C.E.M.TUITION

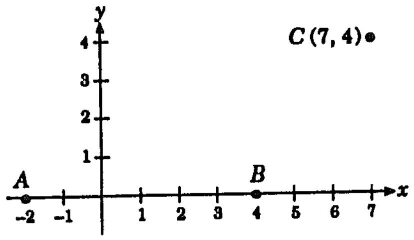
Name :________________________________

Review Topic : Coordinate Geometry

(HSC - PAPER 1)

Year 12 - Mathematics

1.



A, *B*, *C* and *D* have coordinates (−2, 0), (4, 0), (7, 4) and (1, 4) respectively.

(a) Draw this diagram and label the point *D*. Draw the lines *BC* and *CD*.

(b) If $\angle BAD = \alpha°$, show that $\tan\alpha = \frac{4}{3}$.

(c) Prove that $\angle ABC = (180 - \alpha)°$. Why is $AD \parallel BC$?

(d) (i) Find the equation of the line *AC* and the midpoint of the interval *BD*.

(ii) Show that *AC* bisects *BD*. Why is this an expected result?

(e) Calculate the area of figure *ABCD* and hence write down the area of ΔACD.

2. A, B and C are the points (2, 2), (4, 8) and (−2, 5) respectively. Plot these points on your number plane.

(a) Find:
- **(i)** distance between B and C;
- **(ii)** equation of the line containing B and C;
- **(iii)** equation of the line perpendicular to BC passing through A;
- **(iv)** the perpendicular distance from A to BC;
- **(v)** the midpoint, L, of AB.

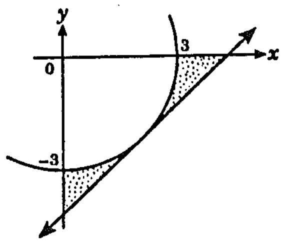
(b) Calculate the area of ΔABC.

(c) Derive the equation of the median, CL, of the ΔABC.

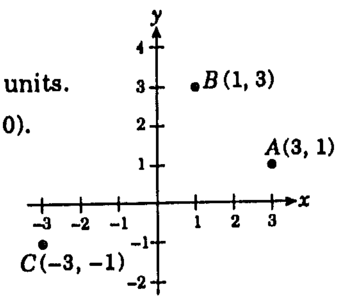
3. (a) Prove that the line $3x - 4y = 15$ is a tangent to the circle $x^2 + y^2 = 9$.

(b) Derive the equation of the line perpendicular to the tangent at the point of contact.

(c) Calculate the area in the fourth quadrant between the tangent and the circle (shaded on diagram).



4. A, B and C are the points (3, 1), (1, 3) and (–3, –1) respectively.



(a) Show that the length of BC is $4\sqrt{2}$ units.

(b) Show that the midpoint of AC is (0, 0).

(c) Derive the equation of the line BC.

(d) Show that the equation, ℓ, of the line through A parallel to BC has equation $x - y - 2 = 0$.

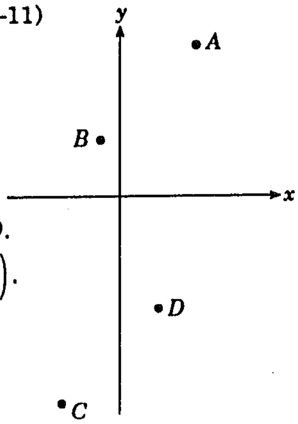
(e) **(i)** From the diagram plot the point D, the fourth vertex of parallelogram $ABCD$ (with diagonal AC). Write down the coordinates of D.

(ii) Show that D lies on ℓ.

(f) Find the area of parallelogram $ABCD$.

(g) Prove that the diagonals AC and BD bisect each other.

5. Consider the points $A(4, 8)$, $B(-1, 3)$, $C(-3, -11)$ and $D(2, -6)$ on the number plane.



(a) Show that the gradient of AD is 7.

(b) Show that the length of AD is $10\sqrt{2}$.

(c) By investigating the gradient and length of BC name the type of quadrilateral that best describes $ABCD$.

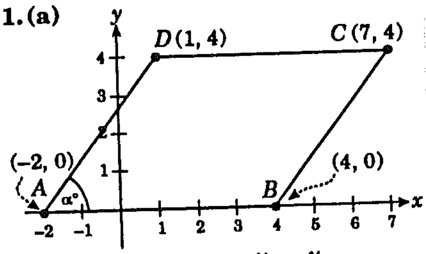
(d) Show that the midpoint of AC is $\left(\frac{1}{2}, \frac{-3}{2}\right)$.

(e) Prove that the diagonals AC and BD bisect each other.

(f) Show that the equation of the line AD is given by $7x - y - 20 = 0$.

(g) By first calculating the perpendicular distance of the point B from the line AD, find the area of $ABCD$.

1. (a)



(b) Gradient $AD = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{4-0}{1-(-2)} = \dfrac{4}{3}.$

Now $m = \tan\alpha$, $\therefore \tan\alpha = \dfrac{4}{3}.$

(c) Gradient $BC = \dfrac{4-0}{7-4} = \dfrac{4}{3}.$

Then $\tan C\hat{B}X = \dfrac{4}{3}$, i.e. $C\hat{B}X = \alpha°$

Then $A\hat{B}C = (180 - \alpha)°$
(angle sum of st. line)

Also, $AD \parallel BC$ as lines have the same gradient.

Alternatively, first state $AD \parallel BC$ because of gradients, and then follow with:

$\therefore A\hat{B}C = \alpha°$ (corr. ∠s, $AD \parallel BC$)

(d) (i) Equation of AC is of form:

$$\boxed{\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}}$$

$\therefore \quad \dfrac{y-4}{x-7} = \dfrac{0-4}{-2-7} = \dfrac{4}{9}$

$\therefore \; 9y - 36 = 4x - 28$

i.e. $4x - 9y + 8 = 0.$

Midpoint BD

$= \left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$

$= \left(\dfrac{4+1}{2}, \dfrac{0+4}{2}\right) = \left(\dfrac{5}{2}, 2\right).$

Midpoint BD is $\left(\dfrac{5}{2}, 2\right).$

(ii) AC bisects BD if AC passes through midpoint of BD *or* if midpoint of BD lies on AC.

Subst. midpoint into $4x - 9y + 8 = 0.$

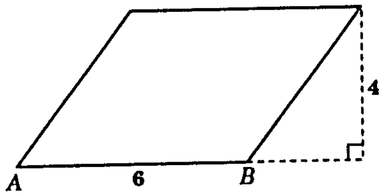
$\therefore$ LHS $= 4\left(\dfrac{5}{2}\right) - 9(2) + 8$

$= 10 - 18 + 8 = 0.$

Then midpoint satisfies eqn., i.e. AC bisects BD.

This is expected, as $ABCD$ is a parallelogram (both pairs of opposite sides parallel, $AB \parallel DC$ is obvious); diagonals of a parallelogram bisect each other.

(e) We need length of AB, which is 6 units from diagram, and height which is 4 from diagram.



Area $= b \times h$

$= 6 \times 4 = 24.$

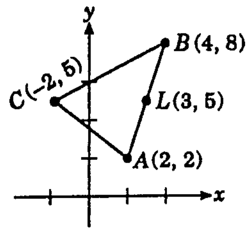
Area is 24 units^2.

Area $\Delta ACD = \dfrac{1}{2} \times 24$

$= 12$ units^2.

(Diagonals cut parallelogram into two equal triangles.)

2.



(a) (i) Distance BC

$= \sqrt{(4 - -2)^2 + (8-5)^2}$

$= \sqrt{45}$

$= 3\sqrt{5}$ units.

(ii) Equation of BC is of form:

$\dfrac{y-y_1}{x-x_1} = \dfrac{y_2-y_1}{x_2-x_1}$

i.e. $\dfrac{y-5}{x+2} = \dfrac{8-5}{4+2}$

$= \dfrac{3}{6}$

$= \dfrac{1}{2}$, *

$\therefore \; 2(y-5) = 1(x+2)$ $\boxed{\times}$

i.e. $2y - 10 = x + 2$

$\therefore \; x - 2y + 12 = 0$ is eqn.

(iii) $m_{BC} = \dfrac{1}{2}$ (from *),

$\therefore$ gradient perpendicular $= -2$.

$\boxed{m_1 \times m_2 = -1}$

Eqn. is of form:

$y - y_1 = m(x - x_1)$ $\boxed{(2, 2)}$

$\therefore \quad y - 2 = -2(x-2)$

$= -2x + 4,$

$\therefore\ 2x+y-6=0.$

(iv) $p=\left|\dfrac{Ax_1+By_1+C}{\sqrt{A^2+B^2}}\right|$

$A=1 \quad x_1=2$
$B=-2$
$C=12 \quad y_1=2$

$=\left|\dfrac{1(2)+(-2)(2)+12}{\sqrt{1^2+(-2)^2}}\right|$

$=\left|\dfrac{2-4+12}{\sqrt{5}}\right|$

$=\dfrac{10}{\sqrt{5}}=\dfrac{10\sqrt{5}}{5}=2\sqrt{5}.$

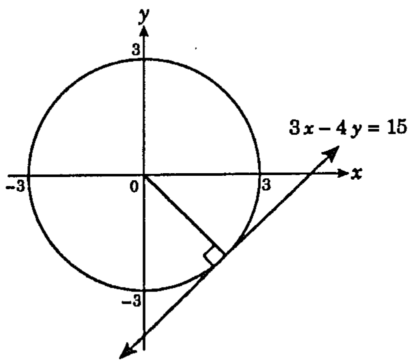
Perp. distance is $2\sqrt{5}$ units.

(v) $L=\left(\dfrac{2+4}{2},\ \dfrac{2+8}{2}\right)$
$=(3,\ 5).$

(b) Area
$=\dfrac{1}{2}\times BC\times$ perp. distance
$=\dfrac{1}{2}\times 3\sqrt{5}\times 2\sqrt{5}$
$=15$ units^2.

(c) By observation, equation of CL is $y=5$ (check diagram).

3. **(a)** Circle has centre (0, 0), radius = 3 units. Radius drawn to point of contact of tangent is right angle.



If perpendicular distance from (0, 0) to $3x-4y=15$ is 3 units, then $3x-4y=15$ is a tangent.

$p=\left|\dfrac{Ax_1+By_1+C}{\sqrt{A^2+B^2}}\right|$

$A=3 \quad x_1=0$
$B=-4$
$C=-15 \quad y_1=0$

$=\left|\dfrac{0+0-15}{\sqrt{9+16}}\right|$

$=\left|\dfrac{-15}{\sqrt{25}}\right|$

$=\left|\dfrac{-15}{5}\right|=3$ (= radius).

$\therefore\ 3x-4y=15$ is tangent to circle.

(b) $3x-4y=15$
$\therefore\ 4y=3x-15$
$y=\dfrac{3}{4}x-\dfrac{15}{4}$
$\therefore\ m=\dfrac{3}{4}.$

Perpendicular gradient
$=\dfrac{-4}{3}$ $\quad\boxed{m_1m_2=-1}$

Eqn. of perpendicular is of form:
$y-0=\dfrac{-4}{3}(x-0)$
$\therefore\ 3y=-4x,$
i.e. $4x+3y=0$ is required equation.

(c) Shaded A
= area of Δ − area of $\dfrac{1}{4}$ circle
$=\left(\dfrac{1}{2}\times 5\times\dfrac{15}{4}\right)-\dfrac{1}{4}\pi(3)^2$
$=\dfrac{3}{8}[25-6\pi]$ units^2.

Note Line $3x-4y=15$ cuts x axis at $5(y=0)$ and y axis at $\dfrac{-15}{4}(x=0)$.

4. $A(3,1),\ B(1,3),\ C(-3,-1)$

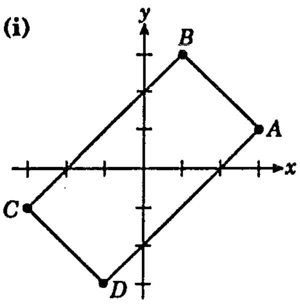
(a) $BC=\sqrt{(1--3)^2+(3--1)^2}$
$=\sqrt{32}$
$=4\sqrt{2}$ units.

(b) Midpoint of AC
$=\left(\dfrac{3+-3}{2},\ \dfrac{1+-1}{2}\right)$
$=(0,\ 0).$

(c) Equation of BC has form:
$\dfrac{y-3}{x-1}=\dfrac{-1-3}{-3-1}=\dfrac{-4}{-4}=1$ *
$\therefore\ y-3=x-1$
i.e. $x-y+2=0.$

(d) Line parallel to BC has same gradient, i.e. $m=1$ *.
Eqn. is of form:
$y-1=1(x-3)\quad [A(3,1)]$
$\therefore\ y-1=x-3,$
i.e. $x-y-2=0$ is equation of ℓ.

(e) **(i)**



From diagram, coords. of D are (−1, −3).

→ 2   ↓ 2 B to A

(ii) Substitute (−1, −3) into $x-y-2=0.$
LHS $=-1+3-2$
$=0=$ RHS.
$D(-1,-3)$ lies on ℓ.

(f) Gradient $AB=\dfrac{3-1}{1-3}$
$=\dfrac{2}{-2}=-1.$

Hence $AB\perp BC$ $(1\times -1=-1)$,
$\therefore$ figure $ABCD$ is a rectangle.
Area $=L\times W$
$=BC\times AB$
$=4\sqrt{2}\times 2\sqrt{2}$
$=16$ units^2.

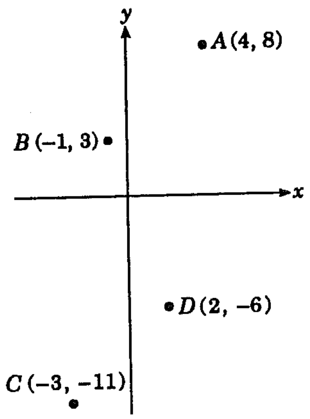
$$\boxed{\begin{aligned}AB&=\sqrt{(3-1)^2+(1-3)^2}\\&=\sqrt{4+4}=\sqrt{8}\\&=2\sqrt{2}.\end{aligned}}$$

(g) Midpoint $AC = (0, 0)$ from (b).
Midpoint BD

$$= \left(\frac{1+-1}{2}, \frac{3+-3}{2}\right)$$
$$= (0,\ 0).$$

Diagonals AC and BD both have midpoints at $(0, 0)$,
$\therefore$ diagonals bisect each other.

5.



(a) $m_{AD} = \dfrac{8-(-6)}{4-2} = \dfrac{14}{2} = 7.$

(b) $AD = \sqrt{(4-2)^2 + (8+6)^2}$
$= \sqrt{4+196} \quad = \sqrt{200}$
$= 10\sqrt{2}.$

(c) $m_{BC} = \dfrac{3-(-11)}{-1-(-3)} \quad = \dfrac{14}{2}$
$= 7 \qquad = m_{AD}.$

$BC = \sqrt{(-1+3)^2 + (3+11)^2}$
$= \sqrt{200} = 10\sqrt{2} = AD.$

$ABCD$ is a parallelogram as one pair of opposite sides is both parallel and equal.

(d) $MP_{AC} = \left(\dfrac{4-3}{2}, \dfrac{8-11}{2}\right)$
$= \left(\dfrac{1}{2}, \dfrac{-3}{2}\right).$

(e) $MP_{BD} = \left(\dfrac{-1+2}{2}, \dfrac{3-6}{2}\right)$
$= \left(\dfrac{1}{2}, \dfrac{-3}{2}\right).$

As midpoints of both diagonals are the same, the diagonals bisect each other.

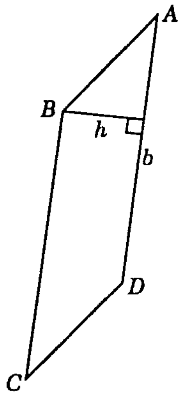
(f) Eqn. of AD is of form:
$y - 8 = 7(x-4)$
$= 7x - 28,$
$\therefore\ 7x - y - 20 = 0$ is eqn. of AD.

(g) $p = \left|\dfrac{Ax_1 + By_1 + C}{\sqrt{A^2+B^2}}\right|$

$A = 7$, $B = -1$, $C = -20$, $x_1 = -1$, $y_1 = 3$

$$= \left|\frac{7(-1)+(-1)(3)-20}{\sqrt{7^2+(-1)^2}}\right|$$
$$= \left|\frac{-7-3-20}{\sqrt{50}}\right|$$
$$= \frac{30}{\sqrt{50}} \qquad = \frac{3\sqrt{50}}{5}$$
$$= \frac{3.\cancel{5}\sqrt{2}}{\cancel{5}} = 3\sqrt{2}.$$

$$\boxed{\frac{30}{\sqrt{50}} \times \frac{\sqrt{50}}{\sqrt{50}} = \frac{3\sqrt{50}}{5} = \frac{3}{5}\sqrt{50}}$$



Area of parallelogram
= base × height
= AD × perp. ht.
$= 10\sqrt{2} \times 3\sqrt{2}$
$= 60.$
Area is 60 units^2