C.E.M.TUITION

Student Name :________________________________

Review Topic : Rates of change
(HSC - Paper 1)

1. When Nicolium crystals are placed in solution, they dissolve in such a way that their volume, V mm^3, is given by

$$V = 20t - \frac{t^2}{4} + 2.$$

(a) At what rate do the crystals dissolve at the end of 7 minutes?

(b) What is the initial rate at which the crystals dissolve?

(c) After how many minutes are the crystals dissolving at the rate of 10 cubic millimetres per minute?

(d) How long does it take for the crystals to dissolve completely? (Give your answer correct to the nearest minute.)

2. Water was poured into a tank for 10 hours until it was full. At any time, t hours, the volume, V litres, of water in the tank was given by $V = 2(20t - t^2 + 100)$.

(a) How much water was in the tank initially?

(b) How many litres of water were in the tank when it was full?

(c) At what rate was water poured into the tank at the end of 5 hours?

3. Suppose that oil flows out of a pipe at the rate, R cubic metres per minute, where $R = 2t + 25$ and t is time in minutes.

(a) Find the rate of flow of oil at the end of 10 minutes.

(b) Draw a sketch of R as a function of t.

(c) Calculate the total volume of oil that flows through the pipe in the first 10 minutes.

4. A water tank had 5000 litres of water in it. Water is flowing into the tank at the rate R litres per minute, where $R = 1 \cdot 4t$ and t is time in minutes.
   (a) Find the formula for the volume, V litres of water in the tank at any time t.
   (b) How much water is in the tank after 8 minutes?

5. The rate, R grams per minute, at which a certain chemical compound is formed during a chemical reaction is given by the relationship

$$R = 20 + \frac{10}{2t+2}, \text{ where } t \text{ is time in minutes.}$$

(a) At what rate is the compound formed at the end of 4 minutes?

(b) What is the value of R as t becomes very large?

(c) Draw a sketch of R as a function of T.

(d) How many grams of the compound are formed in the first 5 minutes of the chemical reaction?

6. A hazardous chemical leaks out from the bottom of a storage tank at the rate

$$\frac{dV}{dt} = 30e^{-0\cdot006t},$$

where V is the volume of the chemical remaining in the tank in litres and t is time in hours.

(a) At what rate is the chemical leaking out of the tank after 5 hours?

(b) If $R = \frac{dV}{dt}$, draw a sketch of R as a function of t.

(c) How much chemical leaks out of the tank in the first 20 hours?

1. (a) $V = 20t - \frac{t^2}{4} + 2$

Note $\frac{t^2}{4} = \frac{1}{4}t^2$

$= 20t - \frac{1}{4}t^2 + 2$

$\frac{dV}{dt} = 20 - \frac{1}{2}t \quad \left(\text{Rate} = \frac{dV}{dt}\right)$

when $t = 7$, $\frac{dV}{dt} = 20 - \frac{1}{2} \times 7$
$= 16{\cdot}5.$

Therefore, the rate at which the crystals dissolve at the end of 7 minutes is 16·5 mm^3 per minute.

(b) The initial rate $\Rightarrow$ find $\frac{dV}{dt}$, when $t = 0$.

$t = 0, \ \frac{dV}{dt} = 20 - \frac{1}{2}t$
$= 20 - \frac{1}{2} \times 0 = 20.$

Therefore, the initial rate is 20 mm^3 per minute.

(c) In this question we are asked to find t, given $\frac{dV}{dt} = 10$.

$\frac{dV}{dt} = 20 - \frac{1}{2}t$, when $\frac{dV}{dt} = 10$

$20 - \frac{1}{2}t = 10$

(Multiply both sides by 2.)

$40 - t = 20$

$t = 20.$

Therefore, after 20 minutes the crystals dissolve at the rate of 10 mm^3 per minute?

(d) The crystals dissolve completely when $V = 0$.

$V = 20t - \frac{t^2}{4} + 2$

When $V = 0$,

$20t - \frac{t^2}{4} + 2 = 0$

(Multiply both sides by 4.)

$80t - t^2 + 8 = 0$

(Multiply both sides by -1.)

$t^2 - 80t - 8 = 0$

(Use Quadratic Formula to solve)

$a = 1, \ b = -80, \ c = -8$

$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-80) \pm \sqrt{(-80)^2 - 4(1)(-8)}}{2 \times 1}$

$= \frac{-(-80) \pm \sqrt{6432}}{2}$

$= \frac{80 + \sqrt{6432}}{2}$ or $\frac{80 - \sqrt{6432}}{2}$

$= 80{\cdot}099\,8$ or $-0{\cdot}099\,8$

$\approx 80{\cdot}1$ or $-0{\cdot}1$ (1dp)

$t = -0{\cdot}1$ is meaningless, and discarded. Therefore the crystals dissolve completely after 80 minutes.

2. $V = 2(20t - t^2 + 100)$

$V = 40t - 2t^2 + 200$

(a) Initially (i.e. $t = 0$),

$V = 40 \times 0 - 2 \times (0^2) + 200$
$= 200$

Therefore, initially there were 200 litres of water in the tank.

(b) The tank is completely full after 10 hours.

When $t = 10$,

$V = 40t - 2t^2 + 200$

$V = (40 \times 10) - 2 \times (10^2) + 200$
$= 400.$

Therefore, there were 400 litres of water when the tank was full.

(c) $V = 40t - 2t^2 + 200$

$\frac{dV}{dt} = 40 - 4t \quad \left[\frac{dV}{dt} = \text{rate}\right]$

When $t = 5$,

$\frac{dV}{dt} = 40 - 4 \times 5$
$= 20.$

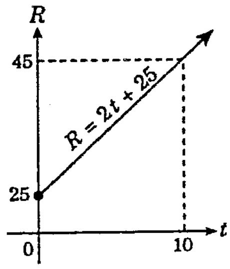
$\therefore$ After 5 hours water poured in the tank at the rate of 20 litres/h.

3. (a) $R = 2t + 25$

When $t = 10$, $R = 2 \times 10 + 25$
$= 45.$

Therefore, the rate of flow of oil at the end of 10 minutes is 45 m^3 per minute.

(b)



(c) R represents the rate of flow, i.e. $\frac{dV}{dt} = R = 2t + 25$

Therefore, the total volume of oil that flows through the pipe in the first 10 minutes is given by:

$\int_0^{10} R\, dt = \int_0^{10} (2t + 25)\, dt$

$= \left[t^2 + 25t\right]_0^{10}$

$= \left[10^2 + 25(10)\right] - [0 + 0]$

$= (100 + 250) - (0)$

$= 350.$

Therefore, 350 litres of oil flows through the pipe in the first 10 minutes.

4. (a) $R = 1{\cdot}4t$ *Note* $R = \frac{dV}{dt}$

$\frac{dV}{dt} = 1{\cdot}4t$

and when $t = 0$, $V = 5000$.

$\frac{dV}{dt} = 1{\cdot}4t$

$V = \int 1{\cdot}4t\, dt$

$V = 0{\cdot}7t^2 + c,$

when $t = 0$, $V = 5000$,

$5000 = 0 + c \Rightarrow c = 5000$

$\therefore \quad V = 0{\cdot}7t^2 + 5000.$

(b) When $t = 8$,

$V = 0{\cdot}7t^2 + 5000$
$= 0{\cdot}7 \times 8^2 + 5000$
$= 5044{\cdot}8.$

Therefore, after 8 minutes the tank contains approximately 5045 litres of water.

5. (a) $R = 20 + \frac{10}{2t + 2}$

when $t = 4$, $R = 20 + \frac{10}{8 + 2}$

$= 20 + \frac{10}{10}$
$= 20 + 1$
$= 21.$

Therefore, at the end of 4 minutes the compound is formed at the rate of 21 grams per minute.

(b) As t becomes very large, (i.e. $t \to \infty$)

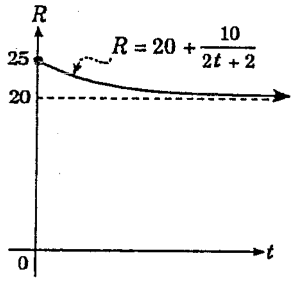
$R \to 20 + 0 \to 20$

Note $\lim_{t \to 0} \frac{10}{2t + 2} = 0$

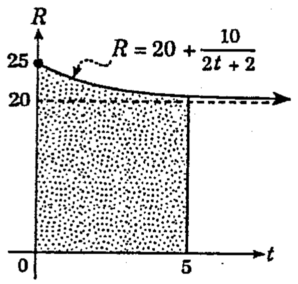
Therefore, as t becomes very large, R approaches 20 grams/minute.

(c) To sketch $R = 20 + \dfrac{10}{2t+2}$

> *Note* When $t = 0$
> $R = 20 + \dfrac{10}{0+2}$
> $= 20 + 5 \quad = 25$
> as $t \to \infty, \quad R \to 20$
> $t \geq 0, \quad R \geq 20.$



(d)



The amount of compound formed is given by the shaded area under the curve,

$= \int_0^5 R\,dt$

$= \int_0^5 \left(20 + \dfrac{10}{2t+2}\right) dt$

$= \left[20t + 5\ln(2t+2)\right]_0^5$

$= (100 + 5\ln 12) - (0 + 5\ln 2)$

$= 100 + 5\ln 12 - 5\ln 2$

$= 100 + 5(\ln 12 - \ln 2)$

$= 100 + 5\ln\left(\dfrac{12}{2}\right)$

> *Note* $\log\dfrac{a}{b} = \log a - \log b$

$= 100 + 5\ln 6.$

Therefore, the amount of compound formed in the first 5 minutes of the reaction is $100 + 5\ln 6 \approx 109$ grams.

6. (a) $\dfrac{dV}{dt} = 30e^{-0{\cdot}006t}$

When $t = 5$, $\dfrac{dV}{dt} = 30e^{-0{\cdot}006 \times 5}$

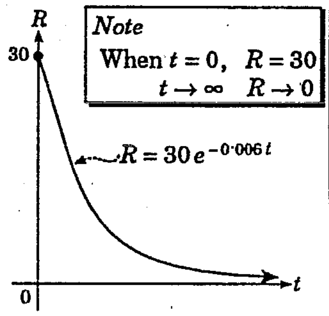
$\approx 29{\cdot}1.$

Therefore, after 5 hours, the chemical is leaking out of the tank at the rate of 29·1 litres per hour.

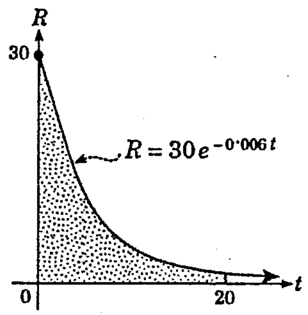
(b) To sketch

$R = \dfrac{dV}{dt} = 30e^{-0{\cdot}006t}$

> *Note*
> When $t = 0, \quad R = 30$
> $t \to \infty \quad R \to 0$



(c) The amount of chemical that leaks out of the tank in the first 20 hours is represented by the area under the curve $R = f(t)$ between $t = 0$ and $t = 20$.



Area under curve

$= \int_0^{20} R\,dt$

$= \int_0^{20} 30e^{-0{\cdot}006t}\,dt$

$= \left[\dfrac{30e^{-0{\cdot}006t}}{-0{\cdot}006}\right]_0^{20}$

$= \left[-5000e^{-0{\cdot}006t}\right]_0^{20}$

$= -5000\left[e^{-0{\cdot}006t}\right]_0^{20}$

$= -5000\left[e^{-0{\cdot}006 \times 20} - e^{-0{\cdot}006 \times 0}\right]$

$= -5000\left[e^{-0{\cdot}12} - e^0\right]$

$= -5000\left[e^{-0{\cdot}12} - 1\right]$

> *Note* $e^0 = 1$

$= 565{\cdot}397\,82$

≈ 565 (nearest whole number).

Therefore 565 litres of the compound leaks out of the tank in the first 20 hours.