C.E.M.TUITION

Student Name :________________________________

Review Topic : Rates of change
(HSC - Paper 2)

Year 12 - 2 Unit

7. The atmospheric pressure, P kilopascals, at a height h kilometres above sea-level, is given by the formula $P = 100e^{-0{\cdot}13h}$.

(a) Find the atmospheric pressure at sea-level.

(b) At what rate is the atmospheric pressure decreasing 2 kilometres above sea-level?

8. The rate of sales, R in dollars per day, of a certain Australian company is given by $R = 2t + 120$, where t is the time in days.

(a) What is the rate of sales at the end of 10 days?

(b) Draw a sketch of R as a function of t.

(c) Calculate the total amount of sales in the first 8 days.

(d) On what day will the accumulated sales exceed $10 000?

9. Water flows from a full tank at the rate given by $\frac{dh}{dt} = -1{\cdot}5\sqrt{t}$ where h is the depth of water in metres at any time, t minutes. The height of the tank is 20 metres.

(a) Find an equation for h in terms of t.

(b) How long does it take for the tank to empty?

10. The rate of water flow into a tank which initially contains 900 litres of water is given by the relationship

$$\frac{dV}{dt} = 6(6 - t),$$

where V litres is the volume of water in the tank after t minutes.

(a) Find the volume of water in the tank after 10 minutes.

(b) How long does it take for the tank to empty? (Give your answer to the nearest minute.)

7. $P = 100e^{-0\cdot13h}$

(a) At sea-level, $h = 0$.
When $h = 0$,

$$P = 100e^{-0\cdot13\times0} = 100e^0 \quad \boxed{\text{Note } e^0 = 1} = 100.$$

Therefore the atmospheric pressure at sea-level is 100 kilopascals.

(b)

$$P = 100e^{-0\cdot13h}$$

$$\frac{dP}{dh} = 100e^{-0\cdot13h} \times -0\cdot13 = -13e^{-0\cdot13h}.$$

When $h = 2$,

$$\frac{dP}{dh} = -13e^{-0\cdot13\times2} = -10\cdot023\,671 \approx -10 \text{ (nearest whole no.).}$$

> The negative value indicates that as height (h) increases, the pressure (P) decreases,
> i.e. the rate is –ve.

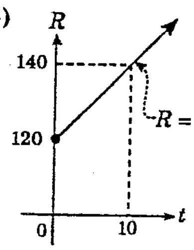
Therefore, at a height of 2 kilometres above sea-level the atmospheric pressure decreases at the rate of approximately 10 kilopascals per kilometre.

8. (a) $R = 2t + 120$

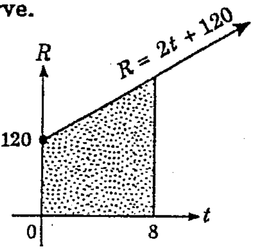
When $t = 10$, $R = 2\times10+120 = 20+120 = 140.$

Therefore, after 10 days the rate of sales is $140 per day.

(b)



(c) The total amount of sales in the first 8 days is represented by the shaded area under the curve.



Therefore, accumulated sales for the first 8 days is given by:

$$\int_0^8 R\,dt = \int_0^8 (2t+120)\,dt = \left[t^2+120t\right]_0^8 = \left[8^2+120(8)\right]-[0+0] = 1024.$$

Therefore, the total amount of sales in the fist 8 days is equal to $1024.

(d) To find the day which the accumulated sales exceed $10 000 we need to find the value of k such that

$$\int_0^k (2t+120)\,dt > 10\,000$$

where k is the required day.

$$\int_0^k (2t+120)\,dt > 10\,000$$

$$\left[t^2+120t\right]_0^k > 10\,000$$

$$\left(k^2+120k\right)-(0+0) > 10\,000$$

$$k^2+120k > 10\,000$$

$$k^2+120k-10\,000 > 0.$$

> Solve $k^2+120k-10\,000=0$ by using the Quadratic Formula:
>
> $$k = \frac{-120\pm\sqrt{120^2-4(1)(-10\,000)}}{2\times1} = \frac{-120\pm\sqrt{54\,400}}{2} = \frac{-120+\sqrt{54\,400}}{2} \text{ or } \frac{-120-\sqrt{54\,400}}{2}$$
>
> $\approx 56\cdot6$ or $-176\cdot6$.

Neglect the negative value since $k > 0$ (since it represents time).

Therefore $k = 56\cdot6$. But k is a required day, therefore it must have an integral value ($k = 57$).

Therefore, on the 57th day the accumulated sales exceed $10 000?

9. (a) Need to find h where $\frac{dh}{dt} = -1\cdot5\sqrt{t}$, and when $t = 0$, $h = 20$.

$$h = \int -1\cdot5\sqrt{t}\,dt = \int -1\cdot5t^{\frac{1}{2}}\,dt \quad \boxed{\text{Note } \sqrt{t} = t^{\frac{1}{2}}}$$

$$= \frac{-1\cdot5t^{\frac{3}{2}}}{\frac{3}{2}} + C = -t^{\frac{3}{2}} + C$$

i.e. $h = -\sqrt{t^3} + C$. $\boxed{\text{Note } t^{\frac{3}{2}} = \sqrt{t^3}}$

But when $t = 0$, $h = 20$,

$\therefore\ 20 = -\sqrt{0^3} + C$

$20 = 0 + C \Rightarrow C = 20$

$\therefore\ h = -\sqrt{t^3} + 20$

i.e. $h = 20 - \sqrt{t^3}$.

(b) The tank is empty when $h = 0$, $h = 20 - \sqrt{t^3}$.

When $h = 0$,

$0 = 20 - \sqrt{t^3}$

$\sqrt{t^3} = 20$

(square both sides)

$t^3 = 400$

(take cube root on both sides)

$t = \sqrt[3]{400} \approx 7\cdot4$ (1dp).

Therefore, it takes approximately 7·4 hours for the tank to empty.

10.(a) First we need to find V,

where $\frac{dV}{dt} = 6(6-t)$, and

when $t = 0$, $V = 900$.

$\frac{dV}{dt} = 6(6-t)$ expand first

$= 36 - 6t$

$V = \int (36-6t)\,dt$

$V = 36t - 3t^2 + C$

when $t = 0$, $V = 900$

$\therefore\ 900 = 0 - 0 + C \Rightarrow C = 900$

$\therefore\ V = 36t - 3t^2 + 900$

V represents the volume of water in the tank at any time t.

When $t = 10$, and

$V = 36t - 3t^2 + 900$

$= 36(10) - 3(10)^2 + 900$

$= 360 - 300 + 900$

$= 960.$

Therefore, after 10 minutes the tank will contain 960 litres of water.

(b) The tank is empty when $V = 0$ (i.e. there is no water in the tank).

$V = 36t - 3t^2 + 900$

When $V = 0$,

$0 = 36t - 3t^2 + 900$

$36t - 3t^2 + 900 = 0$

(divide both sides by -3)

$t^2 - 12t - 300 = 0$

(quadratic equation) Solve it by using the Quadratic formula.

$t = \frac{12 \pm \sqrt{(-12)^2 - 4(1)(-300)}}{2}$

$= \frac{12 \pm \sqrt{1344}}{2}$

$= \frac{12 + \sqrt{1344}}{2}$ or $\frac{12 - \sqrt{1344}}{2}$

$= 24{\cdot}330\,303$ or $-12{\cdot}330\,303$

≈ 24 or -12 (nearest whole no.)

Neglect the negative value, since $t \geq 0$, i.e. $t = 24$ (nearest whole number). Therefore, the tank is empty after 24 minutes.