

Applications of calculus to the physical world

TOPIC TEST

Time allowed: 1 hour

Total marks = 100

SECTION I Multiple-choice questions

10 marks

Instructions • This section consists of 10 multiple-choice questions

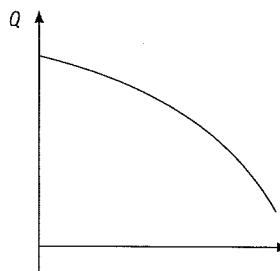
- Each question is worth 1 mark
- Fill in only ONE CIRCLE
- Calculators may be used

1 Velocity is the rate of change of:

- (A) displacement (B) speed (C) acceleration (D) none of these

2 From the diagram

- (A) $\frac{dQ}{dt} > 0$ and $\frac{d^2Q}{dt^2} > 0$
(B) $\frac{dQ}{dt} > 0$ and $\frac{d^2Q}{dt^2} < 0$
(C) $\frac{dQ}{dt} < 0$ and $\frac{d^2Q}{dt^2} > 0$
(D) $\frac{dQ}{dt} < 0$ and $\frac{d^2Q}{dt^2} < 0$



3 If a particle is undergoing motion such that at a particular time T , $x > 0$, $\dot{x} < 0$ and $\ddot{x} < 0$ then at T the particle is:

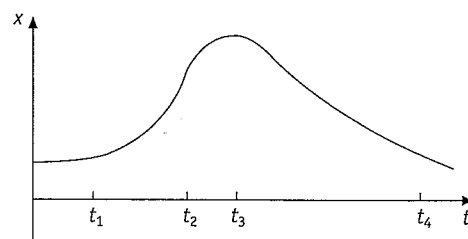
- (A) slowing down (B) speeding up (C) stationary
(D) there is not enough information to determine what the particle is doing

4 The population P of a colony of flying foxes at time t is approximated using the expression $P = 800e^{0.5t}$. Initially the population was approximately:

- (A) 1320 (B) 800 (C) 400
(D) there is not enough information to determine the initial population

5 The graph shows the distance x of a particle (which is moving in a straight line), from a fixed point at time t . At what time is the particle moving fastest?

- (A) t_1 (B) t_2
(C) t_3 (D) t_4



6 If the velocity of a moving particle at time t is given by $v = 9t$ the acceleration is?

- (A) constant (B) zero (C) increasing (D) decreasing

7 The rate, R kg/s at which grain is flowing from a bin is given by $R = 80t - 3t^2$ where t is the time in seconds. For which value of t is R not physically possible?

- (A) $t = 0$ (B) $t = 10$ (C) $t = 20$ (D) $t = 30$

8 Which *must* be true if a particle is stationary?

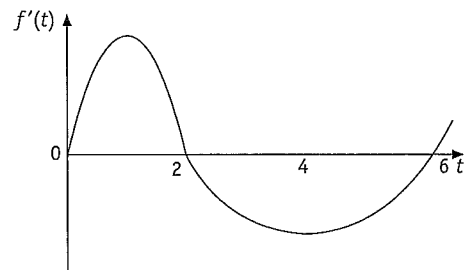
- (A) displacement is zero (B) velocity is zero
(C) acceleration is zero (D) all of these

9 The number of animals on an island, N , at time t is given by the formula $N = 7000e^{-kt}$ where k is a positive constant. Over time the number of animals on the island is?

- (A) increasing exponentially (B) decreasing exponentially
(C) increasing at a constant rate (D) decreasing at a constant rate

10 The diagram shows a sketch of the graph of $y = f'(t)$ at time t . When $t = 2$, $y = f'(t)$ is?

- (A) a maximum
(B) a minimum
(C) zero
(D) there is insufficient information



SECTION II

90 marks

Show all necessary working

11 If $y = 8 - 5t - 3t^2$ find:

2 marks each

a $\frac{dy}{dt}$

b $\frac{d^2y}{dt^2}$

12 If $f(t) = 2 \cos 4t$ find:

2 marks each

a $f'(t)$

b $f''(t)$

13 If $x = t^3 - 3t^2 + 7e^t$ find:

2 marks each

a \dot{x}

b \ddot{x}

14 Find x if $\dot{x} = 8t^2 - 6t + 3$ and, when $t = 2$, $x = 3$

4 marks

15 Find x if $\ddot{x} = -4$ and, when $t = 1$, $\dot{x} = 5$ and $x = 8$

6 marks

16 $Q = 7 + 20t - 2t^2$ ($t \geq 0$). Find:

a Q when $t = 4$

2 marks

b t when $Q = 39$

4 marks

c $\frac{dQ}{dt}$ when $t = 3$

4 marks

d t when $\frac{dQ}{dt} = 0$

4 marks

17 $\frac{dP}{dt} = 17 - 4t$. When $t = 0$, $P = 12$. Find:

a $\frac{dP}{dt}$ when $t = 3$

2 marks

b P when $t = 3$

5 marks

18 Gas is pumped into a previously empty chamber at a rate given by $\frac{dV}{dt} = \frac{e^t}{5}$ where V is the volume of gas in litres and t is the time in minutes.

a What is the initial rate at which the gas is pumped into the chamber?

3 marks

b Find an expression for V in terms of t .

5 marks

19 The number of bacteria in a culture is given by $N = 10\,000e^{kt}$ where k is a positive constant and t the time in hours. After 5 hours the number of bacteria has increased to 20 000.

a What was the initial number of bacteria?

2 marks

b Find the value of k correct to four decimal places.

4 marks

continued ...

19 continued ...

c How many bacteria are in the colony after 24 hours?

4 marks

_____	_____
_____	_____
_____	_____

d How long is it before the number of bacteria reaches 1 million?

5 marks

_____	_____
_____	_____
_____	_____

20 The displacement, x m, of a moving particle at time t seconds is given by $x = \ln(1 + t)$

a Find the velocity when $t = 2$

4 marks

_____	_____
_____	_____
_____	_____

b Find the acceleration when $t = 2$

5 marks

_____	_____
_____	_____
_____	_____

21 The acceleration (a m s⁻²) of a particle moving along the x -axis is given by $a = -1$. Originally the particle is moving with velocity 4 m s⁻¹ at a position 8 m to the left of the origin.

a When is the particle stationary?

5 marks

_____	_____
_____	_____
_____	_____
_____	_____
_____	_____

b When is the particle at the origin?

6 marks

_____	_____
_____	_____
_____	_____
_____	_____
_____	_____

c Briefly explain why the particle will never move to the right of the origin.

4 marks

_____	_____
_____	_____
_____	_____

Pages 136-140 1 A 2 D 3 B 4 B 5 B 6 A 7 D 8 B 9 B 10 A 11 a $-5 - 6t$ b -6

12 a $-8 \sin 4t$ b $-32 \cos 4t$ 13 a $3t^2 - 6t + 7e^t$ b $6t - 6 + 7e^t$

14 $x = \frac{8t^3}{3} - 3t^2 + 3t - 12\frac{1}{3}$ 15 $x = -2t^2 + 9t + 1$ 16 a 55 b 2 or 8 c 8 d 5 17 a 5 b 45 18 a 0.2 L/min b $V = \frac{e^t - 1}{5}$

19 a 10 000 b 0.1386 c 278 576 [nearest whole number] d 34th hour 20 a $\frac{1}{3} \text{ m s}^{-1}$ b $-\frac{1}{9} \text{ m s}^{-2}$ 21 a 4 s b 4 s c The particle

is stationary when $t = 4$, and because $\ddot{x} < 0$, the maximum displacement occurs when $t = 4$. So the maximum displacement is 0 m and the particle never moves right of the origin.