary Assessment Mark)

NAME: _

MARK: / 45

PERCENTAGE: %

RANK ON THIS TASK:

Time Allowed: **50 minutes**

Topics: Number System, Algebra, Equations, Plane Geometry, Functions

Directions:

- There are **THREE** questions on this paper
- Marks have been allocated for each question
- Answer each questions on a separate page
- Show all necessary working
- Marks may not be awarded for careless or badly arranged work

QUESTION 1 **(Start a new page)** **Marks**

a) Calculate $\dfrac{\sqrt[3]{1.35\times 0.479}}{\pi(29.3)^2}$, using scientific notation correct to two significant figures. **1**

b) Simplify $\left(\dfrac{8}{27}\right)^{\frac{1}{3}}\times\left(\dfrac{64}{9}\right)^{\frac{-1}{2}}$ **2**

c) Express $0\cdot\dot{2}\dot{8}$ as a rational number in its simplest form. **2**

d) Factorise completely over the field of real numbers: **6**

(i) $3m+3y+m^2-y^2$

(ii) x^4-1

(iii) $8-(x-1)^3$

e) Show that $\dfrac{x^3+27}{6x^2+21x+9}\div\dfrac{x^3-3x^2+9x}{2x^2+x}$ is rational. **3**

QUESTION 2 **(Start a new page)** **Marks**

a) Express $\frac{2+\sqrt{5}}{2-\sqrt{5}}$ in the form $a+\sqrt{b}$. 3

b) Show that $1-\sqrt{3}$ is a solution of the equation $x^2-2x-2=0$. 2

c) Solve each of the following:

(i) Solve $2(3a-4)-(a+2)=8a+5$ 6

(ii) $32^{3-x}=\frac{1}{8}$

(iii) $|2x-1|>5$

d) In a rectangle of area 4 cm^2 the width is x cm and the length is 2 cm longer than the width. 4

(i) Show that $x^2+2x-4=0$.

(ii) Find the exact dimensions of the rectangle in simplest form.

e) Solve the following simultaneously, 3

$2x-y-3=0$

$y=x^2-4x+5$

QUESTION 3 **(Start a new page)** **Marks**

a) For the following function, $g(x)=\frac{1}{x-4}+1$ 4

(i) State the domain and range.

(ii) Show whether the function is even, odd or neither.

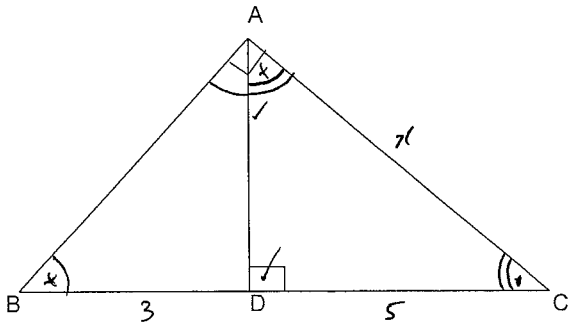
b) (i) Sketch $y=\sqrt{16-x^2}$. 4

(ii) Shade the region defined by:

$\{(x,y): y\le\sqrt{16-x^2}\}\cap\{(x,y): y\ge|x|\}$. Hence,

calculate the exact area of the shaded region giving your answer in terms of π.

c)



ΔABC is a right-angled triangle at A. ΔDAC is a right angled triangle at D.

(i) Prove ΔBAC /// ΔADC. 3

(ii) Given that BD = 3cm, DC = 5 cm and AC = x cm, show that $x=2\sqrt{10}$ cm. 2

Name:

Teacher's initials: A·B

Question: 1

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a) $\frac{\sqrt[3]{1.35 \times 0.79}}{\pi (29.3)^2}$ ✓

$= 3.2 \times 10^{-4}$

b) $\left(\frac{8}{27}\right)^{\frac{1}{3}} \times \left(\frac{64}{9}\right)^{\frac{-1}{2}}$

$= \frac{2}{3} \times \frac{3}{8}$

$= \frac{6}{24}$ ✓

$= \frac{1}{4}$

c) $0.2\dot{8}$

let $0.2\dot{8} = n$

$100n = 28.8888...$

$10n = 2.88888...$ ✓

$100n - 10n = 90n$

$90n = 26$

$n = \frac{26}{90}$

$= \frac{13}{45}$

d) i) $3m + 3y + m^2 - y^2$ →

$3(m+y) + (m-y)(m+y)$

$(m+y)(3+m-y)$ ✓ —1

ii) $x^4 - 1$

$(x^2-1)(x^2+1)$

$(x-1)(x+1)(x^2+1)$ ✓

Name:

Teacher's initials: A·B

Question: 1

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iii) $8 - (x-1)^3$ →

$= 2^3 - (x-1)^3 = (2-(x-1))(2^2 + 2(x-1) + (x-1)^2)$

$= (3-x)(4+2x-1+x^2-2x+1) = (3-x)(4+x^2)$

$8 - x - 2x + 1$

$-x^3 - 2x + 9$ ✗

$x^3 + 2x + 9$

$(x+1)^2 + 9$ —2

e) $\frac{x^3+27}{6x^2+21x+9} \div \frac{x^3-3x^2+9x}{2x^2+x}$

$= \frac{(x+3)(x^2-3x+9)}{(2x+1)(3x+9)} \div \frac{x(x^2-3x+9)}{x(2x+1)}$

$= \frac{(x+3)(x^2-3x+9)}{(2x+1)(3x+9)} \times \frac{(2x+1)}{(x^2-3x+9)}$ ✓ 2

$= \frac{x+3}{3x+9}$

$= \frac{x+3}{3(x+3)}$

$= \frac{1}{3}$ ✗

Name:

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Question: 2

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a) $\frac{2+\sqrt{5}}{2-\sqrt{5}} \times \frac{2+\sqrt{5}}{2+\sqrt{5}}$

$= \frac{4+2\sqrt{5}+5}{4-5}$

$= \frac{9+2\sqrt{5}}{-1}$

$= -9-2\sqrt{5}$

$= -9+\sqrt{20}$

good ✓

b) $x^2-2x-2=0$

$x^2-2x+(-1)^2 = 2+1$ ✓

$(x-1)^2 = 3$

$x-1 = \pm\sqrt{3}$

$x = 1 \pm \sqrt{3}$

$\therefore 1-\sqrt{3}$ is a solution for the equation $x^2-2x-2=0$

c) i) $2(3a-4)-(a+2) = 8a+5$

$6a-8-a-2 = 8a+5$ ✓

$5a-10 = 8a+5$

$-15 = 3a$

$a = -5$

ii) $32^{3-x} = \frac{1}{8} \Rightarrow (2^5)^{3-x} = 2^{-3}$

$32^{3-x} = 8^{-1}$

$2^{15-5x} = 2^{-3}$

$32^{3-x} = 2^{-3}$

$15-5x = -3$

$16(3-x) = -3$ ✗

$48-16x = -3$ —

$-16x = -51$

$\therefore 5x = 18$

$x = \frac{51}{16}$

$x = \frac{18}{5}$

$x = 3\frac{3}{16}$

$= 3\frac{3}{5}$

Name:

Teacher's initials: AB

Question: 2

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iii) $2x-1>5$ or $-2x+1>5$

$2x>6$ or $-2x>4$ ✓

$x>3$ or $x<-2$

d) i) $A = l \times b$

$4 = x \times (x+2)$ ✓

$4 = x^2+2x$

$x^2+2x-4=0$

ii) $(x-2)(x-2)=0$

$x=2$

width $= x = 2$, length $= x+2 = 2+2 = 4$

$\therefore$ width $= 2$ cm

length $= 4$ cm

sillly ✗ —

M

c) $2x-3=y$ ①

$y=x^2-4x+5$ ②

sub ① into ②

$2x-3 = x^2-4x+5$

$x^2-6x+8=0$

$(x-2)(x-4)=0$

$x=2$, $x=4$ —

$y=?$ $y=?$

careless errors

Name:

Teacher's initials: AB

Question: 3

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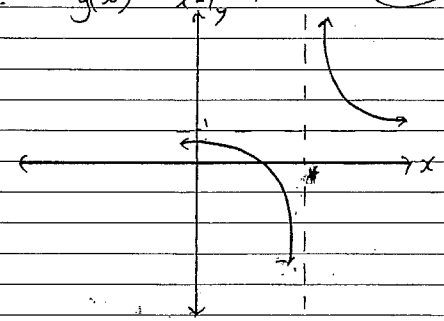
a) i). $g(x) = \frac{1}{x-4} + 1$



D: $\{x : x \in \mathbb{R}, x \neq 4\}$

R: $\{y : y \in \mathbb{R}, y \neq 1\}$

ii) $g(x) = \frac{1}{x-4} + 1$

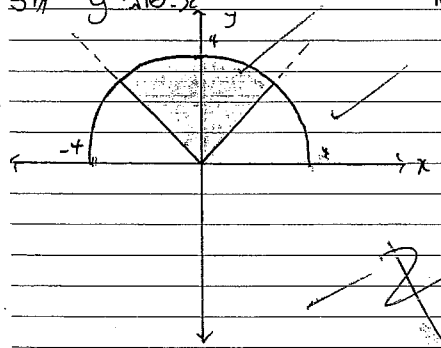
$g(-x) = \frac{1}{-x-4} + 1$

$-g(x) = -\left(\frac{1}{x-4} + 1\right)$

$= -\frac{1}{x-4} - 1$

Since $g(x) \neq g(-x)$ or $-g(x)$, the function is neither odd or even

b) i) $y = \sqrt{16 - x^2}$



ii) + pts

Name:

Teacher's initials: AB

Question: 3

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ii) $A = \frac{1}{4}\pi r^2$

$A = \frac{1}{4} \times \pi \times 4^2$

$A = 4\pi$ units

c) i). $\angle BAC = \angle ADC$ (given)

$\hat{C}$ is common.

$\therefore \triangle BAC \,///\, \triangle ADC$ (equiangular)

(2 pairs of angles in proportion)

ii) $\frac{BC}{AC} = \frac{AC}{DC}$ (corresponding sides in $\triangle BAC /// ADC$ have the same ratio)

$= \frac{8}{x} = \frac{x}{5}$

$x^2 = 40$

$x = \sqrt{40}$

$= 2\sqrt{10}$