10.1 Identify and Shaded The Region On The Graph That Satisfies A Linear Inequality

Remember !!!

The dash line must be drawn when the graph of $y = ax + c$ is $y > ax + c$ or $y < ax + c$.

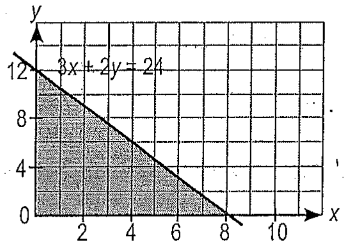
The solid line must be drawn when the graph of $y = ax + c$ is $y \geqslant ax + c$ or $y \leqslant ax + c$.

1 Shade the region that satisfies the inequality $3x + 2y \leqslant 24$.

Solution:

$3x + 2y \leqslant 24$

x	0	8
y	12	0

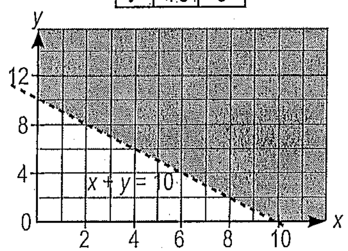


2 Shade the region that satisfies the inequality $x + y > 10$.

Solution:

$x + y > 10$

x	0	10
y	10	0

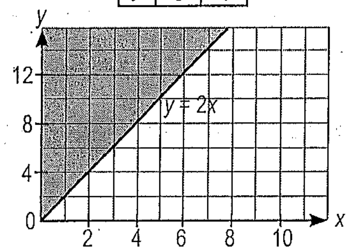


3 Shade the region that satisfies the inequality $y \geqslant 2x$.

Solution:

$y \geqslant 2x$

x	0	2
y	0	4

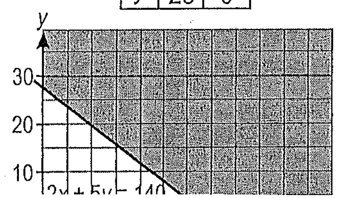


4 Shade the region that satisfies the inequality $2x + 5y \geqslant 140$.

Solution:

$2x + 5y \geqslant 140$

x	0	70
y	28	0

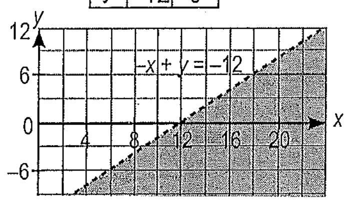


5 Shade the region that satisfies the inequality $x - y < 12$.

Solution:

$x - y < 12 \Rightarrow -x + y > -12$

x	0	12
y	−12	0

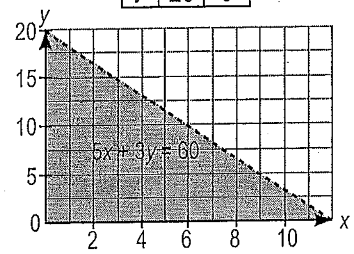


6 Shade the region that satisfies the inequality $5x + 3y < 60$.

Solution:

$5x + 3y < 60$

x	0	12
y	20	0

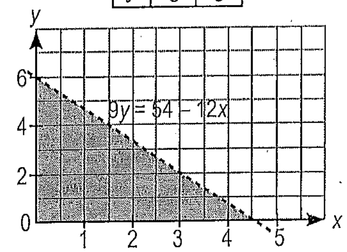


7 Shade the region that satisfies the inequality $9y < 54 - 12x$.

Solution:

$9y < 54 - 12x$

x	0	4.5
y	6	0

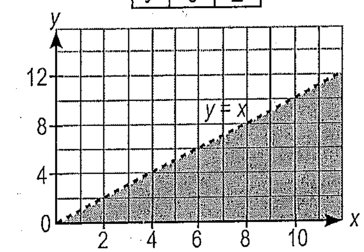


8 Shade the region that satisfies the inequality $y < x$.

Solution:

$y < x$

x	0	2
y	0	2

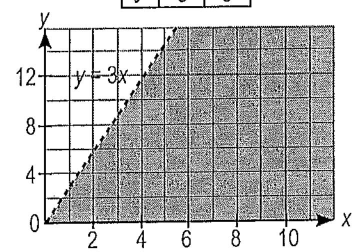


9 Shade the region that satisfies the inequality $x \leqslant 3y$.

Solution:

$y \leqslant 3x$

x	0	3
y	0	9



10 Shade the region that satisfies the inequality $5x \geqslant 400 - 8y$.

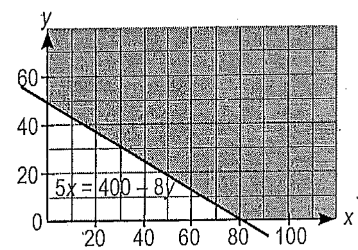
Solution:

$5x \geqslant 400 - 8y$

$400 - 8y \leqslant 5x$

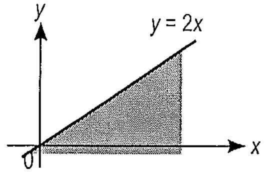
$-400 + 8y \geqslant -5x$

x	0	80
y	50	0



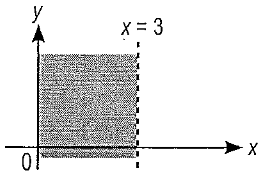
10.2 The Linear Inequality That Defines A Shaded Region

11



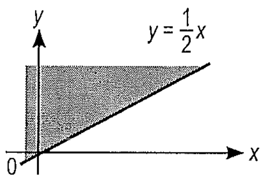
Solution:
$y \leqslant 2x$

12



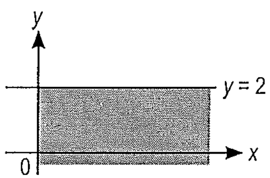
Solution:
$x < 3$

13



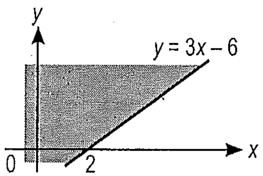
Solution:
$y \geqslant \frac{1}{2}x$

14



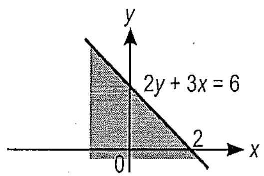
Solution:
$y \leqslant 2$

15



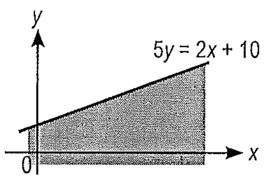
Solution:
$y \geqslant 3x - 6$

16



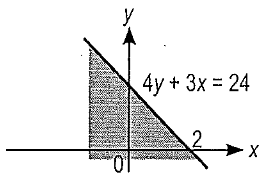
Solution:
$2y + 3x \leqslant 6$

17



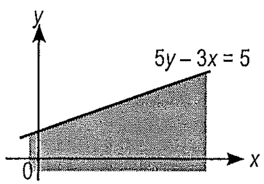
Solution:
$5y \leqslant 2x + 10$

18



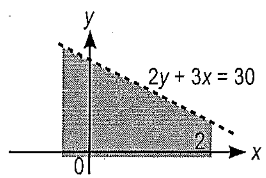
Solution:
$4y + 3x \leqslant 24$

19



Solution:

20

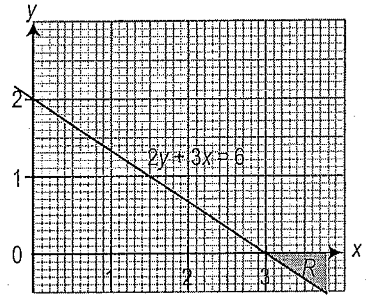


Solution:

10.3 Shaded Region On The Graph That Satisfies Several Linear Inequality

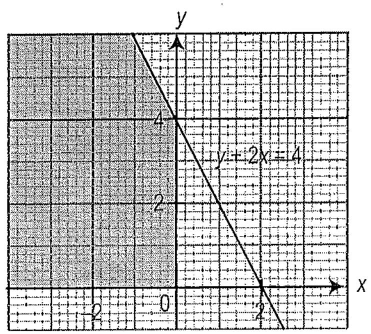
21 The diagram in the answer space shows the graph of the straight line $2x + 3y = 6$. Indicate clearly on the diagram, with the letter R, the region defines by the inequalities $y \geqslant 0$, $x \leqslant 0$, $2x + 3y \geqslant 6$

Solution:



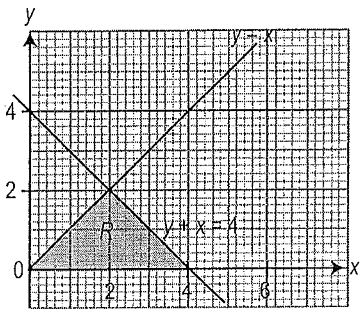
22 The diagram shows the graph of the straight line $2x + y = 4$. Indicate clearly on the diagram, with the letter R, the region defined by the inequalities $x \geqslant 0$, $y \leqslant 0$, $2x + y \leqslant 4$

Solution:



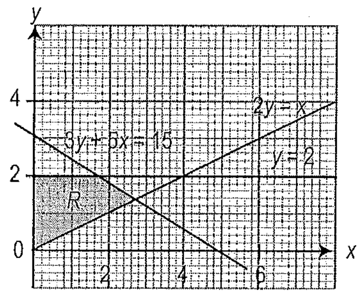
23 Construct and shade the region R that satisfies the inequalities $y \geqslant 0$, $x + y \leqslant 4$ and $y \leqslant x$.

Solution:



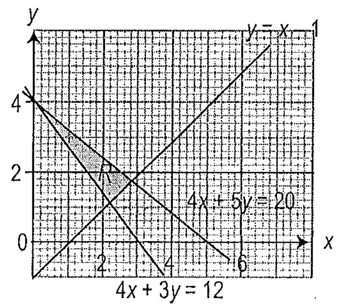
24 Construct and shade the region R that satisfies the inequalities $y \leqslant 2$, $3x + 5y \leqslant 15$ and $2y > x$.

Solution:



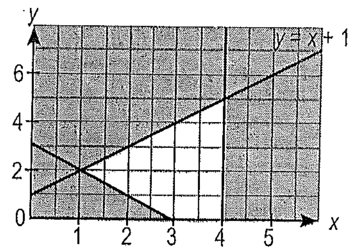
25 Draw the straight line $y = x - 1$, $4x + 5y = 20$ and $4x + 3y = 12$. Shade the region R that satisfies the inequalities $y \geqslant x - 1$, $4x + 5y \leqslant 20$ and $4x + 3y \geqslant 12$.

Solution:



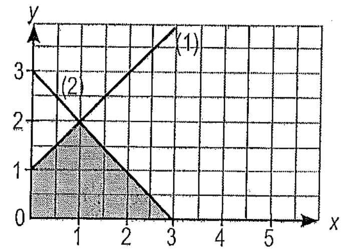
10.4 Finding Linear Inequality That Defines A Shaded Region

26 The unshaded region in the diagram is defined by four inequalities, two of which are $y \geqslant 0$, $y \leqslant x + 1$. Write down the other two inequalities.



Solution:
$x + y \geqslant 3$, $x \leqslant 4$

27 The shaded region in the diagram is defined by four inequalities, two of which are $x \geqslant 0$ and $y \geqslant 0$. Write down the other two inequalities.



Solution:

Linear equation (1):

Gradient, $m = \dfrac{2-1}{1-0} = 1$

$\Rightarrow \quad y - 2 = 1(x - 1)$
$y = x - 1 + 2$
$y = x + 1$

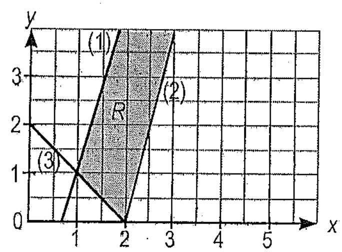
Linear equation (2):

Gradient, $m = \dfrac{0-3}{3-0} = -1$

$\Rightarrow \quad y - 0 = -1(x - 3)$
$y = -x + 3$
$x + y = 3$

$\therefore$ Linear inequalities are $y \leqslant x + 1$ and $x + y \leqslant 3$.

28 The diagram shows a shaded region, R. Write three linear inequalities that satisfies the shaded region.



Solution:

Linear equation (1)

Gradient, $m = \dfrac{3-1}{2-1} = 2$

$\Rightarrow \quad y - 1 = 2(x - 1)$
$y = 2x - 2 + 1$
$y = 2x - 1$

Linear equation (2)

Gradient, $m = \dfrac{4-0}{3-2} = 4$

$\Rightarrow \quad y - 0 = 4(x - 2)$
$y = 4x - 8$

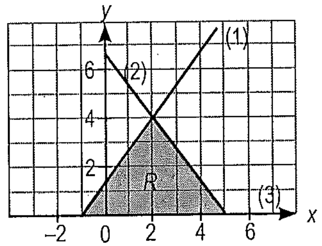
Linear equation (3)

Gradient, $m = \dfrac{0-2}{2-0} = -1$

$\Rightarrow \quad y - 0 = -1(x - 2)$
$y = -x + 2$
$x + y = 2$

$\therefore$ The linear inequalities are ...

29 Write three linear inequalities that satisfy the shaded region, R in the diagram.



Solution:

Linear equation (1):

Gradient, $m = \frac{4-0}{2-(-1)} = \frac{4}{3}$

$\Rightarrow \quad y - 0 = \frac{4}{3}(x - (-1))$

$y = \frac{4}{3}(x + 1)$

$3y = 4x + 4$

Linear equation (2):

Gradient, $m = \frac{4-0}{2-5} = -\frac{4}{3}$

$\Rightarrow \quad y - 0 = -\frac{4}{3}(x - 5)$

$y = -\frac{4}{3}(x - 5)$

$3y = -4x + 20$

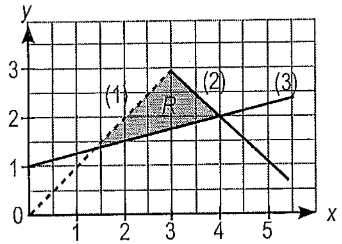
$4x + 3y = 20$

Linear equation (3):

$y = 0$

$\therefore$ Linear inequalities are $3y \leqslant 4x + 4$, $4x + 3y \leqslant 20$ and $y \geqslant 0$.

30 Write down three linear inequalities that satisfy the shaded region in the diagram.



Solution:

Linear equation (1):

Gradient, $m = \frac{3-0}{3-0} = 1$

$\Rightarrow \quad y - 0 = 1(x - 0)$

$y = x$

Linear equation (2)

Gradient, $m = \frac{3-2}{3-4} = -1$

$\Rightarrow \quad y - 3 = -1(x - 3)$

$y = -x + 3 + 3$

$y = -x + 6$

$x + y = 6$

Linear equation (3)

Gradient, $m = \frac{2-1}{4-0} = \frac{1}{4}$

$\Rightarrow \quad y - 1 = \frac{1}{4}(x - 0)$

$4y = x + 4$

$\therefore$ Linear inequalities are $y < x$, $x + y \leqslant 6$ and $4y \geqslant x + 4$.

10.5 Solving Problems Related To Linear Programming

31 Writing linear inequalities and equations describing a situation.

(a) Age of Jaimie (x years) is at least 40 years.

(b) The price for A (RMx) is more than the price of B (RMy) at least RM5.

(c) Maximum value for $2x + 3y$ is 10.

(d) The profit (RMy) gained from item Q is not more than RM20.

(e) The number of male workers (y) is not less than two times the number of female workers (x).

(f) The ratio of the number of male students (y) to the number of female students (x) should not less than 2 : 3.

(g) The number of apples that should be bought (y) must 3 times more than the number of orange (x).

(h) The allowance that should be paid off (RMx) should be less than $\frac{1}{5}$ times the total of expenses (y).

(i) The value of y exceeds the value of x by 5 or less.

(j) The income (y) of Mr Liew is at least RM 30 000 per month.

Solution:

(a) $x \geqslant 40$

(b) $x - y \geqslant 5$

(c) $2x + 3y \leqslant 10$

(d) $y \leqslant 20$

(e) $y \geqslant 2x$

(f) $3y \geqslant 2x$

(g) $y > 3x$

(h) $5x < y$

(i) $y - x \leqslant 5$

(j) $y \geqslant 30\ 000$

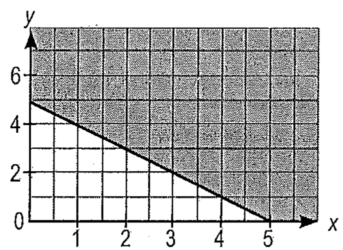
32 The unshaded region in the diagram is defined by three inequalities. Two of these are $x \geqslant 0$ and $y \geqslant 0$.

(a) Write down the third inequality.

(b) On the diagram,

(i) draw the line $y = 2x$,

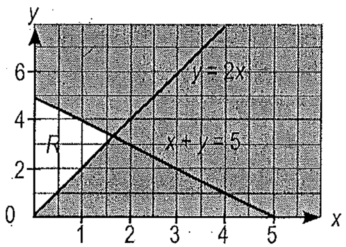
(ii) indicate clearly, with the letter R, that part of the unshaded region for which $y \geqslant 2x$.



Solution:

(a) $x + y \leqslant 5$

(b)



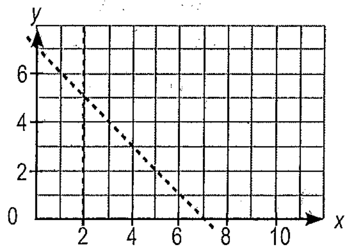
33 It is required to identify the region containing the points (x, y) where $x + y < 7$, $x > 2$, the value is at least half the x value. The diagram shows the lines corresponding to the first two inequalities.

(a) Express the third in equality in symbols.

(b) On the diagram,

(i) draw the line corresponding to the third inequality

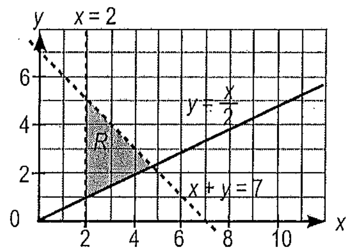
(ii) indicate clearly by shading, the region satisfy all three inequalities.



Solution:

(a) $y \geqslant \frac{x}{2}$

(b)



34 A cafeteria sells two types of coffee flavour, flavour P and flavour Q. In one hour, the number of coffee flavour Q has been sold not more than two times the coffee flavour P. A glass of coffee flavour P is sold at RM2 and a glass of coffee flavour Q is sold at RM3. The total sale of coffee in an hour is not more than RM18. The total coffee flavour P and Q sold in an hour is not more than 8 glass. The cafeteria has sold x glass of coffee flavour P and y glass of coffee flavour Q in an hour.

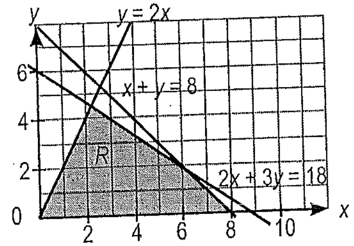
(a) Write down three linear inequalities, apart from $x \geqslant 0$ and $y \geqslant 0$ that satisfy the above constraints.

(b) Draw and shade the region R that satisfies the above constraints.

Solution:

(a) $y \leqslant 2x$,
$2x + 3y \leqslant 18$
$x + y \leqslant 8$

(b)



35 y is the number of apples and x is the number of oranges in a crate. An apple costs RM0.40 and an orange costs RM0.60. The total cost in the crate is not more than RM8 and the number of oranges is at least two times the number of apples.

Write down two linear inequalities, apart from $x \geqslant 0$ and $y \geqslant 0$ that satisfy the above constraints.

Shade the region R that satisfies the above constraints.

Solution:

Linear inequalities

$60x + 40y \leqslant 800 \Rightarrow 3x + 2y \leqslant 40$,

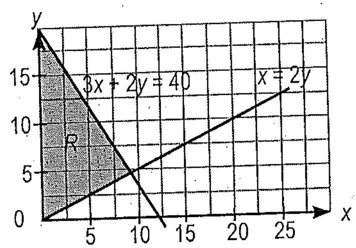
$x \geqslant 2y$

$3x + 2y \leqslant 40$

x	0	2
y	20	17

$x \geqslant 2y$

x	0	10
y	0	5



36 Given x and y are two positive integer with the following conditions:

i $3y + 4x$ is not more than 120.

ii x exceeds y by 20 or less.

iii y is not more than two times of x.

(a) Write down one linear inequality for each of the above situations.

(b) Draw and shade the region R that satisfies the above constraints.

Solution:

(a) $3y + 4x \leqslant 120$,

$x - y \leqslant 20$,

$y \leqslant 2x$

(b) $3y + 4x \leqslant 120$

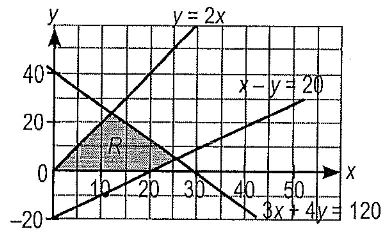
x	0	30
y	40	0

$x - y \leqslant 20$

x	0	20
y	–20	0

$y \leqslant 2x$

x	0	5
y	0	10



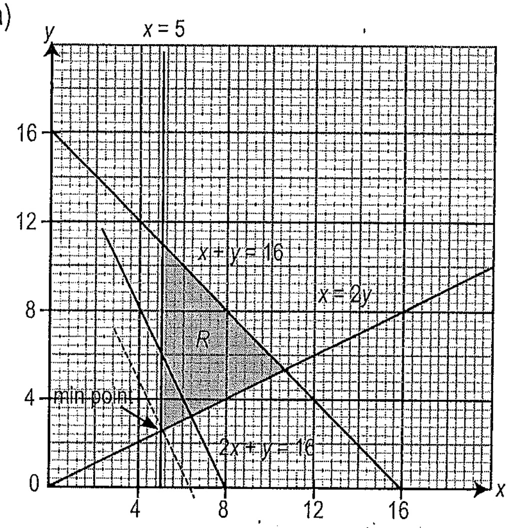
37 Draw straight line $2x + y = 16$, $x = 5$ and $x = 2y$ where x and y are positive integers.

(a) Shade the region R that satisfies the linear equalities $x + y \leqslant 16$, $x \leqslant 2y$ and $x \geqslant 5$.

(b) In the shaded region R, draw the objective function and find the minimum value of $2x + y$.

Solution:

(a)



(b) Let's $2x + y = k$

Draw a straight line $2x + y = 16$.

It is a straight line parallel to $2x + y = 16$ passes the minimum point (5, 3).

$\therefore$ Minimum value for $2x + y$
$= 2(5) + 3 = 13$

38 Pak Dollah buys some lamps and fans from a wholesaler at the unit prices of RM6 and RM4 respectively.

i Pak Dollah has a capital of RM4800

ii The number of lamps has to be at least two times the number of fans bought.

iii Pak Dollah wants to sell the lamps and the fans at the unit prices of RM10 and RM7 respectively and expects the total profits to be not less than RM1200.

Assume that the number of lamps and fans bought and sold by Pak Dollah are x and y respectively.

(a) Write three inequalities, apart from $x \geqslant 0$ and $y \geqslant 0$, which satisfy the above constraints.

(b) Construct and shade the feasible region R that satisfies the above constraints.

(c) Draw the objective function and find the maximum profit he can obtain.

Solution:

(a) $6x + 4y \leqslant 4800$

$3x + 2y \leqslant 2400$

x	0	800
y	1200	0

$x \geqslant 2y$

x	0	600
y	0	300

Profit of each lamp sold = RM(10 – 6) = RM4

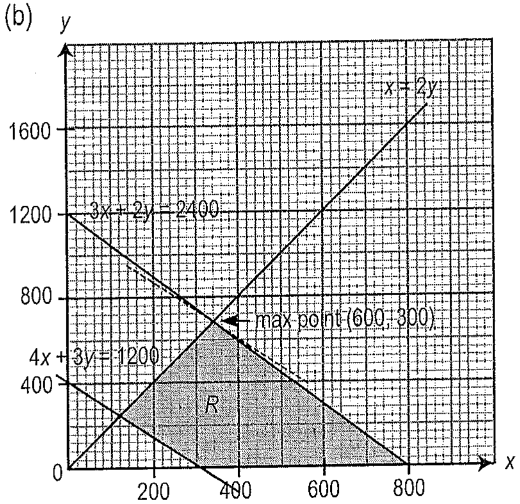
Profit of each fan sold = RM(7 – 4) = RM3

Profit should not be less than RM1200

$\therefore$ $4x + 3y \geqslant 1200$

x	0	300
y	400	0

(b)



(c) Let's $4x + 3y = k$
Draw a straight line
$4x + 3y = 1200$.

x	0	300
y	400	0

$\therefore$ The objective function is $4x + 3y = 1200$.
Using a ruler and a set-square, slide the straight line $4x + 3y = 1200$ in a parallel manner to the furthest point in the feasible region R.
Optimal value = (600, 300)
$\therefore$ Maximum profit = 4(600) + 3(300) = RM3300

39 Given and ay are two positive integers with conditions:
i sums of $x + y$ is 9 or less.
ii minimum value for $5x + 2y$ is 24.
iii the ratio of y is to x is 1: 2 or more.
(a) Write three linear inequalities that satisfy the above conditions.
(b) Draw and shade the region R that satisfies the above conditions.
(c) From the graph, find the greatest value of $4x + 5y$.

Solution:
(a) $x + y \leqslant 9$, $5x + 2y \geqslant 24$, $2y \geqslant x$

(b) $x + y \leqslant 9$

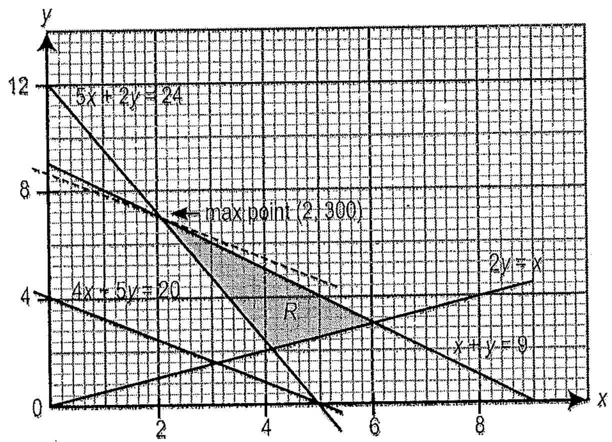
x	0	9
y	9	0

$5x + 2y \geqslant 24$

x	0	48
y	12	0

$2y \geqslant x$

x	0	8
y	0	4

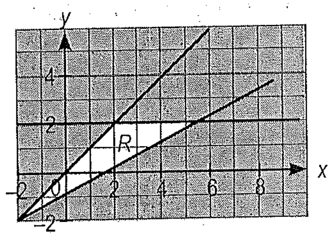


(c) Let's $4x + 5y = k$
Draw a straight line $4x + 5y = 20$.

x	0	5
y	4	0

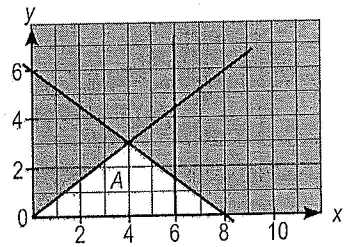
Using a ruler and a set-square, slide the straight line $4x + 5y = 20$ in a parallel manner to the furthest point in the feasible region R.

40 The unshaded region, R in the diagram is defined by three inequalities, one of which is $y > \frac{x}{2} - 1$.
(a) Write down the other two inequalities.
(b) Find the minimum value of $3x + y$, given that x and y satisfy these three inequalities.



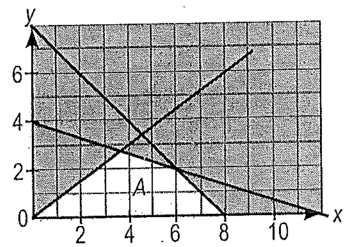
Solution:
(a) $y \leqslant 2$, $y \leqslant x$ (b) −8

41 The region labelled A in the diagram is defined by four inequalities, two of which are $x \leqslant 6$ and $3x + 4y \leqslant 24$.
(a) Write down the other two inequalities.
(b) Find the maximum values of $2x + y$, given that x and y satisfy these four inequalities.



Solution:
(a) $y \geqslant 0$, $4y \leqslant 3x$ (b) 13.5

42 The unshaded region A in the diagram is defined by four inequalities, three of which are $y \geqslant 0$, $x + 3y \leqslant 12$ and $y \leqslant x$.
(a) Write down the other inequality.
(b) Find the greatest value of $5x + 2y$, given that x and y satisfy all four inequalities.

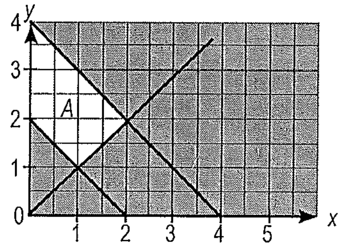


Solution:
(a) $x + y \leqslant 8$ (b) 40

43 The unshaded region R on the diagram is defined by four inequalities. Two of the inequalities are $x + y \leqslant 4$ and $x \geqslant 0$.

(a) Write down the other two inequalities.

(b) Find, for a point (x, y) in the region,
   (i) the maximum value of $3x + y$.
   (ii) the minimum value of $x + 2y$.



Solution:

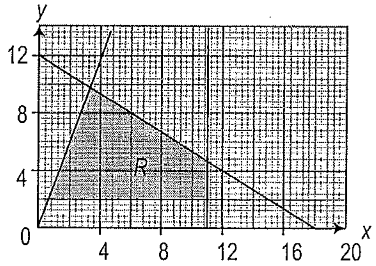
(a) $x + y \geqslant 2$ and $y \geqslant x$ (b) (i) 2 (ii) 3

44 The region R in the diagram shows the set of points (x, y) satisfying four inequalities. Two of the inequalities are $x \leqslant 11$ and $2x + 3y \leqslant 36$. Write down the other two inequalities.

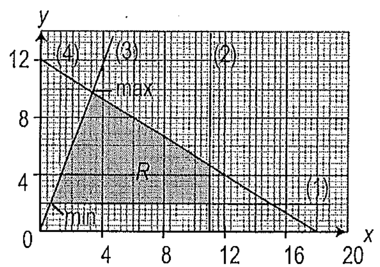
The owner of a toy shop decides to buy x Kiddicars and y Tribikes. The values of x and y are restricted by the four inequalities. He makes RM10 profit on a Kiddicar and RM6 profit on a Tribike. The shopkeeper sells all the toys he buys. Find

(i) the value of x and value of y to give the maximum profit,

(ii) the minimum profit.



Solution:



Linear equation (1), $y = 2$

Linear equation (2), $x = 11$

Linear equation (3),

Gradient, $m = \dfrac{5 - 1}{2 - 0} = 2$

$y - 1 = 2(x - 0)$

$\therefore\ y = 2x + 1$

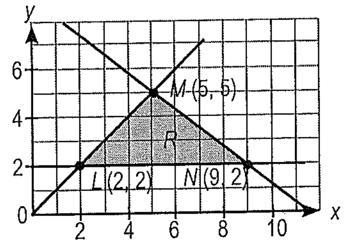
Linear inequality = $y \geqslant 2$, $y \leqslant 2x + 1$

(i) From the graph,
$x = 4.125$, $y = 9.25$

(ii) Minimum profit = $(10)(0.5) + (6)(2) = 17$

SPM Exam Paper

1 The triangle LMN is shown on the diagram. O is the origin, L is the point (2, 2), M is the point (5, 5) and N is the point (9, 2). The shaded region R is defined by three inequalities, one of which is $3x + 4y \leqslant 35$.

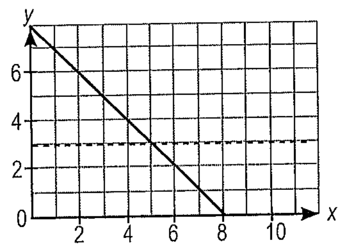


(a) Write down the other two inequalities.

(b) Given that the point (x, y) is in the region R, calculate the maximum value of $3x - y$.

2 To identify the region containing the points (x, y) it is necessary to specify three inequalities. Two of these are $y \geqslant 3$ and $x + y \leqslant 8$. In the third inequality the y value is less than twice the x value.

The diagram in the answer space shows the lines corresponding of the first two inequalities.



(a) Express the third inequality in terms of x and y.

(b) On the diagram in the answer space,
   (i) draw the line corresponding to the third inequality.
   (ii) indicate clearly, by shading , the region satisfying all three inequalities,
   (iii) given that x and y are integers, mark all the points whose coordinates satisfy the three inequalities.

3 A factory produced two types of badminton racquets, model I and model II. The factory produces x unit models I and y unit models II daily. It uses a new machine and an old machine to produce these racquets. An old machine takes 50 minutes and 20 minutes to produce a racquet model I and a racquet model II respectively. A new machine takes 20 minutes and 40 minutes to produce a racquet model I and a racquet model II respectively. The conditions to produce racquets in the factory per day are as follow:

I The total time taken for an old machine must not more than 600 minutes.
II The total time taken for a new machine must at least 400 minutes.
III The number of model II is not more than 6 times the number of model I.

(a) Write linear inequalities that satisfy the above constraints.
(b) Using a suitable scale, draw and shade the region *R* that satisfies the above constraints.
(c) (i) If profit of a racquet model I and a racquet model II is RM20 and RM30 respectively, calculate the maximum profit per day.
(ii) One day, the factory decided to produce an equal number of racquets model I and racquets model II. Find the maximum number of racquets produced on that day.

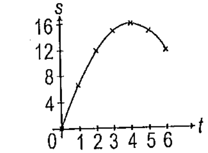
Item	Processing Time	Preparation Time
P	2 hours	4 hours
Q	3 hours	2 hours

4 The table shows the production of item *P* and Q in a factory. The processing time and the preparation time taken to produce a unit of *P* and a unit of Q are shown as in the table. The total processing time taken in a week is 46 hours while the total preparation time taken in a week is 60 hours. The ratio of the number of *P* produced to the number of Q produced should not be more than 2 : 3. The factory produces *x* unit of *P* and y unit of Q per week.

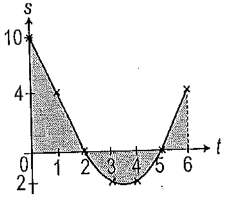
(a) Write three linear inequalities, apart from $x \geqslant 0$ and $y \geqslant 0$, which satisfy the above constraints.
(b) Using a suitable scale, draw and shade the region *R* that satisfies the above constraints.
(c) From your graph, find
(i) the maximum number of Q if 5 unit of P had produced in that week.
(ii) the maximum profit gained in that week if *P* and Q gained a profit of RM3 and RM4 per unit respectively.

ANSWERS

9 (a) $0 \leqslant t < 4$ (b) 16 m
(c) Total distance travelled = 20 m



10 (a) $t_1 = 2, t_2 = 5$
(b) $t_1 = 8\frac{2}{3}, t_2 = 4\frac{1}{6}$
(c)

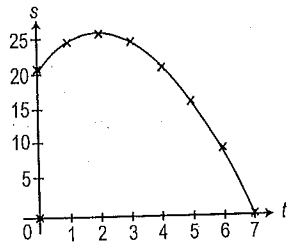


11 (a) -10 m s^{-1} (b) 0 m s^{-2}
(c) $0 \leqslant t < \frac{7}{6}, t > 4$

12 (a) $9 \text{ m s}^{-1}, -12 \text{ m s}^{-2}$
(b) $t = 1, t = 3$
(c) -3 m s^{-1}

13 (a) -10 m s^{-2} (b) 25 m s^{-1}
(c) (i)

t	0	1	2	3	4	5	6	7
v	21	24	25	24	21	16	9	0



(ii) Total distance = 72 m

14 (a) $a = -3, b = 24, c = 60$
(b) -36 cm s^{-1}

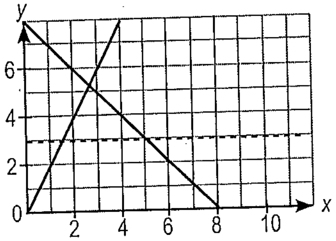
15 (a) $a = -2, b = \frac{16}{3}$
(b) $13\frac{7}{9}$ m

16 (a) 4 s (b) -50 m s^{-1}
(c) 18.96 s

Chapter 10

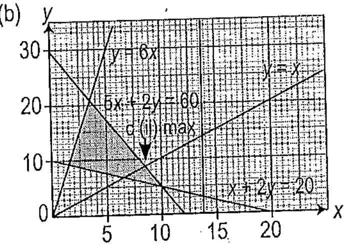
1 (a) $y \leqslant x, y \geqslant 2$ (b) 25

2



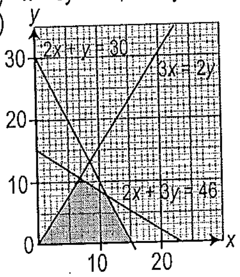
$y \leqslant 2$

3 (a) $5x + 2y \leqslant 60,\ y \leqslant 6x$
(b)



(c) (i) RM60 (ii) 16 racquet

4 (a) $x + 3y \leqslant 46, 2x + y \leqslant 30, 3x \geqslant 2y$
(b)



(c) (i) 7 (ii) RM65