

# Geometrical applications of differentiation

## The second derivative (1)

QUESTION 1 Find  $\frac{d^2y}{dx^2}$  if:

a  $y = 7x^2 - 9x + 3$

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b  $y = x^3 + 2x^2 - 8$

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c  $y = x^7 - x^9$

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d  $y = 5x^6 + 4x^4 - 6x$

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e  $y = 8x - 2$

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f  $y = 3 - 9x - x^2$

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g  $y = (3x - 2)^5$

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h  $y = x^{-3}$

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i  $y = x^{\frac{1}{2}}$

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QUESTION 2 Find  $f''(x)$  if:

a  $f(x) = 8x^2 + 7x + 1$

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b  $f(x) = x^4 - 3x^3 - 2x$

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c  $f(x) = 6x^8$

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d  $f(x) = 2x^{-1}$

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e  $f(x) = (2 - x)^9$

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f  $f(x) = x^{-4} - x^{-7}$

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# Geometrical applications of differentiation



## The second derivative (2)

QUESTION 1 Find  $y''$  if:

a  $y = 4x^5 - 3x^7$

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b  $y = 6x^2 - 3x^{-2}$

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c  $y = (4x + 1)^8$

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QUESTION 2 Find  $f''(-2)$  if:

a  $f(x) = x^4 - 2x^3 + 8$

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b  $f(x) = 3x^3 + 2x^2 - x$

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c  $f(x) = 3 - 5x^2$

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QUESTION 3 Find the value of the second derivative at the given point on the curve.

a  $y = 7x^2 - 9x + 3$  at  $(1, 1)$

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b  $y = x\sqrt{x}$  at  $(9, 27)$

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QUESTION 4  $y = x(3x - 5)^6$  Find:

a  $\frac{dy}{dx}$

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b  $\frac{d^2y}{dx^2}$

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# Geometrical applications of differentiation

## Concavity

QUESTION 1 Complete:

- a If  $f''(x) > 0$  at point P, the curve  $y = f(x)$  is concave \_\_\_\_\_ at P  
b If  $f''(x) < 0$  at point P, the curve  $y = f(x)$  is concave \_\_\_\_\_ at P

QUESTION 2 Determine whether the curve is concave up or concave down at the point where  $x = 0$ .

a  $y = x^2 + 5x - 7$

b  $y = 2x^3 - 5x^2$

c  $y = x^3 + 2x^2 - 7x - 4$

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QUESTION 3  $f''(x) = 12x^2 - 12x - 12$ . Determine whether the curve  $y = f(x)$  is concave up or concave down at the point where:

a  $x = 1$

b  $x = -1$

c  $x = 3$

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QUESTION 4 For what values of  $x$  is the curve concave up:

a  $y = 8 - 2x - 5x^2 - x^3$

b  $y = 4x^3 + 6x^2 - 9x - 3$

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_____	_____
_____	_____



**Page 7** 1 a 14 b  $6x + 4$  c  $42x^5 - 72x^7$  d  $150x^4 + 48x^2$  e 0 f -2 g  $180(3x - 2)^3$  h  $12x^{-5}$  i  $-\frac{1}{4}x^{-\frac{3}{2}}$  2 a 16 b  $12x^2 - 18x$  c  $336x^6$  d  $4x^{-3}$  e  $72(2 - x)^7$  f  $20x^{-6} - 56x^{-9}$

**Page 8** 1 a  $80x^3 - 126x^5$  b  $12 - 18x^{-4}$  c  $896(4x + 1)^6$  2 a 72 b -32 c -10 3 a 14 b  $\frac{1}{4}$  4 a  $(3x - 5)^5(21x - 5)$  b  $18(3x - 5)^4(21x - 10)$

**Page 9** 1 a up b down 2 a concave up b concave down c concave up 3 a concave down b concave up c concave up

4 a  $x < -1\frac{2}{3}$  b  $x > -\frac{1}{2}$

**Page 10** 1 a minimum b maximum 2 a minimum when  $x = 2$  b maximum when  $x = 4$  c maximum when  $x = 1$ , minimum when  $x = 3$  d minimum when  $x = -2$ , maximum when  $x = 0$ , minimum when  $x = 2$