Applications of calculus to the physical world

Velocity

QUESTION **1** Complete:

a Velocity is defined as the rate of change of _____.

b If a particle is at rest its velocity is _____.

QUESTION **2** A particle is moving along the x-axis. Briefly explain:

a the significance of a negative velocity _____

b the difference between speed and velocity _____

QUESTION **3** The displacement from the origin, x m, of a moving particle at time t seconds is given by: $x = t^2 - 11t + 18$. Find:

a the displacement when $t = 6$

b the times when the particle is at the origin

c an expression for the velocity

d the velocity when $t = 6$

QUESTION **4** The velocity (v m s^{-1}) of a particle moving in a straight line is given by $v = 9 - 4t$. Find:

a the velocity when $t = 5$

b when the particle is at rest

c an expression for the displacement if $x = 16$ when $t = 3$

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Acceleration

QUESTION **1** Complete:

Acceleration is defined as the rate of change of _____.

QUESTION **2** State whether a moving particle will be slowing down or speeding up if:

a velocity is negative but acceleration is positive _____

b velocity is positive but acceleration is negative _____

c velocity and acceleration are both positive _____

d velocity and acceleration are both negative _____

QUESTION **3** The displacement x metres at time t seconds of a moving particle is given by $x = 80t^2 - t^3$

Find:

a an expression for the acceleration

b the acceleration when $t = 2$

QUESTION **4** The acceleration, in m s^{-2}, of a particle moving along the x-axis is given by $\ddot{x} = -2$

When $t = 1$, $\dot{x} = 4$ and $x = 3$. Find:

a an expression for the velocity

b the velocity when $t = 5$

c an expression for the displacement

d the displacement when $t = 5$

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Velocity and acceleration (1)

QUESTION **1** A particle is moving in a straight line from a fixed point O. At time t seconds, its displacement from O, x m, is given by $x = 18t^2 - t^3$

a At what times is the particle stationary?

b Find the displacement at the time when the acceleration is zero.

QUESTION **2** The acceleration (a m s^{-2}) of a moving particle is given by $a = 18 - 2t$
The particle starts from rest at the origin.

a Find an expression for the position, x, of the particle.

b Find the acceleration and position of the particle when it is at rest again.

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Velocity and acceleration (2)

QUESTION **1** A body, moving along the x-axis, starts from rest at the origin. Its velocity, $\dot{x}$ m s^{-1}, is given by $\dot{x} = 24t - 3t^2$. Find:

a the velocity after 5 seconds

b the acceleration after 3 seconds

c when the particle comes to rest again

d the displacement when $t = 4$

QUESTION **2** The acceleration a m s^{-2} of a moving particle at time t seconds is given by $a = 1 - \dfrac{8}{(t+1)^3}$. Initially it is at a position 3 metres to the left of the origin travelling to the right with a speed of 16 m s^{-1}. Find an expression for:

a the velocity

b the displacement

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Velocity and acceleration (3)

QUESTION **1** The acceleration of a moving particle, a m s^{-2}, is given by $a = -6$. Initially the particle is at $x = 10$ m with velocity $v = 13$ m s^{-1}.

a Find, as functions of t:

i the velocity

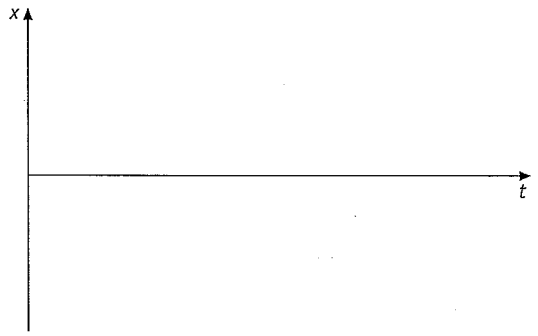
ii the displacement

b When is the particle

i at rest

ii at the origin

c Sketch x as a function of t



d Briefly describe the motion

Applications of calculus to the physical world

Velocity and acceleration (4)

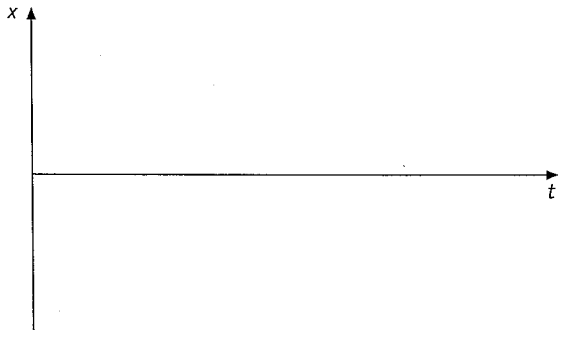
QUESTION **1** A particle moves along a straight line so that its displacement, x metres, from a fixed point, O, is given by $x = 2 \sin \pi t + 1$ where t is the time in seconds ($t \geq 0$).

a What is the position of the particle initially?

b When and where does the particle first come to rest?

c Find the acceleration when the particle is next at rest.

d Sketch a graph of displacement as a function of time.



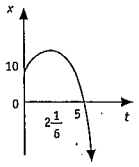
Page 130 1 a displacement b zero 2 a the particle is moving to the left b the velocity gives the direction as well as the magnitude; speed = | velocity | 3 a −12 m b at 2 seconds and at 9 seconds c $v = 2t - 11$ d 1 m s^{-1} 4 a −11 m s^{-1} b 2.25 s c $x = 9t - 2t^2 + 7$

Page 131 1 velocity 2 a slowing down b slowing down c speeding up d speeding up 3 a $a = 160 - 6t$ b 148 m s^{-2} 4 a $v = -2t + 6$ b −4 m s^{-1} c $x = -t^2 + 6t - 2$ d 3 m

Page 132 1 a 0 s and 12 s b 432 m 2 a $x = 9t^2 - \frac{1}{3}t^3$ b −18 m s^{-2}, 972 m

Page 133 1 a 45 m s^{-1} b 6 m s^{-2} c 8 s d 128 m 2 a $v = t + \frac{4}{(t+1)^2} + 12$ b $x = \frac{t^2}{2} - \frac{4}{t+1} + 12t + 1$

Page 134 1 a i $v = -6t + 13$ ii $x = -3t^2 + 13t + 10$ b i $2\frac{1}{6}$ s ii 5 s

c 

d The particle is initially at a position 10 m to the right of the origin travelling right at a speed of 13 m s^{-1}. It stops after $2\frac{1}{6}$ seconds, then moves left passing through the origin after 5 seconds, and continues to travel left at increasing speed.

Page 135 1 a 1 m to the right of the origin b $\frac{1}{2}$ s, 3 m c $2\pi^2$ m s^{-2} d (see right)