Geometrical applications of differentiation

EXCEL HSC MATHEMATICS page 104

Problems involving maxima and minima (1)

QUESTION 1 The height (in metres), of a ball thrown straight up from the ground is given by $h = 27t - 5t^2$ where t is the time in seconds. Find:

a the time when the height is a maximum

b the maximum height

QUESTION 2 The perimeter, in metres, of any rectangle with area 100 m^2 is given by $P = 2x + \dfrac{200}{x}$ where x is the length of one of the sides. Find:

a the value of x for which P will be a minimum

b the minimum perimeter

Problems involving maxima and minima (2)

QUESTION 1 The product P of any two numbers whose sum is 34 is given by $P = 34a - a^2$ where a is one of the numbers. Find:

a the value of a for which P is a maximum

b the maximum product

QUESTION 2 When n items are produced at a certain factory, the cost per item (\$$C$) is given by $C = \frac{8788}{n} + 211 + 2n^2$. Find:

a the value of n for which C will be a minimum

b the minimum cost per item

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Problems involving maxima and minima (3)

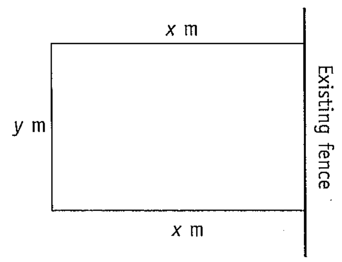
QUESTION 1 The volume of a closed cylindrical aluminium can is 128π cm^3.

a Find an expression for the height (h) in terms of the radius (r).

b Find an expression for the surface area (A) in terms of r.

c Find the least amount of aluminium needed for a can of this size. (Give the answer in terms of π)

QUESTION 2 Ty wants to establish a rectangular vegetable garden. One side will be against an existing fence and he will need to fence the other 3 sides. He has enough materials to fence an additional 10 m. If the length of the garden is x m and the width y m as shown in the diagram:



a show that the area is given by $A = 10x - 2x^2$

b find the value of x if the garden is to have maximum area

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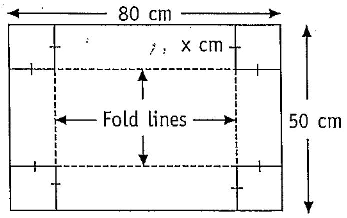
Problems involving maxima and minima (4)

QUESTION **1** A piece of cardboard 80 cm long and 50 cm wide will be used to make an open box. A square of side x cm will be cut from each corner and the sides then folded up to form the box.



a What condition must be placed on x in order for the box to exist? Justify your answer.

b Show that the volume of the box is given by $V = 4000x - 260x^2 + 4x^3$

c Find the value of x for which the box will have maximum volume.

d Find the maximum volume of the box.

Page 22 1 a 2.7 seconds b 36.45 m 2 a $x = 10$ b 40 m
Page 23 1 a $a = 17$ b 289 2 a $n = 13$ b \$1225
Page 24 1 a $h = \frac{128}{r^2}$ b $A = 2\pi r^2 + \frac{256\pi}{r}$ c 96π cm^2 2 b $x = 2.5$
Page 25 1 a $x < 25$, x cannot be longer than half the shortest side c $x = 10$ d 18 000 cm^3